RHIC-PG-40

THE EFFECT OF LANDAU DAMPING ON THE LONGITUDINAL PHASE SPACE INSTABILITY ACROSS THE TRANSITION ENERGY

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As pointed out by J.M. Wang (RHIC-PG-32) that the microwave ~~in~~coherent instability may play an important role in the design of RHIC machine across the transition energy region. In this note we shall examine the growth rate in the presence of the beam momentum and energy spread. We shall proceed in three sections:

(I) The equation of longitudinal motion across the transition region and derive an invariant of the longitudinal phase space

(II) Discuss the dispersion integral of the instability.

(III) Conclusion and Recommendation.

I Equation of Synchrotron motion near transition energy.

$$\theta = \varphi - \varphi_s$$

$$(h\,\Delta\omega) \Rightarrow \quad \dot{\theta} \equiv \frac{h\eta\omega_0^2}{E\beta^2} W \qquad W = \frac{\Delta E}{\omega_0}$$

$$\dot{W} = \frac{eV}{2\pi}(\sin\varphi - \sin\varphi_s) \simeq \left(\frac{eV}{2\pi}\cos\varphi_s\right)\theta$$

$$H = \frac{h\eta\omega_0}{2pR} W^2 + \frac{eV}{2\pi}\left[\cos\varphi - \cos\varphi_s + (\varphi-\varphi_s)\sin\varphi_s\right]$$

$$\frac{d}{dt}\left(\frac{1}{\Omega_s^2(t)}\dot{\theta}\right) + \theta = 0$$

$$\omega_\infty = \frac{c}{R} = \frac{\omega_0}{\beta}$$

$$\Omega_s^2(t) = \frac{\omega_\infty^2 h}{E(t)} \frac{eV}{2\pi} |\eta(t)\cos\varphi_s|$$

$$= \frac{\omega_\infty^2 h}{E_0} \frac{eV}{2\pi} |\cos\varphi_s| \left|\frac{2\dot{\gamma}}{\gamma_t^4} t\right| \equiv \frac{t}{T^3}: \qquad |t| \ll \tau = \frac{\gamma_t}{\dot{\gamma}}$$

$$\eta(t) = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \sim + \frac{2\dot{\gamma}}{\gamma_t^3} t$$

Assumptions

(1) Acceleration is an adiabatic process

$\gamma(t)$; $\eta(t)$ is a smooth function of t

$$\gamma(t) = \gamma_t + \dot{\gamma} t$$

$$T^{-3} = 2h \frac{\omega_\infty^2 eV}{2\pi E_0} \frac{\dot{\gamma}}{\gamma_t^4} |\cos\varphi_s|$$

$$\frac{d}{dt}\left(\frac{1}{\Omega_s^2(t)}\dot{\theta}\right) + \theta = 0$$

$$t > 0 \qquad y = \int_0^t dt'\,\Omega_s(t') = \frac{2}{3}\left(\frac{t}{T}\right)^{3/2}$$

$$\theta = y^{2/3}\,\Phi \qquad\qquad x \equiv t/T$$

$$\frac{d^2\Phi}{dy^2} + \frac{1}{y}\frac{d\Phi}{dy} + \left[1 - \frac{\left(\frac{2}{3}\right)^2}{y^2}\right]\Phi = 0$$

$$\theta = a\,x\left[\cos\psi\, J_{2/3}(y) + \sin\psi\, N_{2/3}(y)\right]$$

$$\boxed{\frac{\theta}{xa} = \cos\psi\, J_{2/3}(y) + \sin\psi\, N_{2/3}(y)}$$

$$\frac{h\eta\omega_0^2}{E\beta^2}W = \dot{\theta} = \frac{\theta}{Tx} + \frac{x^{3/2}a}{T}\left[\cos\psi\left(\frac{2}{3y}J_{2/3} - J_{5/3}\right) + \sin\psi\left(\frac{2}{3y}N_{2/3} - N_{5/3}\right)\right]$$

$$\frac{1}{ax^{3/2}}\left(\frac{2\theta}{x} - \frac{hT\eta\omega_0^2}{E\beta^2}W\right) = \cos\psi\, J_{5/3}(y) + \sin\psi\, J_{5/3}(y)$$

$$\begin{pmatrix}\cos\psi\\ \sin\psi\end{pmatrix} = M^{-1}\begin{pmatrix}\theta/xa\\ \left[\frac{2\theta}{x} - \frac{hT\eta\omega_0^2}{E\beta^2}W\right]/x^{3/2}a\end{pmatrix}$$

$$M \equiv \begin{pmatrix} J_{2/3} & N_{2/3}\\ J_{5/3} & N_{5/3}\end{pmatrix}$$

$$\cos^2\psi + \sin^2\psi = 1 \equiv \alpha_{\theta\theta}\theta^2 + 2\alpha_{\theta W}\theta W + \alpha_{WW}W^2 = \underline{\text{invariant}}$$

$$\alpha_{\theta\theta} = \left[\left(\tfrac{3}{2} y N_{5/3} - 2N_{2/3}\right)^2 + \left(2J_{2/3} - 1.5\, y J_{5/3}\right)^2\right] \Big/ a^2 (\det)^2 x^5$$

$$\alpha_{\theta W} = \frac{h T \eta \omega_0^2}{a^2 (\det)^2 x^4 E \beta^2} \left[N_{2/3}\left(\tfrac{3}{2} y N_{5/3} - 2N_{2/3}\right) - J_{2/3}\left[2J_{2/3} - \tfrac{3}{2} y J_{5/3}\right]\right]$$

$$\alpha_{WW} = \frac{h^2 T^2 \eta^2 \omega_0^4}{a^2 (\det)^2 x^3 E^2 \beta^2} \left[J_{2/3}^2 + N_{2/3}^2 \right]$$

Asymptotically:

$$\alpha_{\theta\theta} = 1/\hat{\theta}^2 \qquad \hat{\theta} = \left(\frac{2A h \omega_0^2 \dot{\gamma} T^2}{\pi E_0 \beta^2 \gamma_t^4}\right)^{1/2} \left(\frac{t}{T}\right)^{1/4}$$

$$\alpha_{WW} = 1/\hat{W}^2 \qquad \hat{W} = \frac{1}{h}\left(\frac{A E_0 \beta^2 \gamma_t^4}{2\pi h^2 \omega_0^2 \dot{\gamma} T^2}\right)^{1/2} \left(\frac{T}{t}\right)^{1/4}$$

$$a^2 = \frac{2A h^2 \omega_0^2 \dot{\gamma} T^2}{3 E_0 \beta^2 \gamma_t^4}$$

$$[A] \equiv \text{eV sec.}$$

Gaussian Bunch. $\xi_0 \cong \sqrt{3}$

$$\rho(\theta, W) = N \frac{\xi_0^2 \left[\alpha_{\theta\theta}\alpha_{WW} - \alpha_{\theta W}^2\right]^{1/2}}{\pi} e^{-\xi_0^2\left(\alpha_{\theta\theta}\theta^2 + 2\alpha_{\theta W}\theta W + \alpha_{WW} W^2\right)}$$

$$= N\, G(\theta)\, g_\theta(W)$$

$$g_\theta(W) = \frac{\xi_0 \sqrt{\alpha_{WW}}}{\sqrt{\pi}} e^{-\xi_0^2 \alpha_{WW}\left(W + \frac{\alpha_{\theta W}}{\alpha_{WW}}\theta\right)^2}$$

$$G(\theta) = \frac{\xi_0}{\sqrt{\pi}} \left[\frac{\alpha_{\theta\theta}\alpha_{WW} - \alpha_{\theta W}^2}{\alpha_{WW}}\right]^{1/2} e^{-\xi_0^2\left[\frac{\alpha_{WW}\alpha_{\theta\theta} - \alpha_{\theta W}^2}{\alpha_{WW}}\right]\theta^2}$$

II Coherent Instability.

a Coasting beam.

Small perturbation in current $\quad I = I_0 + I_1 e^{in[(\theta - \Delta\Omega t) - \omega_0 t]}$

$$\longrightarrow \quad \omega = \omega_0 + \omega_1 e^{in[(\theta - \Delta\Omega t) - \omega_0 t]}$$

$\Rightarrow$ Voltage change$/_{\text{turn}}$ $= - I_1 Z\, e^{in[(\theta - \Delta\Omega t) - \omega_0 t]}$

$$\frac{\partial E}{\partial t} = - \frac{\partial E}{\partial \theta}\omega + e U_s \frac{\omega}{2\pi}$$

$$\Rightarrow \quad \frac{\partial \omega}{\partial t} = - \frac{\partial \omega}{\partial \theta}\omega - \frac{\partial \omega}{\partial E}\frac{\partial E}{\partial t} = - \frac{\partial \omega}{\partial \theta}\omega_0 + \frac{e\eta\omega_0^2 Z}{\beta^2 E \cdot 2\pi} I_1 e^{i(n\theta - n\Omega t)}$$

$$\omega_1 \Delta\Omega = i e \eta \omega_0^2 I_1 \left(\frac{Z}{n}\right) \Big/ 2\pi\beta^2 E$$

Equation of continuity: $\quad \dfrac{\partial I}{\partial t} = - \dfrac{\partial I}{\partial \theta}\omega_0 - \dfrac{\partial \omega}{\partial \theta} I_0$

$$I_1 \Delta\Omega = \omega_1 I_0$$

$$\Delta\Omega^2 = i \, \frac{e\eta\omega_0^2 I_0}{2\pi\beta^2 E}\left(\frac{Z}{n}\right)$$

$$e^{-i n \Delta\Omega t} \quad : \quad \begin{cases} \Delta\Omega_i > 0 & \text{unstable} \\ \Delta\Omega_i < 0 & \text{stable.} \end{cases}$$

$$\text{Growth} = e^{\int n \Delta\Omega_i \, dt}$$

with energy spread

$$1 = - \Delta\Omega^2 \int \frac{g'(\omega)}{\delta\Omega - \omega} d\omega$$

b Bunched Beam with energy spread [Wang - Pellegrini]

$$1 = -(\Delta\Omega)^2 \cdot \int \frac{\left(\frac{dg}{d\omega}\right)}{(\delta\Omega - \omega)} \cdot d\omega$$

normal modes $\longleftrightarrow \left(\begin{matrix} \text{TDA} \\ \text{RPA} \end{matrix} \right.$ equations)

(phonon)

$$\int g \, d\omega = 1$$

$$\Delta\Omega^2 = i \, \frac{e \eta \omega_0^2 I_{peak}}{2\pi \beta^2 E} \left(\frac{Z}{n}\right) \qquad \propto \left[\frac{\left(\frac{N}{B}\right) \cdot e^2}{A} \right]$$

$$= i \, \frac{e^2 \eta \omega_0^3 N}{(2\pi)^2 \beta^2 E} G(\theta) \left(\frac{Z}{n}\right)$$

see p. 4

$$\zeta \equiv \frac{Z}{n} = \zeta_r + i\,\zeta_i$$

$$\langle n \rangle = \frac{R}{b} \simeq 2.0 \times 10^4 \quad \text{(RHIC)}$$

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	p,	^{2}H	C	S	Cu	I	Au
N/B [10^9]	10^3	10^2	22	6.4	4.5	2.6	1.1
e	1	1	6	16	29	53	79
A	1	2	12	32	63	127	197
		50	66	51.2	60.1	57.5	34.8
					3.8		3.4

$\frac{Z}{n} = 2 + i$ [illegible]

Growth

III Conclusions and Recommendation

From the following table and figure, the following conclusions can be drawn:

(1) less instability for higher harmonic number h

(2) The phase space area should be at least 0.4 evsec/amu before the transition.

(3) Calalation indicates that there is an interesting correlation between the total growth

$$G = e^{\int \delta\Omega_i \, dt} \qquad (\delta\Omega_i > 0)$$

and the parameters $\left|\frac{Z}{n}\right| * N_B$.

$^{197}_{79}Au$

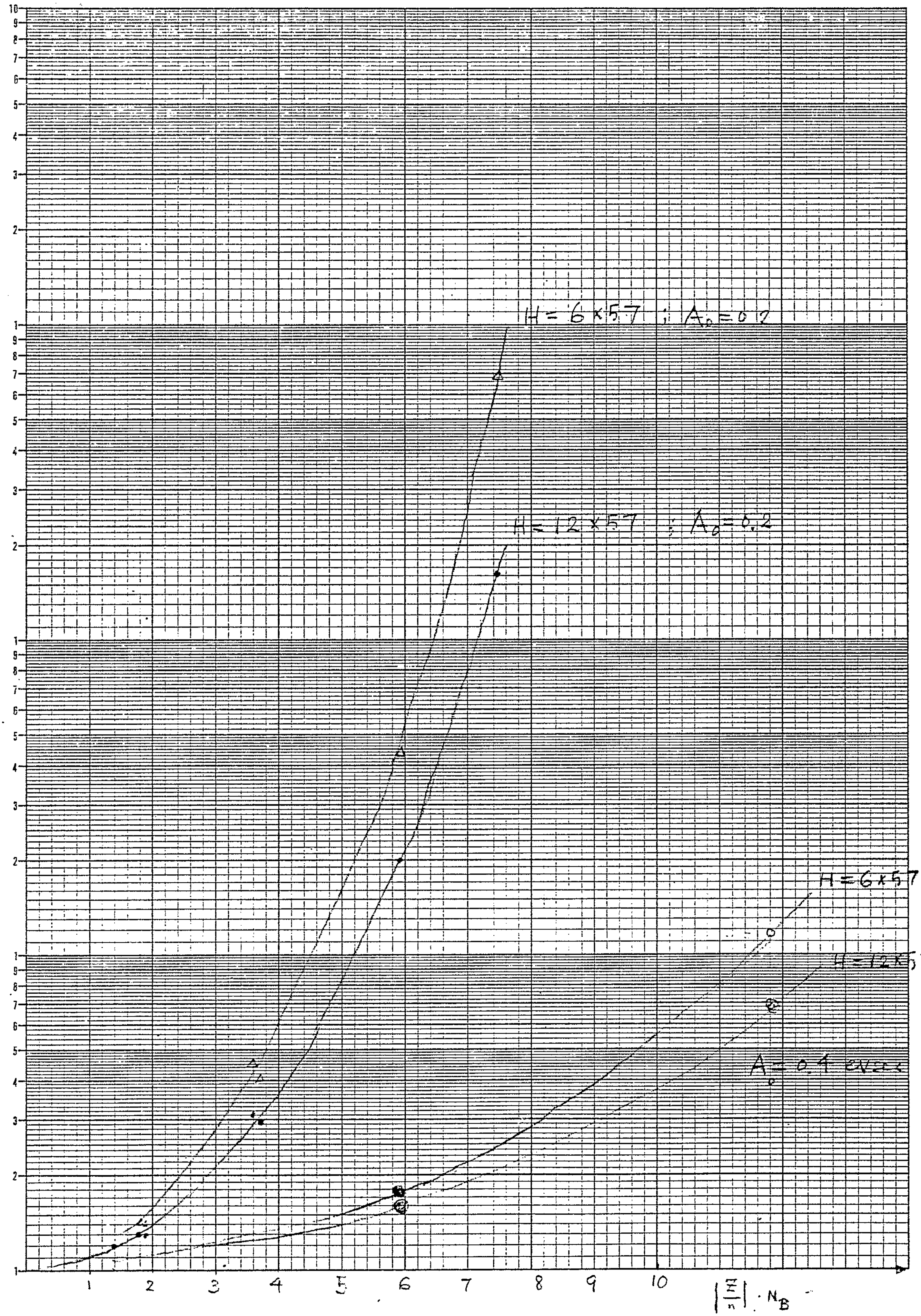
$\gamma_T = 26.4$ $\langle n \rangle = 2.034 \times 10^4$

$V = 1.MV$ $\bar{\beta} = 29.5^m$

$\phi_s = 5.45°$ $\bar{x}_p = 1.0$

$\dot{\gamma} = 3.2$ /sec $R = 610.168^m$, $a = 0.03^m$ [tube radius]

A_0 [ev sec]	H	Re $[\dot{Z}/n]$	$N_B [10^9]$	$G = e^{\int \Omega_i dt}$	$\lvert Z/n \rvert \times N_B$
0.2	12×57	0	1.15	1.17	1.38
		5	1.15	19.7	5.91
		1	1.15	1.29	1.80
			2.30	3.05	3.59
		3	0.58	1.29	1.87
			1.15	2.97	3.72
			2.30	162	7.43
	6×57	1	1.15	1.42	
			2.30	4.59	
		3	0.58	1.39	
			1.15	4.05	
			2.30	692	
		5	1.15	43.76	
0.4	12×57	5	1.15	1.56	
			2.30	6.88	
	6×57	5	1.15 2.30	1.75 11.63	
$^{63}_{29}Cu$ 0.4	12×57	5	4.5	2.91	


H = 6×57 ; A₀ = 0.2
H = 12×57 ; A₀ = 0.2
H = 6×57
H = 12×57
A = 0.4
1 2 3 4 5 6 7 8 9 10
N_B