

APPENDIX B.

Electron Acceleration for the eRHIC with the Non-Scaling Fixed Field Alternating Synchrotrons

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B.1 Introduction

The scaling Fixed Field Alternating Gradient (FFAG) synchrotron has been introduced in 1954 by K. R. Symon [1] and few example electron rings were built at that time. During the last decade, the concept has been revived due to requests for high intensity machines, and has been applied in building the proton and electron FFAG, especially in Japan. The proof of principle proton FFAG was built and commissioned in June 2000 at the KEK, Japan [2]. Presently a 150 MeV proton FFAG synchrotron is being commissioned at the KEK [3].

A principle of the scaling FFAG

The magnetic field in regular synchrotrons varies during acceleration in accordance to the beam momentum such that the average radius and therefore the path length around the machine are constant. A concept of the scaling FFAG lattice is presented in fig. 1.

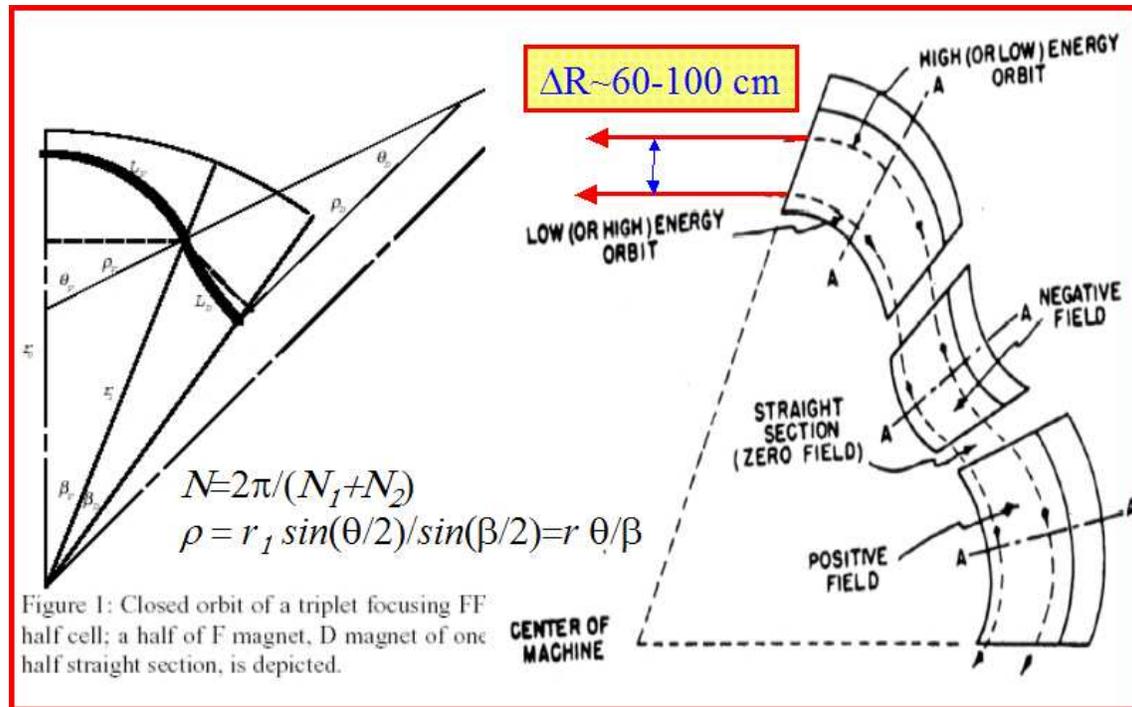


Figure 1. Concept of the scaling FFAG lattice by K. R. Symon.

- The transverse magnetic field in the scaling FFAG is non-linear and fixed:

$$B = B_o \left(\frac{r}{r_o} \right)^k$$

- The radius of particles as they are accelerated, is increasing with momentum: the revolution frequencies vary with momentum, unlike in the cyclotrons where the frequency is constant.
- The scaling FFAG has strong transverse focusing with alternating gradients. The stability of the betatron oscillations is obtained by optimization of betatron tunes. The tunes are constant which makes the chromaticity throughout acceleration equal to zero.
- The momentum acceptance of the FFAG is as large as $\delta p/p \sim \pm 50\%$. The largest momentum offsets in the regular synchrotrons are of the order of few percent. This depends on dispersion function (as the transverse offset is $\Delta x = D_x \delta p/p$, where D_x is the horizontal dispersion function).
- The transition energy is outside of region of energies during acceleration.
- Relatively large apertures are required for the scaling FFAG due to the large radius excursions. The original MARK-I design for the output energy of 10 MeV, required a radius of almost $\Delta x \sim 1$ meter. The present FFAG 10 MeV synchrotron at KEK also has the large variation of radius from $r_o = 4.4$ m to $r_m = 5.3$ m, $\Delta x \sim 0.9$ meter, it requires large aperture magnets.
- A relatively large circumference is required due to large opposite bends.

The non-scaling FFAG

We had introduced the idea of non-scaling FFAG at the Montauk Muon Collider meeting 1999 [4]. The collaboration accepted this concept and a design for the higher energy muon acceleration. We followed the basic idea of the scaling FFAG with a combination of the previously known “minimum emittance lattice” concept for the electron storage rings. Several disadvantages of the original FFAG design: like large aperture requirements, are reduced, the required aperture in the non-scaling FFAG is at least one order of magnitude smaller. The transverse magnetic field in the alternating gradient magnets is linear and particle with different momentum follow orbits oscillating around the ring. The average radius corresponds to the value of the momentum.

The basic lattice cell consists of the **two combined function elements**: the large bending magnet with a defocusing gradient, and a smaller opposite bend with focusing gradient. There is a drift space large enough for the accelerating cavity between two focusing magnets.

The principle of the non-scaling FFAG is based on the basic relation between the radial offset and dispersion function: $\Delta x = D_x \delta p/p$. For example, if the dispersion function were smaller than 10 cm ($D_x < 10$ cm) during acceleration, the orbit offsets for the very large momentum offsets ($\delta p/p = \pm 50\%$) would not be larger than ± 5 cm. The non-scaling design does not have zero chromaticity during acceleration and the betatron tunes change within a range $\nu_{x,y} \sim 0.4 - 0.1$.

B.2 Application of the FFAG Lattice for eRHIC Electron Acceleration

Courant-Snyder functions and magnets in the FFAG non-scaling lattice cell for acceleration from energy of $E_o=3.2$ GeV up to $E_{max}=10$ GeV (or in the momentum range of from $\Delta p/p = -52\%$ to $\Delta p/p = +50\%$): The basic cell with the betatron functions at the central energy of $E_o = 6.7$ GeV and magnets is presented in figure 2.

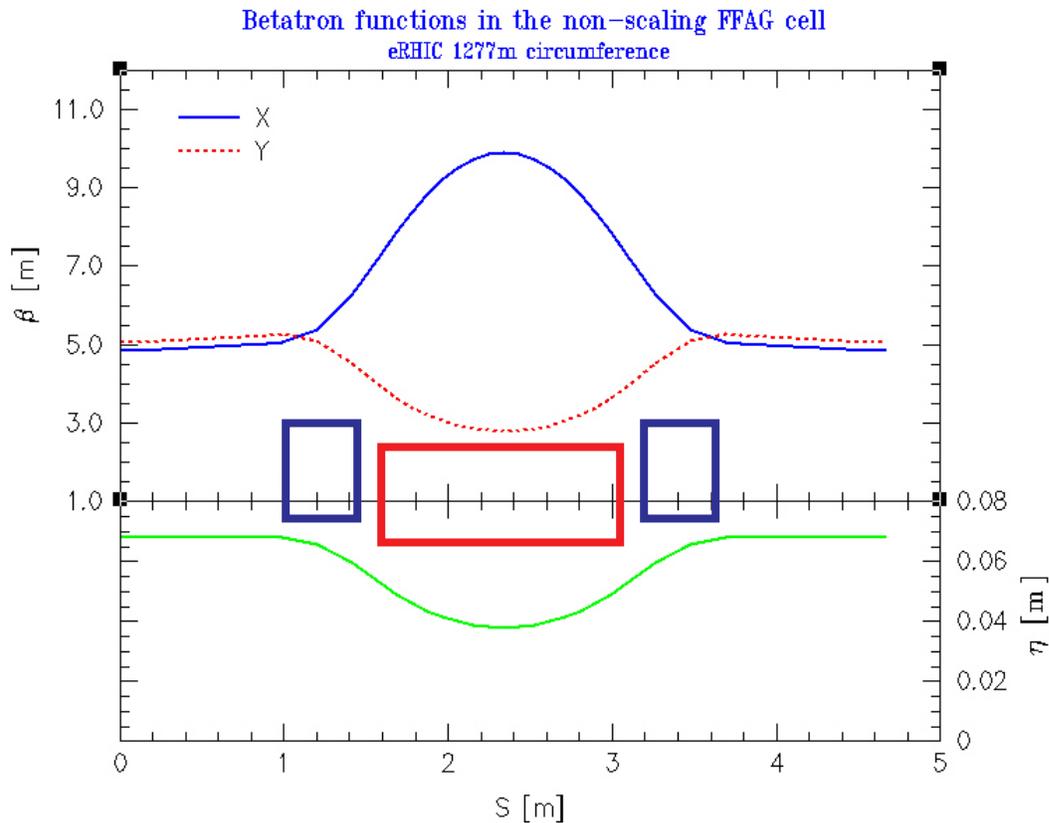


Figure 2. Basic FFAG cell with betatron functions at the central energy of $E_o = 6.7$ GeV.

The central 1.5 meter long magnet has a bending angle of $\theta=45.56$ mrad, and it is a combined function magnet with a defocusing gradient of $G_d=-11.7$ T/m. The opposite bend is 0.42 meter long with the bending angle of $\theta=-11.27$ mrad, with a focusing gradient of $G_f= 23$ T/m. The cell length is 4.678 m. The available drift length for the RF cavity is 2.335 m. There are 273 cells. The required aperture in a transverse direction depends on the orbit offsets during acceleration. The synchrotron radiation from electron depends on the value of the bending angle ($\rho = l/\theta$). The bending angle of the major bend in this example is $\theta=0.045$ rad. The bending radius is estimated as $\rho=l/\theta\sim 33$ m.

Orbits During Acceleration: At the start of acceleration, the energy is equal to $E_o = 3.22$ GeV, corresponding momentum offset from the central energy of 6.7 GeV is $\delta p/p = -52\%$. The maximum orbit offset at the beginning of acceleration, as presented in figure 2, is equal to $\Delta x = -15.5$ mm. It is passing parallel to the central circular orbit through a drift space assigned to the cavity. As

acceleration proceeds, the orbits move towards the central path. When the momentum offset becomes positive orbits move outside of the central path reaching the maximum value of $\Delta x = +49.27$ mm. The aperture of the beam pipe should be wider than ~ 70 mm. The magnetic field strength of the major bending magnet is $B_p = 0.679$ T while the opposite bending field, of the 42 cm long combined function magnet, is $B_{\text{neg}} = 0.6$ T.

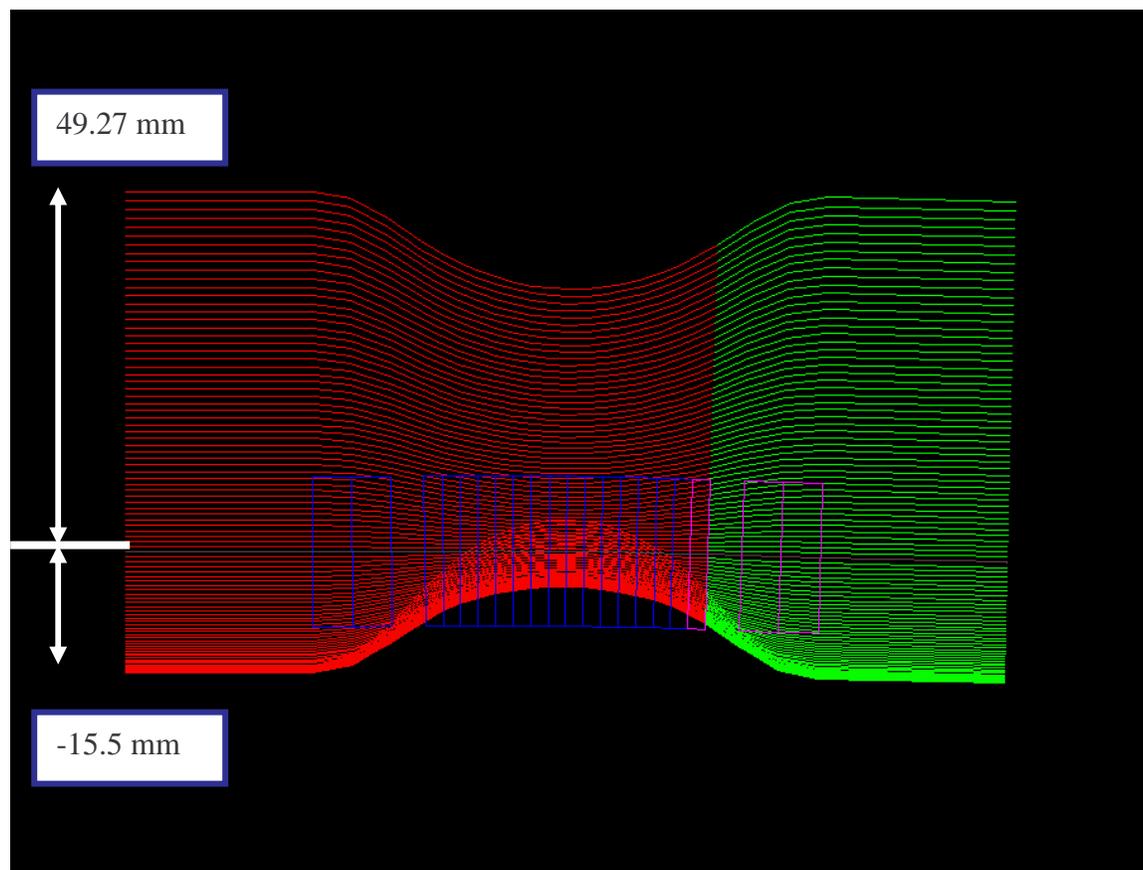


Figure 3. Particle orbits during acceleration. The magnets are presented by the “blue” boxes.

Betatron tunes during acceleration: The betatron tunes in the non-scaling FFAG vary with energy, to the contrary of the scaling FFAG. It is important to avoid half and full integer tunes within the single cell because accelerating particles might be lost due to repetition of the same cells along the circumference path. The vertical and horizontal tune dependence on energy or momentum is presented in figure 4.

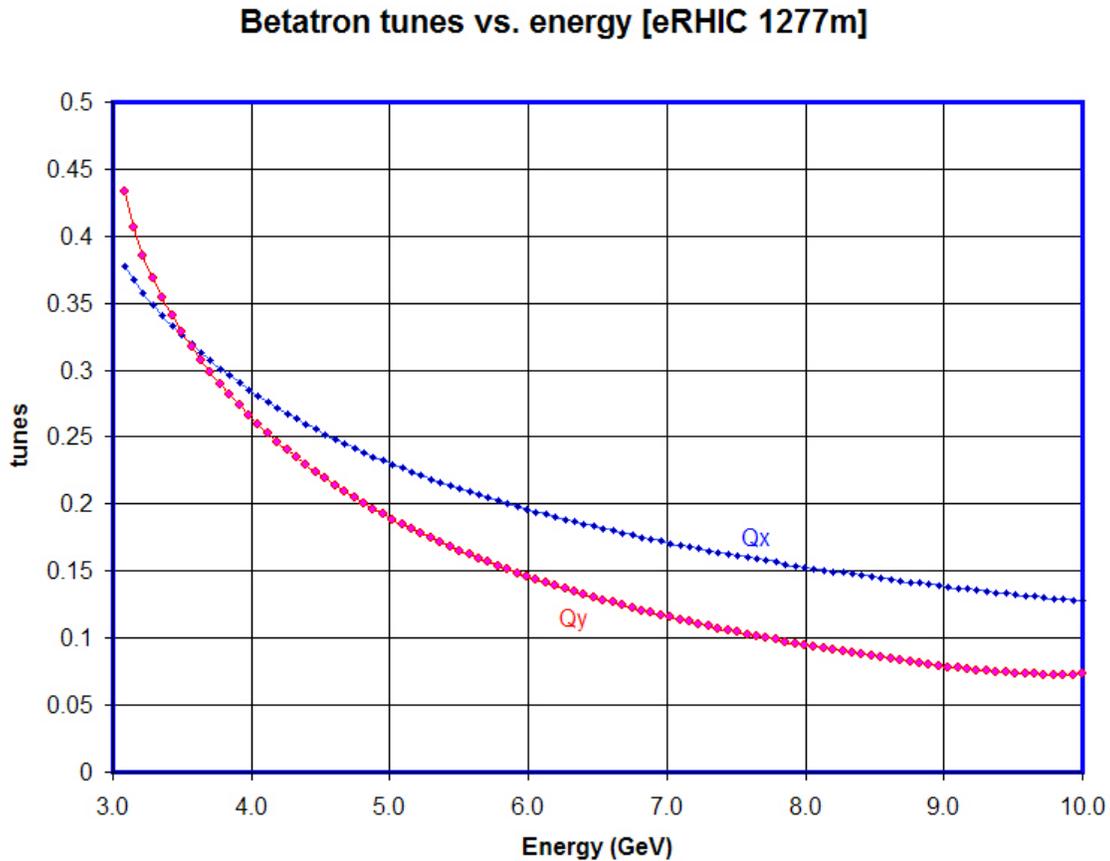


Figure 4. Betatron tunes dependence on energy.

The Path Length and momentum compaction dependence on energy

The path length dependence on energy (fig.5) is very close to a parabolic function. At the lowest energy, α , the momentum-compaction is negative (the transition energy γ_t is an imaginary number). During acceleration from $\delta p/p = -52\%$ up to a value of momentum close to the central value, the absolute value of the transition energy gradually increases. The momentum compaction is also a parabolic function and it crosses the zero value at the same position. The momentum compaction dependence on energy is shown in figure 6.

1277 Path Length

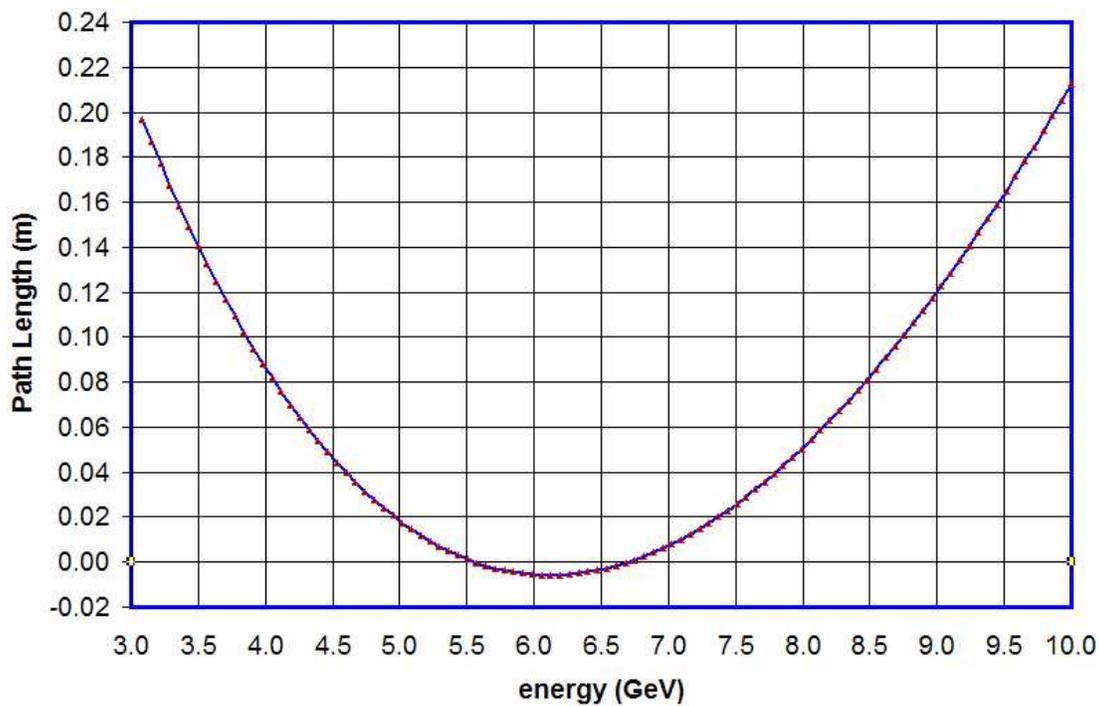


Figure 5. The path length dependence on energy

Momentum Compaction vs. Energy - eRHIC 1277m

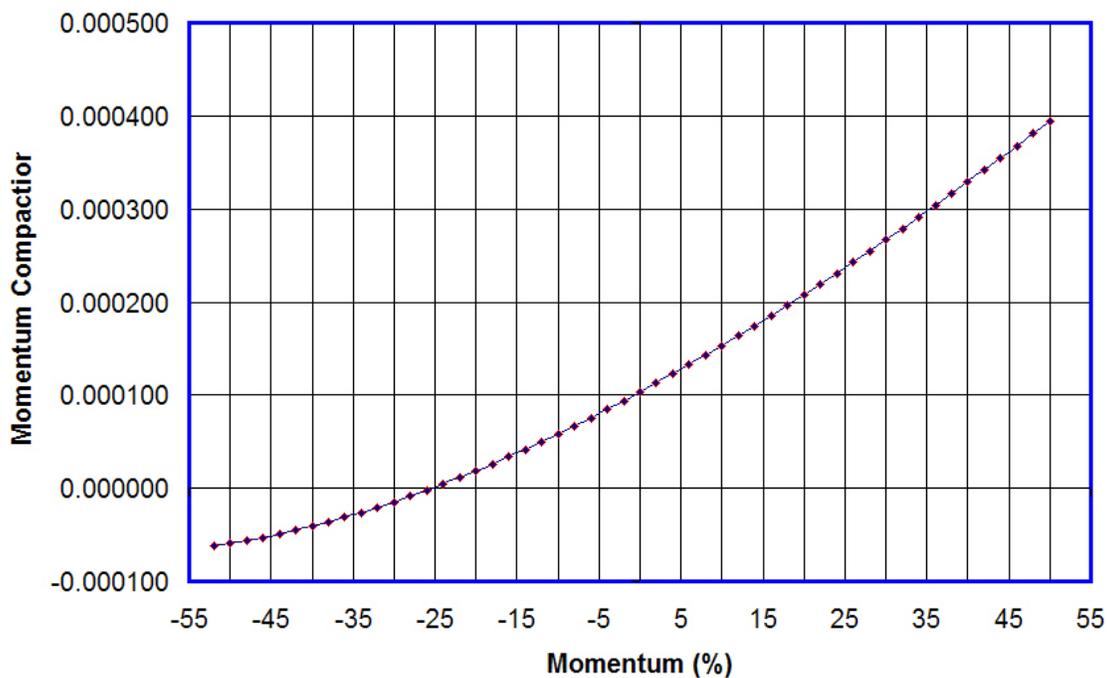


Figure 6. Energy dependence of the momentum compaction factor.

Acceleration

The major problem of applying the FFAG concept for the high electron energy acceleration *is the loss of energy due to synchrotron radiation*. Energy loss for a single electron per turn in the circular orbit is defined as:

$$\frac{\Delta E}{m_o c^2} = \frac{4\pi}{3} \left(\frac{r_o}{\rho_d} \right) \beta^3 \gamma^4, \quad r_o \text{ is the classical electron radius.}$$

The average radius of the 1/3 of the RHIC circumference is of the order of 200 m, but within the FFAG magnets electrons would be bend much harder even in the opposite way and the radius of curvature is much smaller. The energy loss of electron at the last turn, reaching the highest energy of 10 GeV, is calculated for this example to be 12.891 MeV. This energy needs to be compensated by the RF system. A detail calculation of the synchrotron radiation energy loss is presented in the appendix.

It is assumed that the cavity voltage in the non-scaling FFAG should be at least twice higher at the last turn (energy reaches the largest value equal to 10 GeV). This implies that the total required RF voltage is ~24 MV. If cavities to be used are similar to the RHIC storage cavities, were each reaches a voltage of 2 MV, then twelve cavities are necessary to fulfill the acceleration requirement. This also shows that from 3.2 up to 10 GeV there are required ~560 turns.

The parabolic dependence of the path length difference on momentum requires adjustment of the RF voltage in time. The 20cm path length difference corresponds to a fractional frequency difference of $1.5 \cdot 10^{-4}$ or a quality factor of $Q=6000$ for no cavity tuning. This is a fairly small Q so we will need to tune the cavity. The synchrotron tune with 20MV/turn with a 700MHz cavity frequencies shown in Fig. 7. The large tunes imply adiabaticity except near transition. Figure 8 shows the initial and final longitudinal phase space distributions for an initial (full) emittance of $1 \cdot 10^{-3}$ eV-s per bunch.

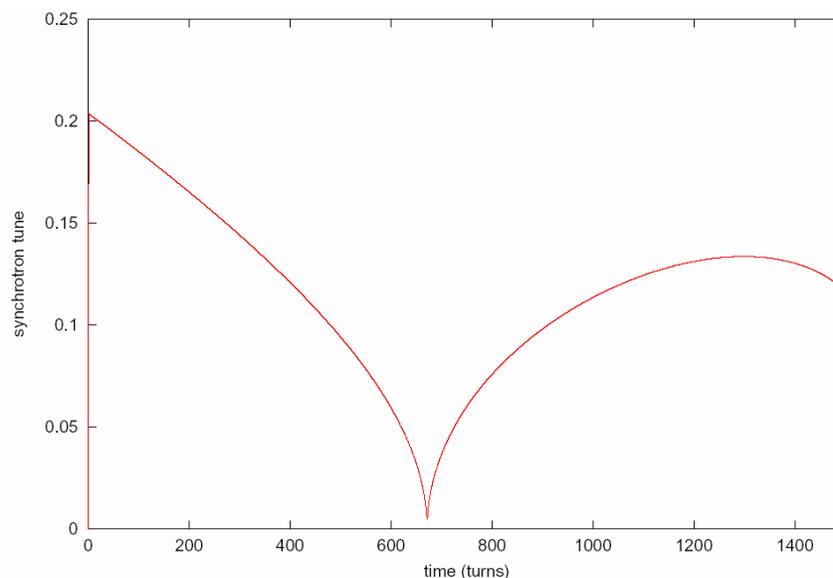


Figure 7. Synchrotron tune versus time for 3-10 GeV acceleration with 20MV/turn and 700MHz cavity frequency

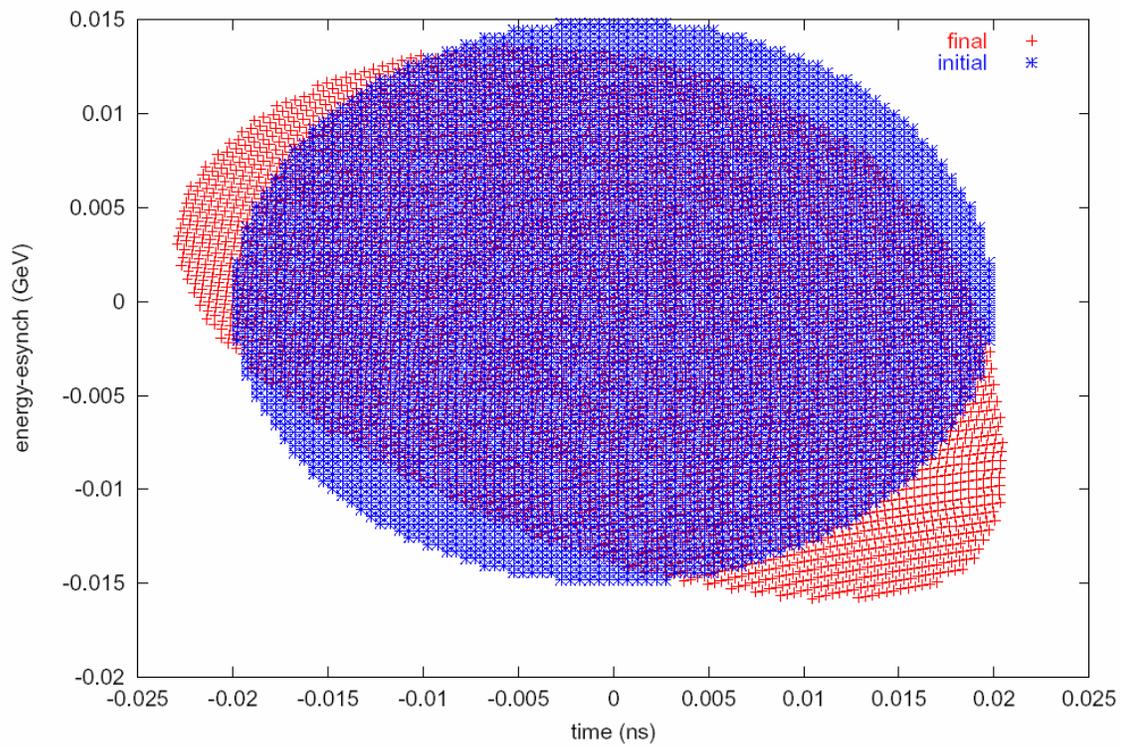


Figure 8. Initial (blue) and final (red) longitudinal phase space for 3-10 GeV acceleration with 20MV/turn and 700MHz cavity frequency.

Energy loss per turn

The energy loss of an electron in this non – scaling FFAG ring during the last turn at the 10 GeV energy is calculated by separating three part :

The classical electron radius is : $r_o = 2.817940285 \cdot 10^{-15} \text{ m}$,

$$C_1 = m_o c^2 \frac{4\pi}{3} r_o \beta^3 \gamma^4 = 510998.902 \text{ eV} \cdot 4.18879 r_o \cdot 1.4666310^{17} = 8.8463 \cdot 10^8 \text{ eVm}$$

A total length in one turn of the major bending magnets : $L_{QD} = 273 \cdot 1.5 \text{ m}$

$$L_{QD} = 409.5 \text{ m. A relation to the total circumference : } k_1 = \frac{L_{qd}}{C_o} = 0.321$$

$$\text{The bending radius : } \rho_d = \frac{l_d}{\theta} = \frac{1.5 \text{ m}}{0.045566883} = 32.92 \text{ m}$$

$$\text{Energy loss in one turn due to the major bends : } \Delta E_{QD} = \frac{C_1}{\rho_d} k_1 = 8.626 \text{ MeV}$$

A length in one turn of the opposite bends : $L_{QF} = 273 \cdot 2 \cdot 0.42 \text{ m} = 229.32 \text{ m}$.

$$\text{A relation to the total circumference : } k_2 = \frac{L_{QF}}{C_o} = 0.1796$$

$$\text{The opposite bend radius : } \rho_d = \frac{l_d}{\theta} = \frac{0.42 \text{ m}}{0.011275776} = 37.25 \text{ m.}$$

The synchrotron radiation energy loss from the opposite bends in one turn :

$$\Delta E_{QF} = 4.265 \text{ MeV}$$

The total energy loss per turn is equal to $E_{\text{tot}} = 8.626 + 4.265 = 12.891 \text{ MeV}$.

References:

- [1] K.R. Symon, “The FFAG Synchrotron – MARK I”, MURA-KRS-6, pp. 1-19, November 12, 1954.
- [2] R. Ueno et al, “Multi Orbit Synchrotron with FFAG Focusing For Acceleration of High Intensities Hadron Beams”, PAC 99, New York, 1999, 2271-2273
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