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The Dependence of the Dynamic Aperture on Momentum and Synchrobetatron Coupling

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1. Introduction

Long term tracking studies reported below, about one millon turns with synchrotron oscillations, indicate that the momentum aperture may be considerably reduced by the presence of the synchrotron oscillations. These studies were done with a RHIC lattice. The effect is most pronounced at lower energies in RHIC, where large momentum apertures are required. At $\gamma = 30$ in RHIC, with a synchrotron oscillation amplitude of $\Delta p/p = 0.005$, the dynamic aperture is reduced from 12.5 mm to 8.5 mm by the presence of the synchrotron oscillations. This reduction in aperture may be due to a non-linear coupling between the longitudinal and transverse motions. In RHIC, the longitudinal phase space is much larger than the transverse phase space by a factor of several thousand, and a small amount of coupling can cause considerable growth in the transverse motion.

This effect may be more important for RHIC than for other superconducting proton colliders, because of its relatively low energy and because of the importance of intrabeam scattering for heavy ions. This results in larger dynamic aperture requirements for RHIC both in transverse space and in momentum spread.

2. Tracking Results

The largest effect is seen at $\gamma = 30$ for RHIC, where the beam momentum spread can grow to $\Delta p/p = \pm 0.005$ after 10 hours. Results are shown in Fig. 1 for the dynamic aperture as a function of the size of the synchrotron oscillation amplitude, $\Delta p/p$. The dynamic aperture drops from $A_{SL} = 15.5$ mm at $\Delta p/p = 0$ to $A_{SL} = 8.5$ mm at $\Delta p/p =$

0.005. Also shown in Fig. 1 is the dynamic aperture found when the particle momentum is held fixed at some level, and no synchrotron oscillations are present. This curve drops more slowly and at $\Delta p/p = 0.005$, there is a difference of 4 mm between the two curves.

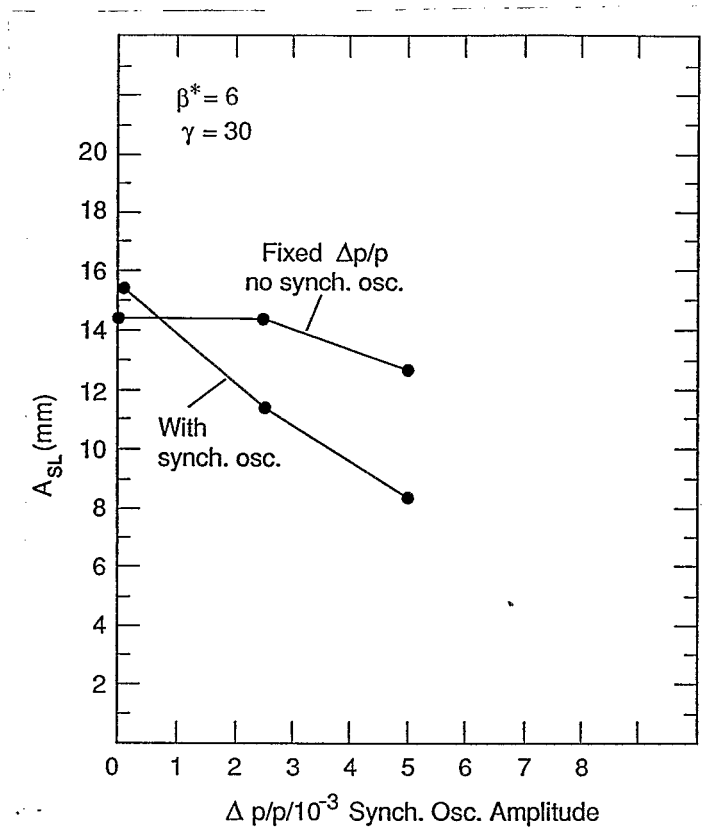

Fig. 1: Comparison of results with and without synchrotron oscillations.

These tracking runs were done for 800,000 turns at $\gamma = 30$ using the RHIC91 lattice with 6 insertions having $\beta^* = 6$ m. One RF cavity is present when the synchrotron oscillations are included.

The effect of the synchrotron oscillations on the dynamic aperture is studied further in Figs. 2 and 3. In Fig. 2, the survival time in turns is plotted against the initial betatron amplitude x_0 for three different synchrotron oscillation amplitudes, $\Delta p/p = 0$, 0.0025, and 0.005, for a particular distribution of field errors which gave the smallest dynamic aperture of 10 distributions studied. The particle is started out with $p_{x0} = p_{y0} = 0$ and $\epsilon_{x0} = \epsilon_{y0}$. The lattice is the same as was used for the results in Fig. 1. One sees that the survival curve moves towards lower x_0 as the synchrotron oscillation amplitude increases. The x_0 to survive 800,000 turns decreases roughly linearly with $\Delta p/p$. Note that x_0 is the initial

betatron oscillation, defined as the initial x minus the closed orbit position for the initial $\Delta p/p$, which is computed for a fixed $\Delta p/p$.

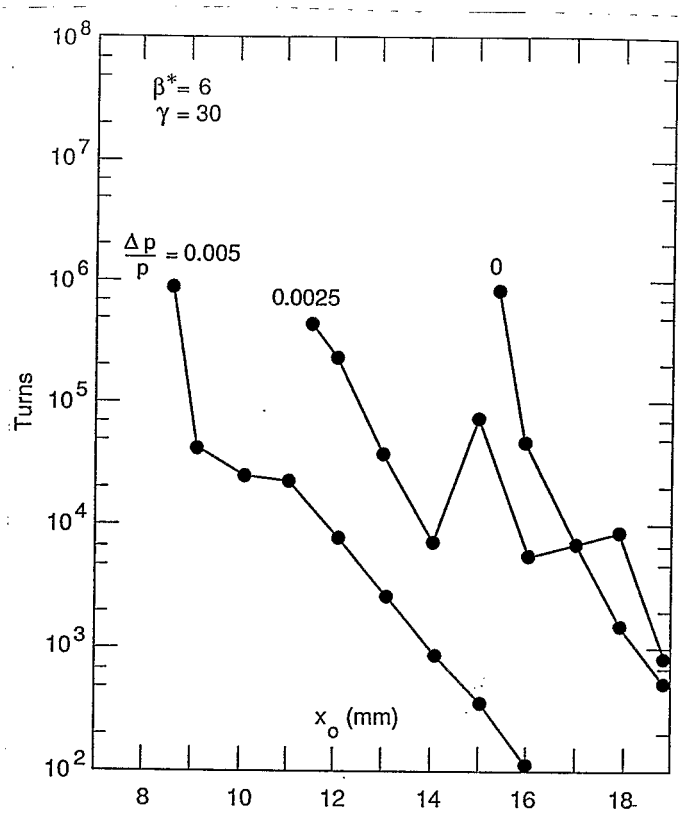

Fig. 2: Comparison of results for different $\Delta p/p$.

Fig. 3 compares survival plots, for this same worst distribution of field errors, for two cases. In one case, the RF is on and the particle has a certain synchrotron oscillation amplitude $\Delta p/p$. In the second case the RF is off, and the particle momentum is fixed. At $\Delta p/p = 0$, the curves for the two cases do not differ greatly. For $\Delta p/p = 0.005$, the two cases differ considerably; the x_0 to survive 800,000 turns is $x_0 = 8.5$ mm for the case with the synchrotron oscillation amplitude $\Delta p = 0.005$, compared to $x_0 = 12.5$ mm for the case with no synchrotron oscillations and $\Delta p/p$ is fixed at $\Delta p/p = -0.005$.

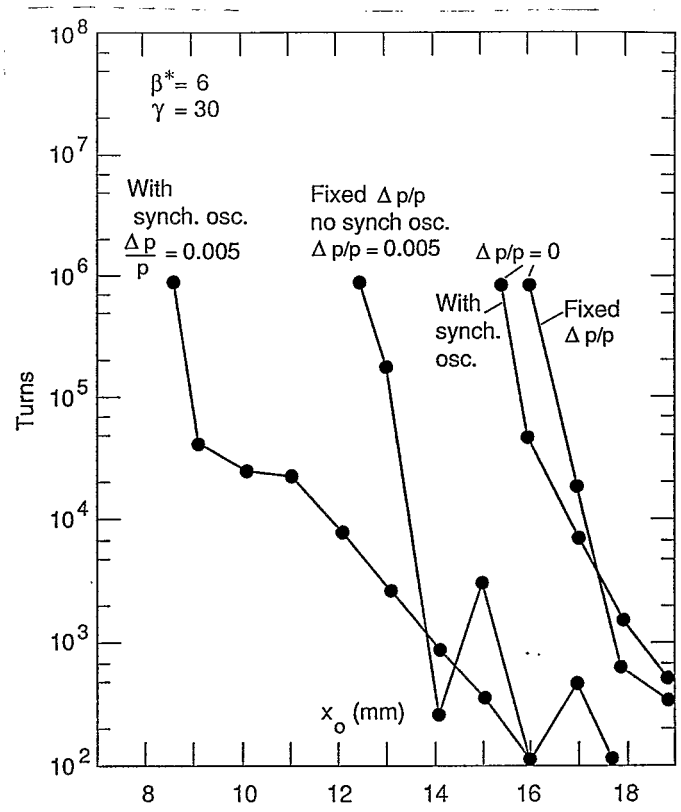

Fig. 3: Comparison of results with and without synchrotron oscillations.

The above results are for RHIC at $\gamma = 30$ with six $\beta^* = 6$ insertions. At $\gamma = 100$, one would like to run at a lower β^*. The momentum spread required is $\Delta p/p = \pm 0.002$ which is just the height of the RF bucket at $\gamma = 100$. Fig. 4 shows the dynamic aperture as a function of the synchrotron oscillation amplitude at $\gamma = 100$ and for a RHIC lattice with six $\beta^* = 2$ insertions. The results for $\beta^* = 6$ and $\gamma = 30$ are also shown. The decrease of A_{SL} with $\Delta p/p$ is not as severe for $\beta^* = 2$ and $\gamma = 100$. One reason is the smaller momentum spread required. Another contributing factor may be the smaller transverse aperture required, which may reduce the synchrobetatron coupling.

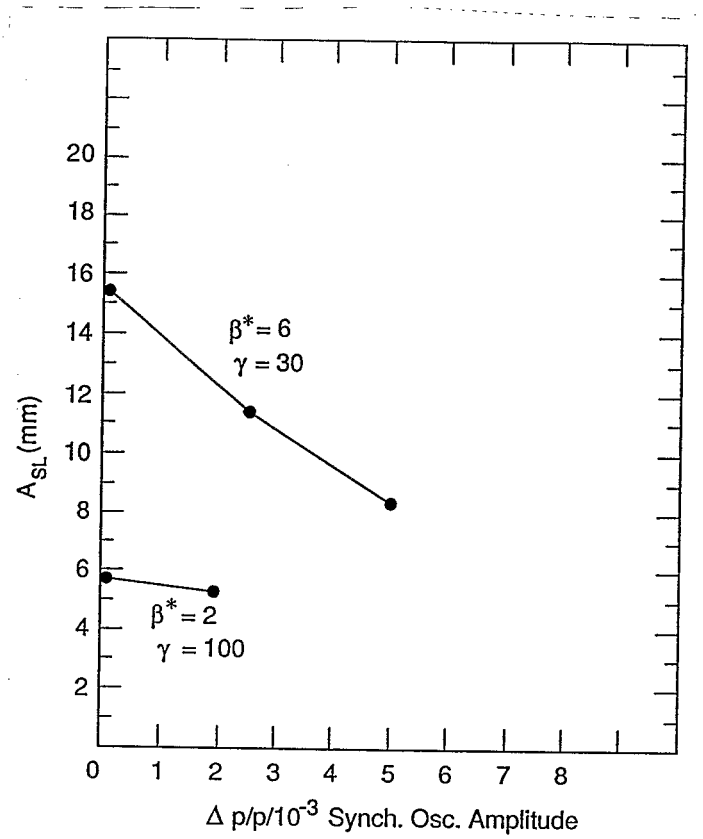

Fig. 4: Comparison of results for $\gamma = 100$, $\beta^* = 2$ with those for $\gamma = 30$, $\beta^* = 6$.

2.1 Comments on the Tracking

It is important that the tracking be symplectic. To achieve this, the ORBIT program was changed to allow the use of point magnets. The methods used are similar to those used in the TEAPOT[1] program, with some modifications. The RHIC91 lattice was used in the above studies. One 160 MHz RF cavity with an RF voltage of 4.5 MV was placed between Q8 and Q9 in one insertion region. The bucket height is $\Delta p/p = 6 \times 10^{-3}$ at $\gamma = 30$ and $\Delta p/p = 2 \times 10^{-3}$ at $\gamma = 100$. Random and systematic field errors were present in each magnet at the level expected for RHIC. Field error multipoles up to order 10 were included. The nominal operating tune is $\nu_x = 28.826$, $\nu_y = 28.821$. To establish the dynamic aperture, the stability limit for 800,000 turns was examined for ten different distributions of the random field errors.

The validity of using point magnets to represent the magnets was checked by comparing the survival plots found using point magnets with those found using thick magnet transfer

matrices based on the usual approximations. These results are shown in Fig. 5. In these tracking results for 800,000 turns, the RF is off and the momentum is fixed. Results are shown for $\Delta p/p = 0$ and $\Delta p/p = -0.005$ for a RHIC lattice with six $\beta^* = 6$ insertions for a particular distribution of field errors. The agreement between the results for point magnets and thick magnets is fairly good. The dynamic aperture depends on the survival time, and for some survival times the dynamic aperture as found by these two methods can differ by about 1 mm out of about 15 mm.

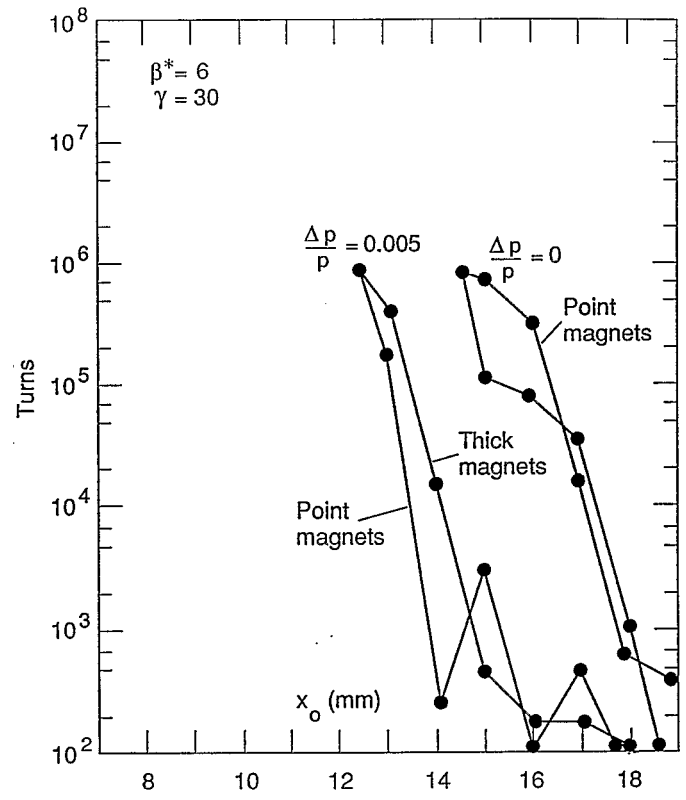

Fig. 5: Comparison of results for point magnets and thick magnets.

3. Synchrobetatron Coupling Studies

The decrease in the dynamic aperture for higher synchrotron oscillation amplitudes appears to be due to a coupling between the longitudinal and transverse oscillations. Several runs were done to explore the mechanism of the coupling. In one study the synchrotron oscillation tune, ν_s, was reduced by a factor 4. The dynamic aperture found for 800,000

turns, for $\Delta p/p = 0.005$ was 7.5 mm. This is to be compared with the 8.5 mm found for the normal ν_s. The loss of 1 mm does not appear large enough to be significant.

In another study, the chromaticity was changed from $C_x = C_y = 0$ to $C_x = C_y = 7$. The dynamic aperture for 800,000 turns for $\Delta p/p = 0.005$ was again reduced by 1 mm.

A tentative conclusion is that changing ν_s and changing the chromaticity did not have an appreciable effect on the dynamic aperture. Fig. 6 compares the results found in these studies with the results found with the unchanged ν_s and chromaticity.

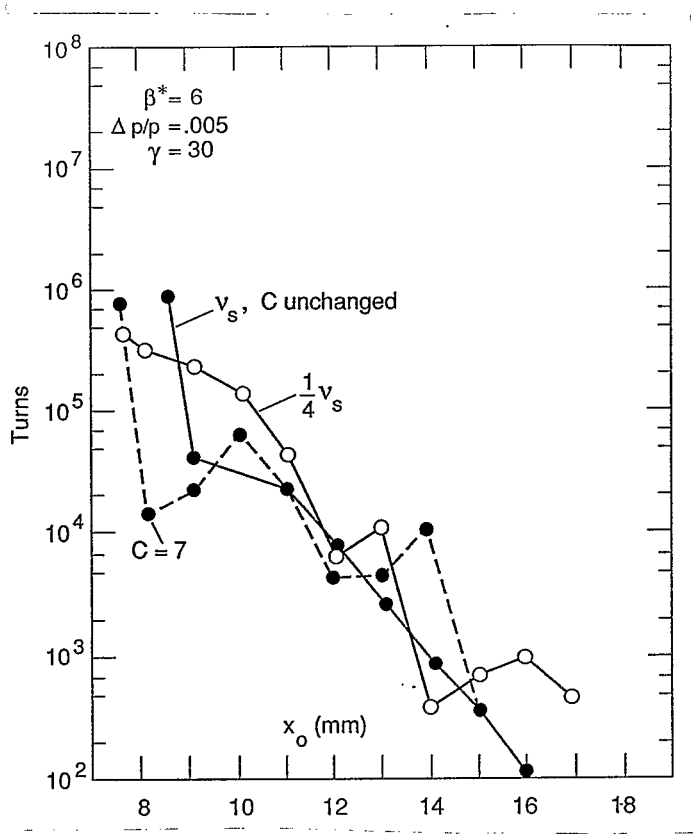

Fig. 6: Effect of changing ν_s and chromaticity.

In a third study, the RF cavity was moved from a location where the horizontal dispersion is about 1.5 m to a location where the dispersion is zero. The dynamic aperture for 800,000 turns for $\Delta p/p = 0.005$ did not change.

An interesting question is at what synchrotron oscillation amplitude, $\Delta p/p$, does the synchrobetatron coupling start to have an appreciable effect on the dynamic aperture, A_{SL}. The tracking studies indicate that it is the magnitude of $\Delta p/p$, rather than how close $\Delta p/p$ is to the edge of the RF bucket that is important. The results for the $\beta^* = 2$

lattice at $\gamma = 100$ show little decrease in A_{SL} at $\Delta p/p = 1.8 \times 10^{-3}$, although the $\Delta p/p$ is quite close to the edge of the RF bucket at $\gamma = 100$, which is at $\Delta p/p = 2 \times 10^{-3}$. The tracking results at $\gamma = 30$ and $\gamma = 100$ suggest the following criterion for the $\Delta p/p$ where the synchrobetatron coupling becomes significant,

$$X_p \frac{\Delta p}{p} \simeq A_{SL} \tag{1}$$

where X_p is some average horizontal dispersion around the accelerator. $X_p \simeq 1$ m in RHIC. Eq. (1) says that the synchrotron coupling becomes important when the particle transverse displacement due to $\Delta p/p$ is about equal to the betatron oscillation amplitude. The tracking results suggest that the loss in A_{SL} is roughly given by $X_p \Delta p/p$.

4. Reference

1. L. Schachinger and R. Talman, TEAPOT, A Thin Element Tracking Program, SSC-52 (1985).