RHIC-PG-27

TABLES OF VALUES FOR THE COMPARISON OF 3 DIFFIERENT LATTICES FOR RHIC

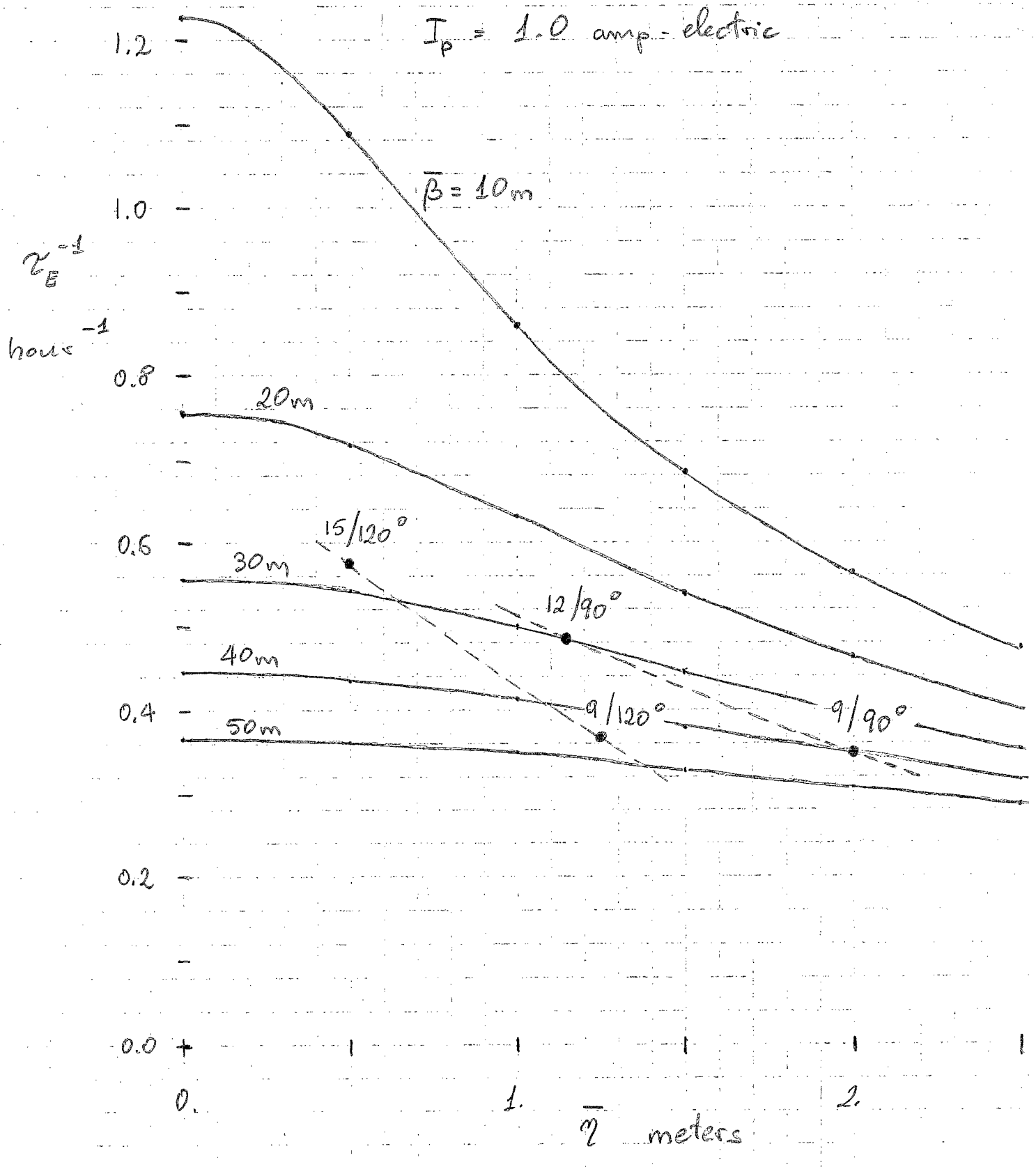
A. G. Ruggiero

(BNL, January 16, 1984)

$\gamma = 100$ Au: $A = 197$ $Z = 79$

$\varepsilon_N = 10\pi$ mm·mrad, $\sigma_E/E = 0.5 \times 10^{-3}$

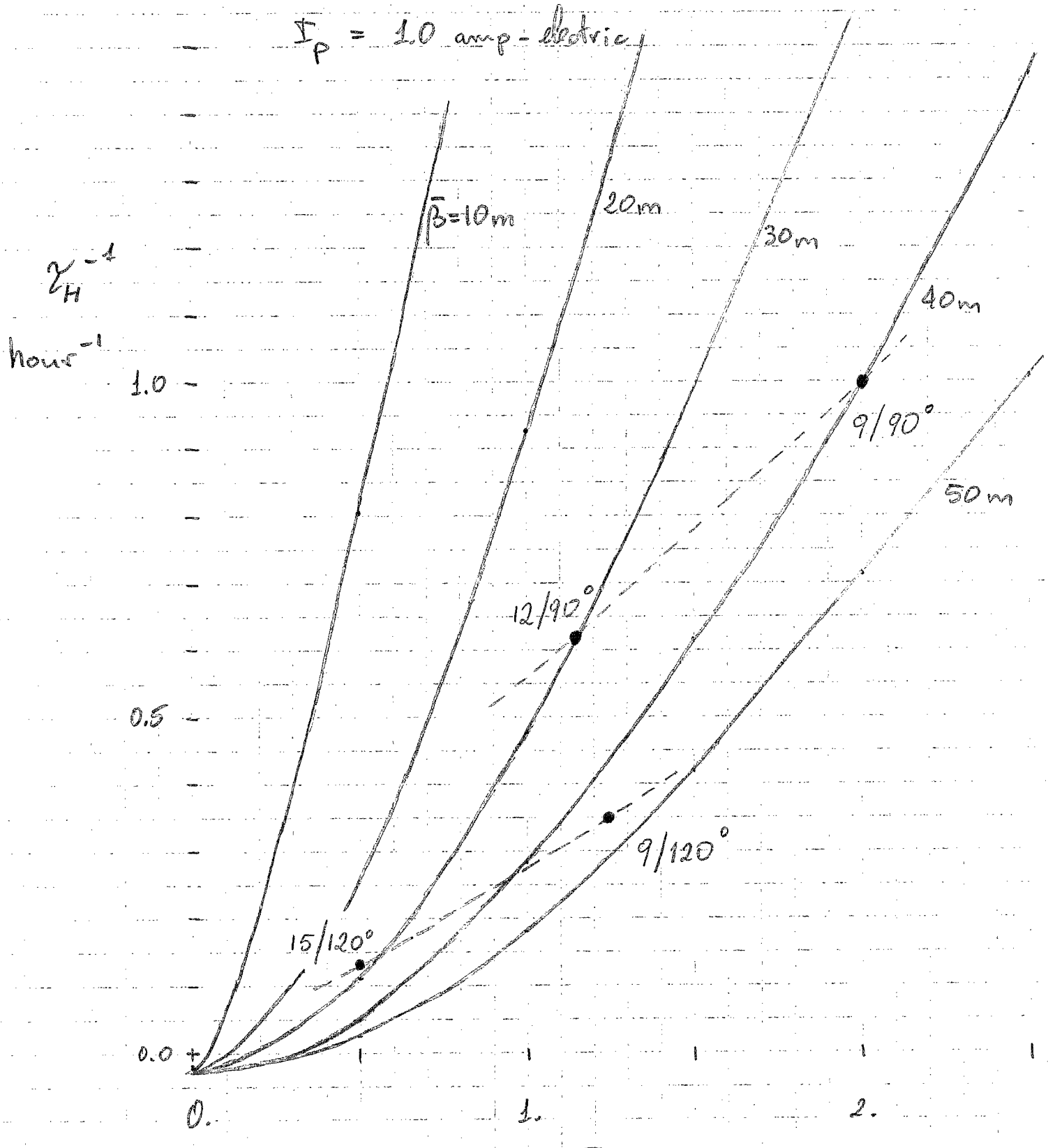
$I_p = 1.0$ amp - electric



$\gamma = 100$ Au: $A = 197$ $Z = 79$

$\varepsilon_N = 10\ \pi$ mm-mrad , $\sigma_E/E = 0.5 \times 10^{-3}$

$I_p = 1.0$ amp - electric



GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.
LOGARITHMIC 2 X 9 CYCLES AL-2461-00

$\tau_E^{-1} \sim \frac{1}{(\sigma_E/E)^3}$

$\gamma = 100$

A_{cl} : $A = 197$, $Z = 79$

$\varepsilon_N = 10\pi$ mm. mrad

$T_0 = 10$ amp - electric

τ_E^{-1}

hour^{-1}

1.0

0.1

0.01

σ_E/E

10.

1.0

0.1

$\times 10^{-3}$

GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.
LOGARITHMIC 2 X 3 CYCLES AL-2461-60

$\gamma = 100$

Au : $A = 197$, $Z = 79$

$\sigma_{e/5} = 1. \times 10^{-3}$

$I_p = 4.0$ amps - electron

π_E - 1.0 - 0.1 - 0.01 hour^{-1}

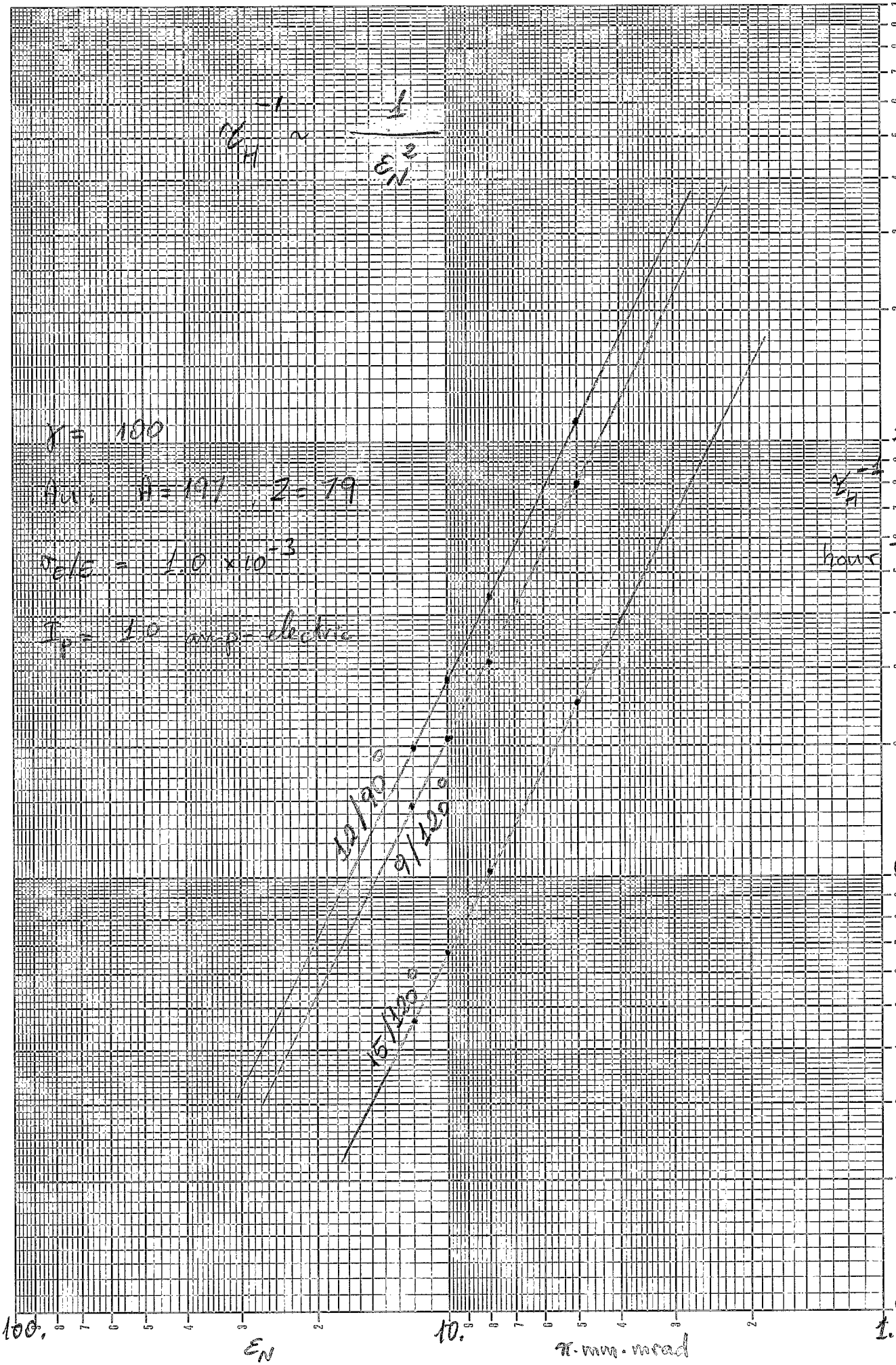
15/120°

12/90°

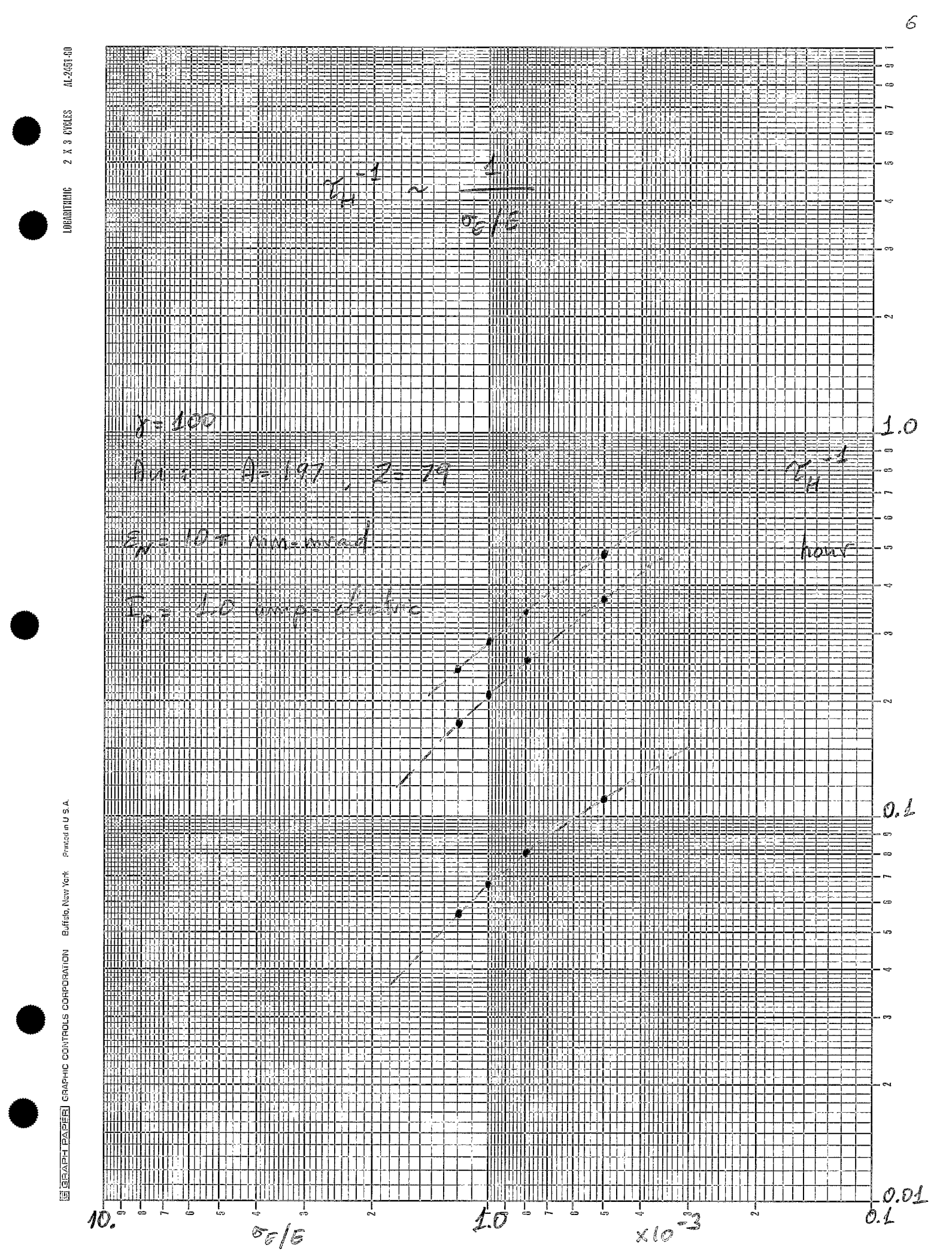
9/120°

100. 10. 1.0

E_N $\pi \cdot mm \cdot mrad$



GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.
LOGARITHMIC 2 X 3 CYCLES AL-2451-60



GRAPH PAPER GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.
LOGARITHMIC 2 X 3 CYCLES AL-2461-00

	15/120°	12/90°	9/120°
τ_E^{-1}	0.54 h^{-1}	0.50 h^{-1}	0.37 h^{-1}
τ_H^{-1}	0.11	0.48	0.37

ε_N = 10 π mm·mrad $\quad \sigma_E/E = 0.5 \times 10^{-3}$

Au: A=197, Z=79 $\quad \gamma = 100 \quad I_p$ = 10 Amp-electric

Define $\delta = \sigma_E/E \rightarrow I_p = NeZ/\sqrt{2\pi}\,\sigma_\tau$

$$\tau_E^{-1} = \frac{1}{\delta}\frac{d\delta}{dt} \qquad \tau_H^{-1} = \frac{1}{\varepsilon_N}\frac{d\varepsilon_N}{dt}$$

$$\tau_E^{-1} = \frac{A_E I_p}{\varepsilon_N \delta^3} \qquad \tau_H^{-1} = \frac{A_H I_p}{\varepsilon_N^2 \delta}$$

$A_E = 1.25\,\tau_E^{-1} \qquad A_H = 50.\,\tau_H^{-1}$

with τ_E^{-1} and τ_H^{-1} given in the top Table, provided I_p is in Amp-electric, ε_N in π·mm·mrad and δ in °/oo -

Sum of the Diffusion Rates ($\tau_V^{-1} > 0$)

$$\tau^{-1} = \frac{A_E I_p}{\varepsilon_N^2 \delta}\left(\frac{A_H}{A_E} + \frac{\varepsilon_N}{\delta^2}\right) \quad \forall\ 0$$

Integration for the beam dimensions

$$\dot{\delta} = \frac{A_E I_p}{\varepsilon_N \delta^2} \quad \text{and} \quad \dot{\varepsilon}_N = \frac{A_H I_p}{\varepsilon_N \delta} \qquad (1)$$

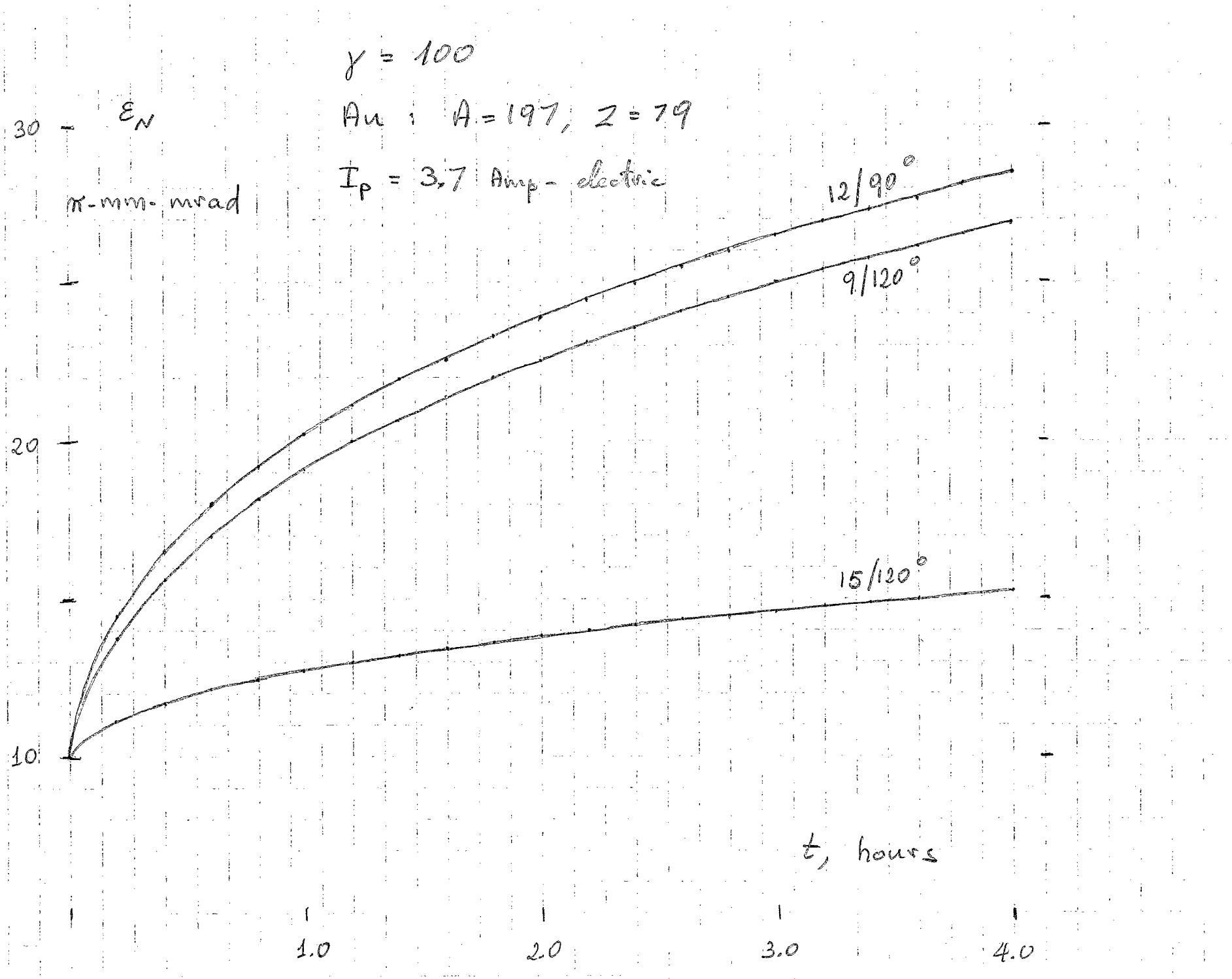
from which

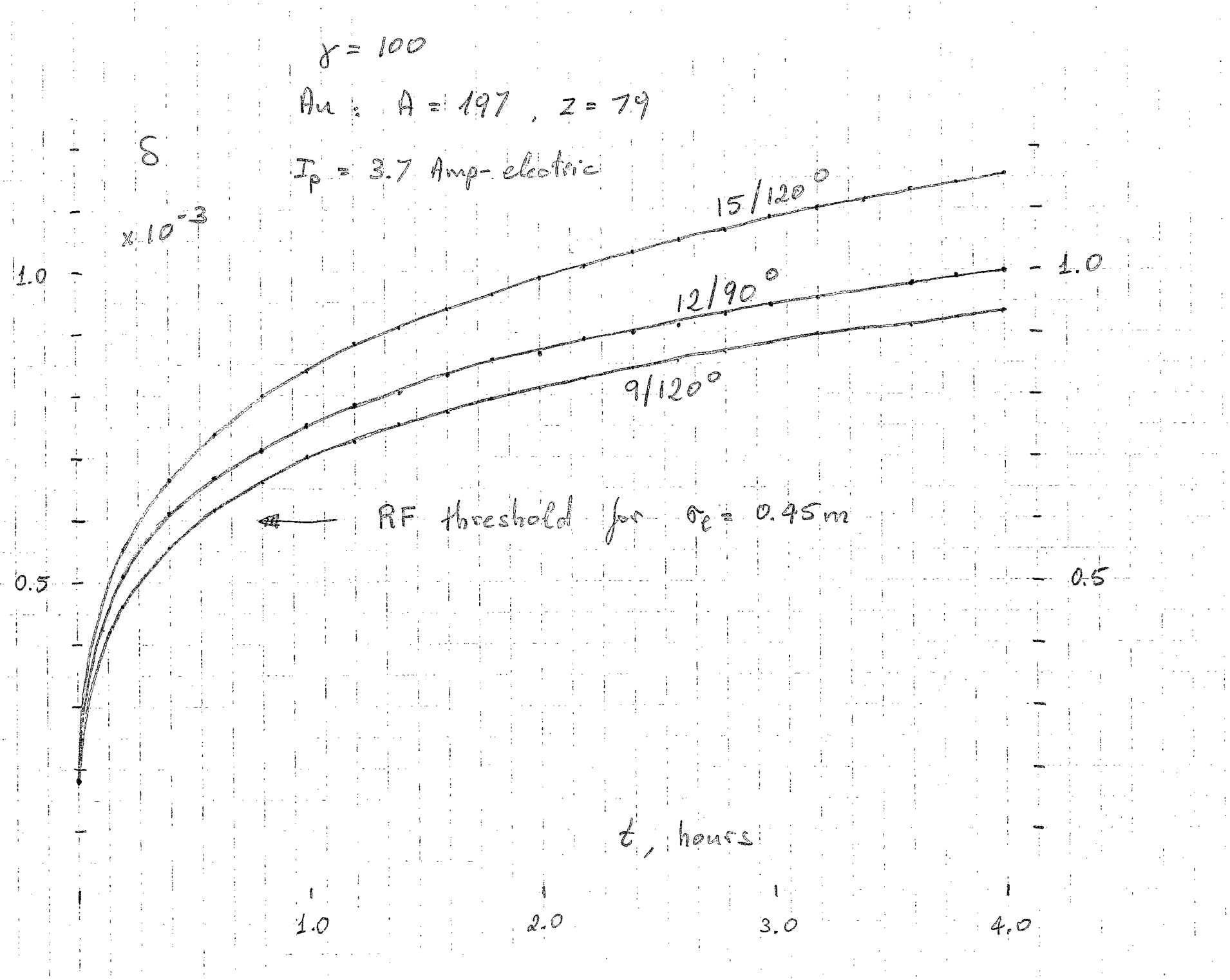
$$S = 2\frac{A_E}{A_H}\varepsilon_N - \delta^2 = \text{invariant} \quad \forall\ 0$$

The system (1) can be reduced down to one equation

$$\dot{\delta} = \frac{2 A_E^2 I_p / A_H}{\delta^2(\delta^2 + S)}$$

This can be easily integrated analytically $\forall\ 0$

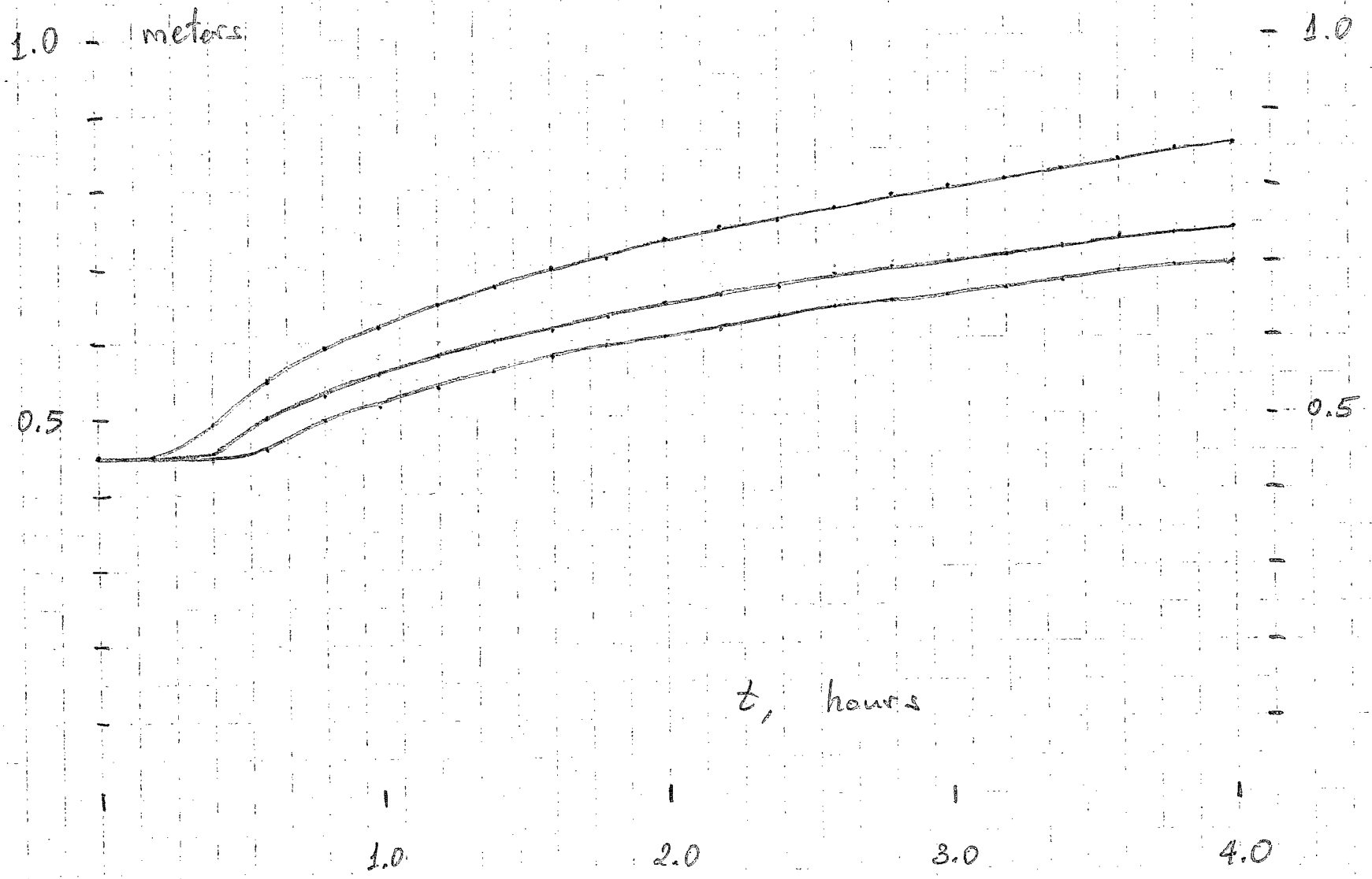

γ = 100
Au : A = 197, Z = 79
I_p = 3.7 Amp - electric
ε_N
π-mm-mrad
30
20
10
12/90°
9/120°
15/120°
t, hours
1.0
2.0
3.0
4.0


γ = 100
Au ; A = 197 , z = 79
I_p = 3.7 Amp- electric
δ
x 10⁻³
15/120°
12/90°
9/120°
RF threshold for σ_ℓ = 0.45m
1.0
0.5
1.0
0.5
t, hours
1.0
2.0
3.0
4.0

$\gamma = 100$

Au : $A = 197$, $Z = 79$

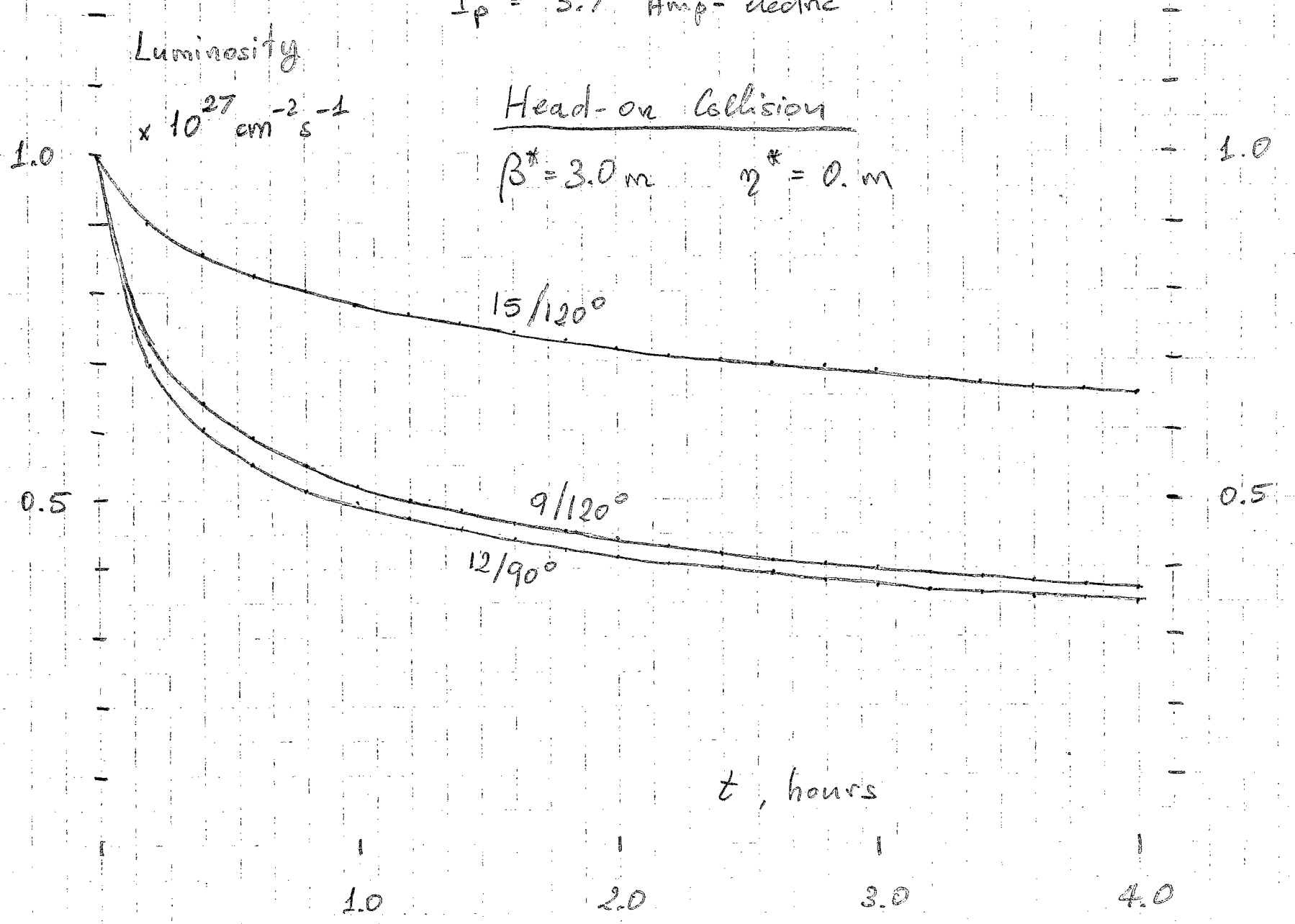
$I_p = 3.7$ Amp - electric



$\gamma = 100$

Au : $A = 197$, $Z = 79$

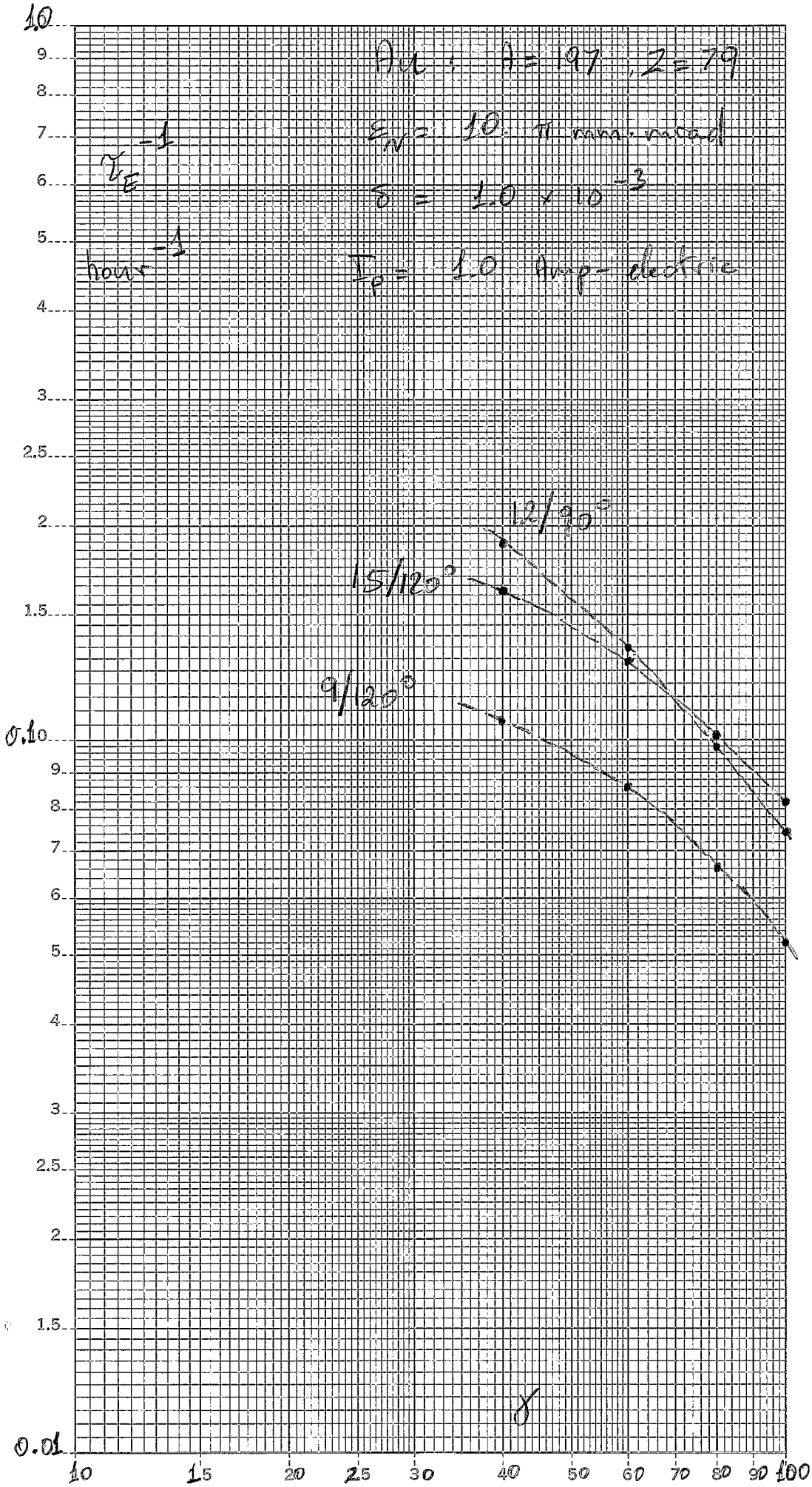
$I_p = 3.7$ Amp - electric



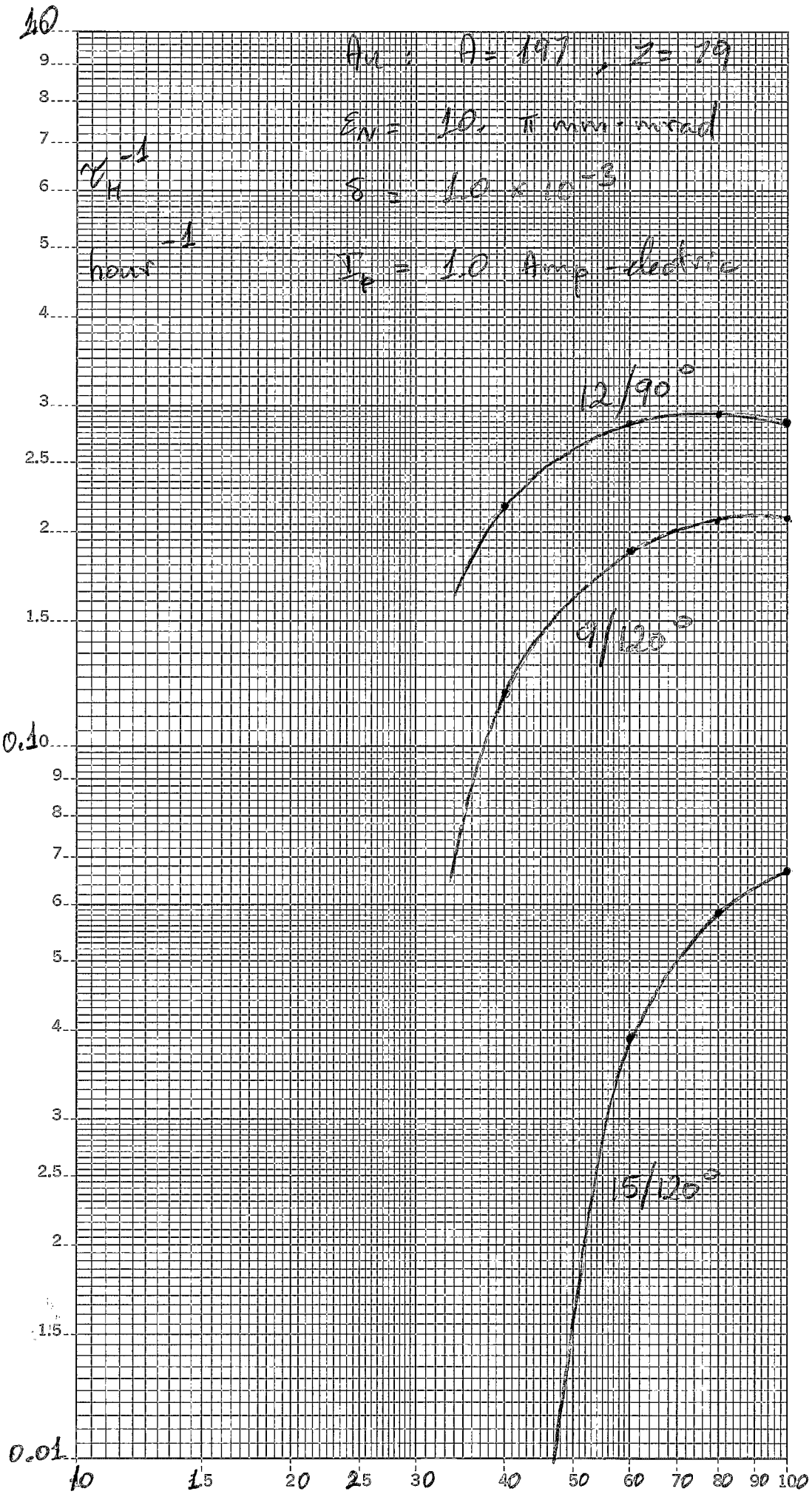
	2 hours - Operation Mode			4 hours - Operation Mode		
	9/120°	12/90°	15/120°	9/120°	12/90°	15/120°
Average luminosity $\times 10^{27}\ cm^{-2}\ s^{-1}$	0.52	0.55	0.80	0.47	0.44	0.73
Normalised Emittance $\pi \cdot mm \cdot mrad$	22.5	23.8	13.8	26.9	28.6	15.3
Rms Energy Spread $\times 10^{-3}$	0.81	0.87	0.98	0.94	1.00	1.16

Beam - Beam Tune Shift is the same for all cases

$\Delta\nu_{BB} = 0.0026$ (initial)



46 7080
LOGARITHMIC 2 X 1 CYCLES
KEUFFEL & ESSER CO. MADE IN U.S.A.



46 7080
LOGARITHMIC 2 X 1 CYCLES
KEUFFEL & ESSER CO. MADE IN U.S.A.

$B\rho$ = 839.5 T·m	15/120°	12/90°	9/120°
Lattice:			
Cell Length, m	23.7	29.6	39.5
β_{max}, m	49.8	49.8	82.8
η_{max}, m	0.70	1.54	1.82 (1.97)
γ_T	39.0	25.1	26.9 (23.6)
$\nu_{H,V}$ appox.	50. (30.0)	38. (18.0)	38. (17.8)
Chromaticity	69. (49.3)	43. (22.8)	49. (28.8)
Dipoles:			
Field, T	3.3	3.05	5.0
Length, m	7.91	10.70	4.36
Sagitta, cm	3.1	5.2	1.4
Radius of Curv., m	254.4	275.4	168.3
Number	180	144	216
Quadrupoles:			
Gradient, KG/cm	6.8	4.4	5.4
Length, m	1.90	1.90	1.40
Number	180	144	108

Comments to the previous Table:

1. The assumed rigidity $B\rho = 839.5$ T-m is equivalent to 100 GeV/A for gold

2. The transition energy value γ_T has been obtained by multiplying the contribution of the arcs by a factor 1.34 to take into account the long-straight sections

3. The numbers in brackets for tune and chromaticity are the exact contribution from the regular cells - A factor of 20 has been added to them to take into account the expected contribution from the long straight sections.

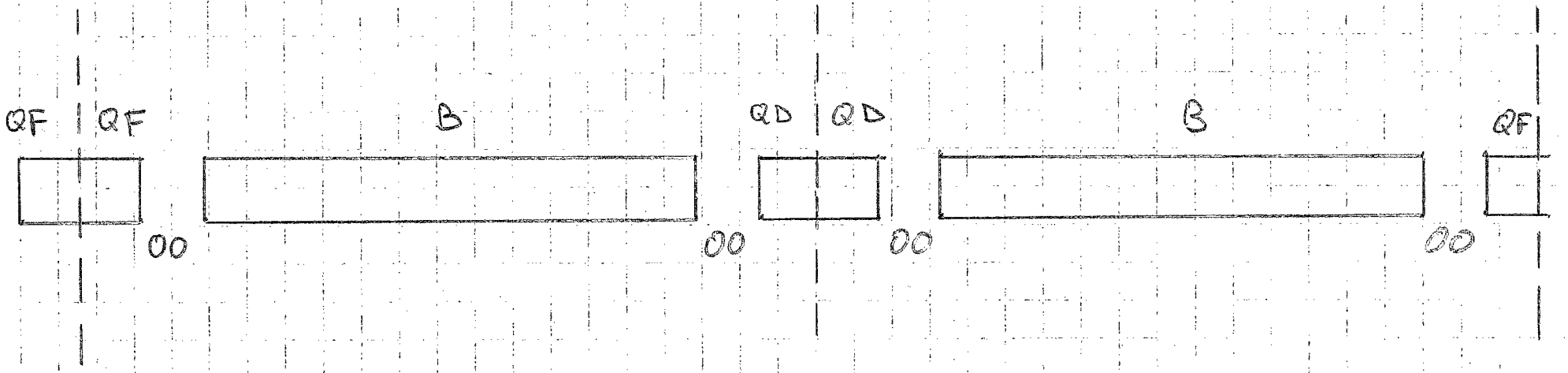
4. There are 24 extra special dipoles for the dispersion killing insertions. There are also 24 more special dipoles to bring the two beams to collide either head-on or at an angle.

5. The quadrupoles in the long straight sections have not been included in the list.

15/120° Lattice

B, dipole: length = 7.906637 m
Field = 3.3 Tesla

OO, drift: length = 1.02109 m



QF, QD, half-quad: length = 0.95 m

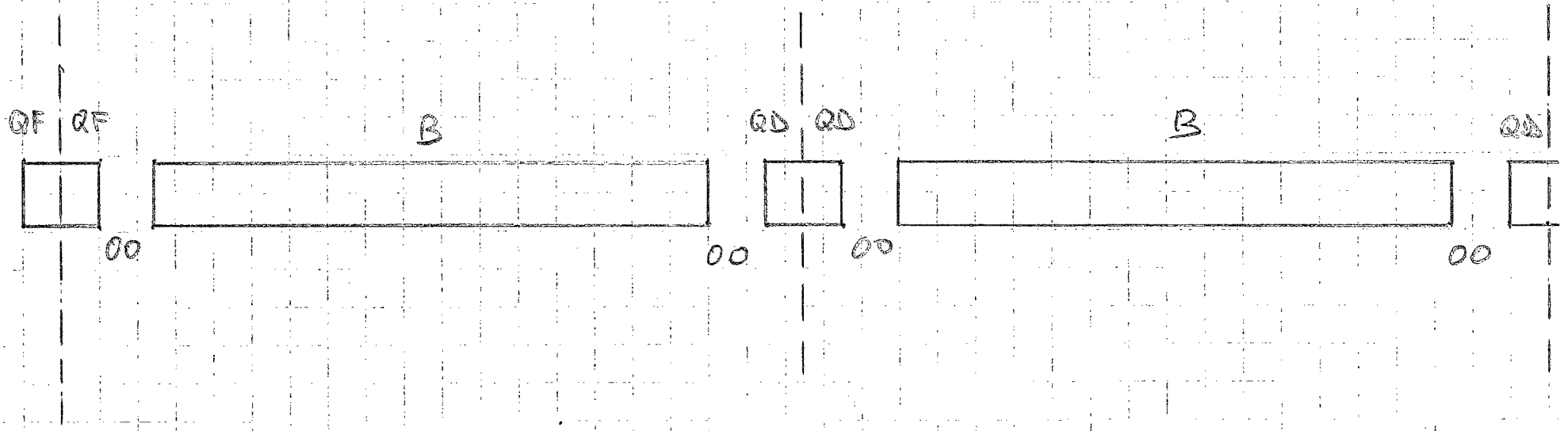
$B'/B\rho$ = 0.081449 m^{-2} for QF
-0.081386 for QD

Total Cell Length = 23.6976 m.

12/90° Lattice

B, dipole : length = 10.7 m
Field = 3.05 Tesla

00, drif : length = 1.1055 m



QF, QD, half-quad : length = 0.95 m

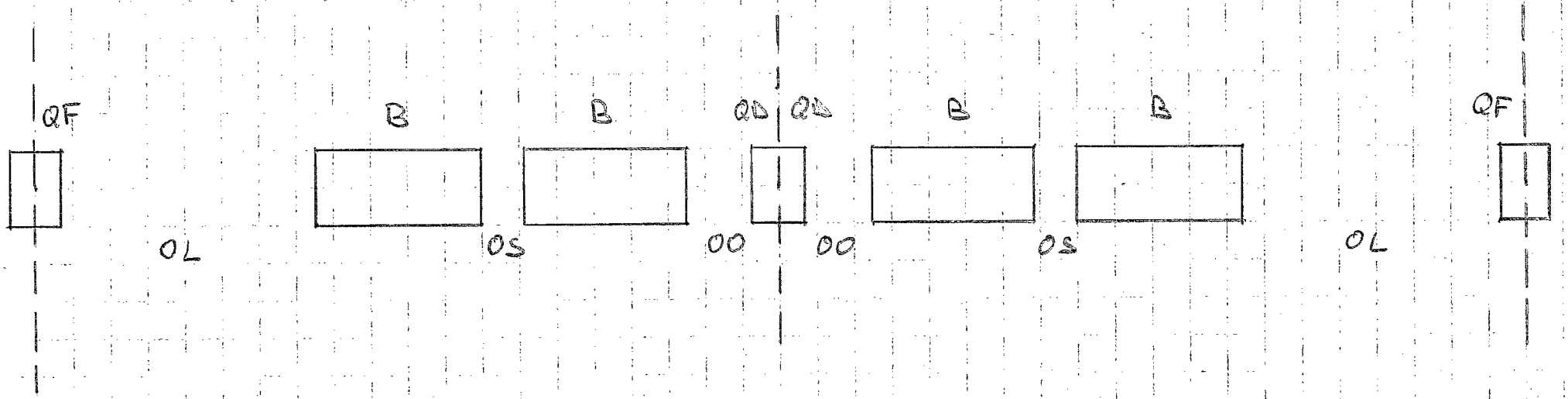
$B'/B\rho$ = 0.052588 m^{-2} for QF
−0.052513 for QD

Total Cell Length = 29.6221 m

9/120° Lattice

B, dipole: length = 4.36 m
Field = 5.0 Tesla

Drifts: OL length = 6.66 m
OS = 1.15
OO = 1.8050



QF, QD, half-quad: length = 0.70 m

B'/Bρ = 0.063823 m^{-2} for QF
−0.063613 for QD

Total Cell Length = 23.6976 m

	15/120°	12/90°	9/120°	
γ_T	39.0	25.1	26.9	23.6
Accelr. Period	30 sec	30 sec	5 min	
Energy Gain/turn	38.1 keV/A	38.1	3.81	
Peak Voltage for Acceleration	1.0 MV	1.0	0.5	
ϕ_s	5.45°	5.45°	1.09°	
Bucket Area: inject.	0.66 eV/A-sec	0.72	0.52	0.50
top energy	5.7	3.5	2.7	2.3
Bucket Height: inject.	± 0.24 %	0.26	0.19	0.18
top energy	0.24	0.15	0.11	0.10
Peak Voltage in Storage Mode at 100 GeV/A	1.0 MV	2.6	2.3	3.0
ν_s @ 100 GeV/A	0.0005	0.0013	0.0013	

The Bucket Area and Height given in this Table are for Stationary Bucket (no Acceleration)

	15/120°	12/190°	9/120° A	9/120° B
γ_T	39.0	25.1	26.9	23.6
S = 0.2 eV/A-sec				
charact. time	31.8 msec	17.7	52.5	44.1
bunch length	± 0.40 nsec	0.54	0.44	0.48
bunch spread	± 0.51 %	0.59	0.67	0.70
S = 0.5 eV/A-sec				
charact. time	31.8 msec	17.7	52.5	44.1
bunch length	± 0.63 nsec	0.85	0.70	0.76
bunch spread	± 0.80 %	0.93	1.06	1.10
Betatron Emittance @ $\gamma = \gamma_T$ 95% of beam	0.39 π·mm·mrad	1.14 π·mm·mrad	1.00 π·mm·mrad	1.14 π·mm·mrad

Shown here are the beam bunch dimensions at transition energy crossing.

BEAM DIMENSIONS

	15/120°	12/90°	9/120° A	9/120° B
Betatron Emittance;	ε_N = 15.3	28.6	26.9	
(95%) injection	1.23 π mm-mrad	2.29	2.16	
transition	0.39	1.14	1.00	1.14
top energy	0.14	0.26	0.25	
δ, rms $\times 10^{-3}$:				
injection	1.16	1.00	0.94	
transition	3.27	3.80	4.33	4.49
top energy	1.16	1.00	0.94	
β_{max}, m	49.8	49.8	82.8	
η_{max}, m	0.70	1.54	1.82	1.97
$6 \times \sigma_V$: mm				
injection	± 19.2	± 26.2	32.8	
transition	10.8	18.5	22.3	23.8
top energy	6.5	8.8	11.1	
$6 \times \sigma_H$: mm				
injection	± 19.8	± 27.8	34.4	34.6
transition	17.5	39.7	52.3	58.2
top energy	8.1	12.8	15.1	15.7

Assume a 2/3-rule for the good field region as compared to the coil i.d.

Lattice	Coil i.d.
15/120°	6.0 cm
12/90°	12.0 cm
9/120° : A	16.0 cm
B	17.5 cm

SEXTUPOLES

$B\rho$ = 839.5 T-m		15/120°	12/90°	9/120° A	9/120° B
η_F	m	0.70	1.55	1.82	1.97
η_D	m	0.29	0.75	0.65	0.80
β_{max}	m	49.8	49.8	82.8	
β_{min}	m	3.8	8.8	6.3	
no. of sextupoles		2×90	2×72	2×54	
natural chromat.		69	43	49	
length, ℓ_s, m		1.9	1.9	1.7	
$B''\ell_s/B\rho$, m^{-2}					
	F	0.276	0.097	0.076	0.070
	D	0.718	0.236	0.228	0.185
Bore Radius, cm		3.0	6.0 (3.81)	(6.35)	
Pole Tip Field at QD, KG		1.43	1.88 (0.76)	(2.27)	(1.84)

$$|Z/n| \lesssim 3 \frac{E|\eta|}{e I_p} \left(\frac{\sigma_E}{E}\right)^2 \frac{A}{Z}$$

$I_p = 3.7$ Amp - electric

$\sigma_E/E = 0.6 \times 10^{-3}$

Gold : $A = 197$, $Z = 79$

$E = 100.9308$ GeV/A

Lattice		$\lvert Z/n \rvert$
15/120°		41.9 ohm
12/90°		110.2
9/120°	A:	95.5
	B:	125.6

Closed Orbit Analysis

For 98% confidential level

$$\hat{y} = 2.1 \sqrt{M \bar{\beta} \beta_{max}} \; \langle \psi \rangle$$

	Horizontal	Vertical
Dipole	$\theta \langle \Delta B/B \rangle$	$\theta \langle \Delta \theta_{rot} \rangle$
Quadrupole	$k l_q \langle \Delta y \rangle$	$k l_q \langle \Delta y \rangle$

θ, bending angle

$\langle \Delta B/B \rangle$, rms field error = 10^{-4}

$\langle \Delta \theta_{rot} \rangle$, rms roll angle = 0.1 mrad

$k = B'/B\rho$

l_q, quadrupole length

$\langle \Delta y \rangle$, rms lateral displacement = 0.1 mm

Max. Expectation of Closed Orbit Distortion

	15/120°	12/90°	9/120°
$\bar{\beta}$ m	26.8	29.3	44.6
β_{max} m	49.8	49.8	82.8
M:			
dipoles	180	144	216
quads	100	144	108
θ mrad	31.1	38.9	25.9
$k l_q$ m^{-1}	0.1545	0.0999	0.0892
$\langle y \rangle$: mm			
dipole	1.6	1.8	2.4
quad	8.0	4.8	5.8
$\langle y \rangle_{total} =$			
$= \sqrt{\langle y^2 \rangle_D + \langle y^2 \rangle_Q}$	8.2	5.2	6.2