RHIC-PG-30

PARAMETERS OF THE RF SYSTEM
FOR THE "WEAK-FOCUSING" LATTICES

H. HAHN

(BNL January 20, 1984)

H. Hahn
January 20, 1984

rf REQUIREMENTS
(Parabolic Distribution)

Bunch half length $= \sqrt{5}\sigma_L$

Bunch phase half width $\phi = \sqrt{5}\,\sigma_L\, h/R$

Bunch half height $\Delta_E = \sqrt{5}\,\delta_E$

Bunch area/amu $S = 5\pi\,\sigma_L\,\delta_E\,\gamma\,E_o/c \quad = \frac{\gamma\,E_o}{2\,h\,f_o}\,\Delta_E\,\phi$

In the small-amplitude approximation and stationary

$$\phi = \left(\frac{8\pi\,|\eta|\,h^3\,f_o^2}{\gamma\,E_o eV}\frac{A}{Q}\right)^{1/4}\sqrt{S}$$

$$\Delta_E = \frac{2h\,f_o}{\gamma\,E_o}\,\frac{S}{\phi}$$

Bucket half height required, stationary

$$\Delta_B = \frac{\Delta_E}{\sin\phi/2}$$

Bucket Area/amu required

$$A_B = \frac{4}{\pi}\frac{\gamma\,E_o}{h\,f_o}\Delta_B$$

Voltage required

$$V = \frac{\pi}{2}\frac{h\,|\eta|\,\gamma\,E_o}{e}\frac{A}{Q}\Delta_B^2$$

$$V \approx 8\pi\,\frac{|\eta|\,h^3\,f_o^2}{e\,\gamma\,E_o}\frac{A}{Q}\frac{S^2}{\phi^4} \quad (\phi \ll \pi)$$

H. Hahn
January 20, 1984

Parameter Variations with h_{rf}
(Equipartition)
γ= 100, Au

Lattice	N_B	h_{rf}	δ_E	S eV·sec	L $cm^{-2}sec^{-1}$	Diamond cm rms	V MV
	$\times 10^9$	×57	$\times 10^{-4}$		$\times 10^{26}$		
			α = 0 mrad				
15/120°	1	12	11.3	3.8	10.5*	29	2.2
	1	6	9.8	6.6	14.*	57	0.85
12/90°	1	12	6.9	2.3	5.6	29	2.1
	1	6	6.0	4.0	7.4	57	0.81
	1	1	4.2	16.9	15.1*	343	0.066
9/120°	1	6	5.9	4.0	8.4	57	0.95
			α = 2 mrad				
15/120°	2	12	12.9	4.4	11.	12	3.0
	2	6	11.3	7.6	6.7	14	1.1
	1	6	9.8	6.6	1.9	13	0.85
12/90°	2	12	7.9	2.7	7.6	14	2.8
	2	6	6.9	4.6	4.8	18	1.1
	2	1	4.8	19.4	1.2	17	0.087
	1	6	6.0	4.0	1.4	16	0.81
9/120°	2	6	6.7	4.6	5.1	17	1.2

* $\Delta\nu_{BB} > 0.003$

H. Hahn
January 20, 1984

MOMENTUM SPREAD AT TRANSITION

The momentum spread at transition scales like

$$\delta_E \propto \left(\frac{h^2}{\gamma_{tr}^2} \frac{V^2}{\dot{B}} \cos^2 \Phi_s\right)^{1/6}$$

with $V \sin \phi_s = 2\pi \, R \, \rho \, \dot{B}$

Assuming the same rf system, the lattices with $\gamma_{tr} \approx 25$ require at transition about 15% more momentum aperture than one with $\gamma_{tr} \approx 38$.

An acceptable rf system for the $\gamma_{tr} \approx 25$ lattices is obtained by using (primed quantities):

$$h' = \frac{1}{2} h; \quad V' = \frac{1}{5} V; \quad \dot{B}' = \frac{1}{4} \dot{B}; \quad \phi_s' \approx \frac{5}{4} \phi_s$$

leading to $\delta_E' = 0.7 \, \delta_E$

The resulting physical aperture requirement due to momentum spread is

($X'_{p\,max} = 1.57$ m, $X_{p\,max} = 0.7$ m)

$$\sigma_H' = 1.57 \, \sigma_H$$

H. Hahn
January 20, 1984

SUGGESTED rf PARAMETERS

$$f_{rf} = 6 \times 57 \times f_o = 26.7 \text{ MHz}$$

$$V_{max} = 1 \text{ MV}$$

$$V_{acceleration} = 200 \text{ kV}$$

Acceleration time = 2 min.

Questions:

- What is dynamics of intrabeam scattering at operating point.

- Parzen will calculate $L = L(t)$
  $\sigma = \sigma_L(t)$

- Slowest beam growth is expected, if full voltage is reached at the end of the acceleration cycle, since

$$\tau_E^{-1} \propto \frac{N_B}{\varepsilon \; S \; \delta_E^2}; \qquad \tau_H^{-1} \propto \frac{N_B}{\varepsilon^2 \; S}$$

H. Hahn

January 20, 1984

CHOICE OF TRANSITION ENERGY

Due to intrabeam scattering the momentum spread of the bunch increases until $\Delta_E = \Delta_B$. If this limit is exceeded, the particles are lost.

At constant voltage

$$\Delta_B^2 \propto \frac{1}{h\,\gamma\,|\eta|} = \frac{\gamma_{tr}}{h\,|\gamma/\gamma_{tr} - \gamma_{tr}/\gamma|}$$

The bucket height requirements vary with energy according to

$$\Delta_E \propto \frac{1}{\sqrt{\gamma}} \qquad \text{(equipartition)}$$

$$\Delta_E(\gamma=12) > 1.4\Delta_E(\gamma=100) \qquad \text{(Parzen)}$$

Equivalent performance over energy range, (i.e. $L \propto \gamma$) requires

$$\gamma_{tr}^2 = \gamma_1\,\gamma_2\,\frac{(\Delta_1/\Delta_2)^2\,\gamma_1 + \gamma_2}{\gamma_1 + (\Delta_1/\Delta_2)^2\,\gamma_2}$$

For $\gamma_1=12$ and $\gamma_2=108$ follows the optimized transition energy $(\Delta_1/\Delta_2 \approx 1.4)$:

$$\gamma_{tr} = 36 \times 0.6 \approx 22$$