C-A/AP/19
July 2000

Measuring "$_1$ in RHIC

M. Blaskiewicz, et al

Collider-Accelerator Department
Brookhaven National Laboratory
Upton, NY 11973

Measuring α_1 in RHIC

M. Blaskiewicz, J.M. Brennan, P. Cameron, A. Drees,
J. Kewish, T. Roser, K. Smith, C. Tang
CAD Brookhaven National Laboratory

Abstract

The chromatic nonlinearity parameter, α_1, has a strong impact on longitudinal dynamics in the vicinity of transition [1]. Measurements of the synchrotron frequency as a function of radius are used to constrain the value of α_1.

The lattice parameters α_0 and α_1 relate the change in closed orbit path length C with the reference value for the center of the beam pipe C_0 and the fractional momentum difference between the closed orbit momentum p and the reference value for the center of the beam pipe p_0 via:

$$\frac{C}{C_0} = 1 + \alpha_0 \delta \left(1 + \alpha_1 \delta\right) + O(\delta^3) \tag{1}$$

where $\delta = (p - p_0)/p_0$ is the fractional momentum difference of the closed orbit and reference orbit momenta. To get the revolution frequency one needs the change in velocity (βc) between the closed and reference orbits.

Define $u = p/mc$ then :

$$\frac{1}{\beta} = \left(1 + u^{-2}\right)^{1/2} \tag{2}$$

$$\frac{d}{du}\frac{1}{\beta} = -u^{-3}\left(1 + u^{-2}\right)^{-1/2} \tag{3}$$

$$\frac{d^2}{du^2}\frac{1}{\beta} = 3u^{-4}\left(1 + u^{-2}\right)^{-1/2} - u^{-6}\left(1 + u^{-2}\right)^{-3/2} \tag{4}$$

Since $\delta = (u - u_0)/u_0$

$$\frac{1}{\beta} = \frac{1}{\beta_0}\left(1 - \frac{\delta}{\gamma_0^2} + \frac{\delta^2}{2}\left[\frac{3}{\gamma_0^2} - \frac{1}{\gamma_0^4}\right]\right) + O(\delta^3) \tag{5}$$

The revolution period is $T = C/\beta c$ so

$$\frac{T}{T_0} = 1 + \left(\alpha_0 - \frac{1}{\gamma_0^2}\right)\delta + \delta^2\left(\alpha_0\alpha_1 - \frac{\alpha_0}{\gamma_0^2} + \frac{3}{2\gamma_0^2} - \frac{1}{2\gamma_0^4}\right) + O(\delta^3) \tag{6}$$

The data are synchrotron frequency versus radius. These were obtained with the 2GHz Schottky cavity and the values at the peaks in the synchrotron spectrum correspond to small amplitude synchrotron oscillations. Therefore, a linear expansion of the equations of motion about the stable fixed point will suffice. Define

$$\hat{\delta} = \frac{p - p_s}{p_0} = \frac{p - p_0}{p_0} - \frac{p_s - p_0}{p_0} = \delta - \delta_s \tag{7}$$

where p_s is the synchronous momentum, and let $\tau = T - T_s$ be the difference in revolution period between a particle and the synchronous particle. The experiment was done at constant magnetic field below transition so:

$$\frac{d\hat{\delta}}{dn} = \frac{qV}{\gamma_0\beta_0\beta_s mc^2}(\omega_{rf}\tau) \tag{8}$$

$$\frac{d\tau}{dn} = T_0\hat{\delta}\left(\frac{d}{d\delta}\frac{T}{T_0}\right)_{\delta\,=\,\delta_s}, \tag{9}$$

where n is the turn number. The synchrotron *frequency* is given by

$$f_s^2 = \frac{qV}{\gamma_0\beta_0 mc^2}\frac{\omega_{rf}^3}{4\pi^2h^2}\frac{T_0}{\beta_s}\left(\frac{d}{d\delta}\frac{T}{T_0}\right)_{\delta\,=\,\delta_s}, \tag{10}$$

where $h = 360$ is the harmonic number. In equation (10) only ω_{rf}, β_s and the derivative term vary with radial steering and they are tightly related since

$$\frac{\omega_{rf}}{\omega_{rf,0}} = \frac{T_0}{T_s}. \tag{11}$$

Taking the logarithm of equation (10), differentiating with respect to δ_s, and evaluating at $\delta_s = 0$ yields

$$\frac{2}{f_s}\frac{df_s}{d\delta_s} = 2\frac{\alpha_1 + \dfrac{3}{2\gamma_0^2\alpha_0} + O(\alpha_0)}{1 - \dfrac{1}{\gamma_0^2\alpha_0}} - 3\left(\alpha_0 - 1/\gamma_0^2\right) - \frac{1}{\gamma_0^2}. \tag{12}$$

In equation (12) the $O(\alpha_0)$ appearing in the numerator of the first term on the right are found in equation (6) and are neglected since they produce a very small correction. Also the second and third terms on the right of equation (12) are very small near transition and will be neglected.

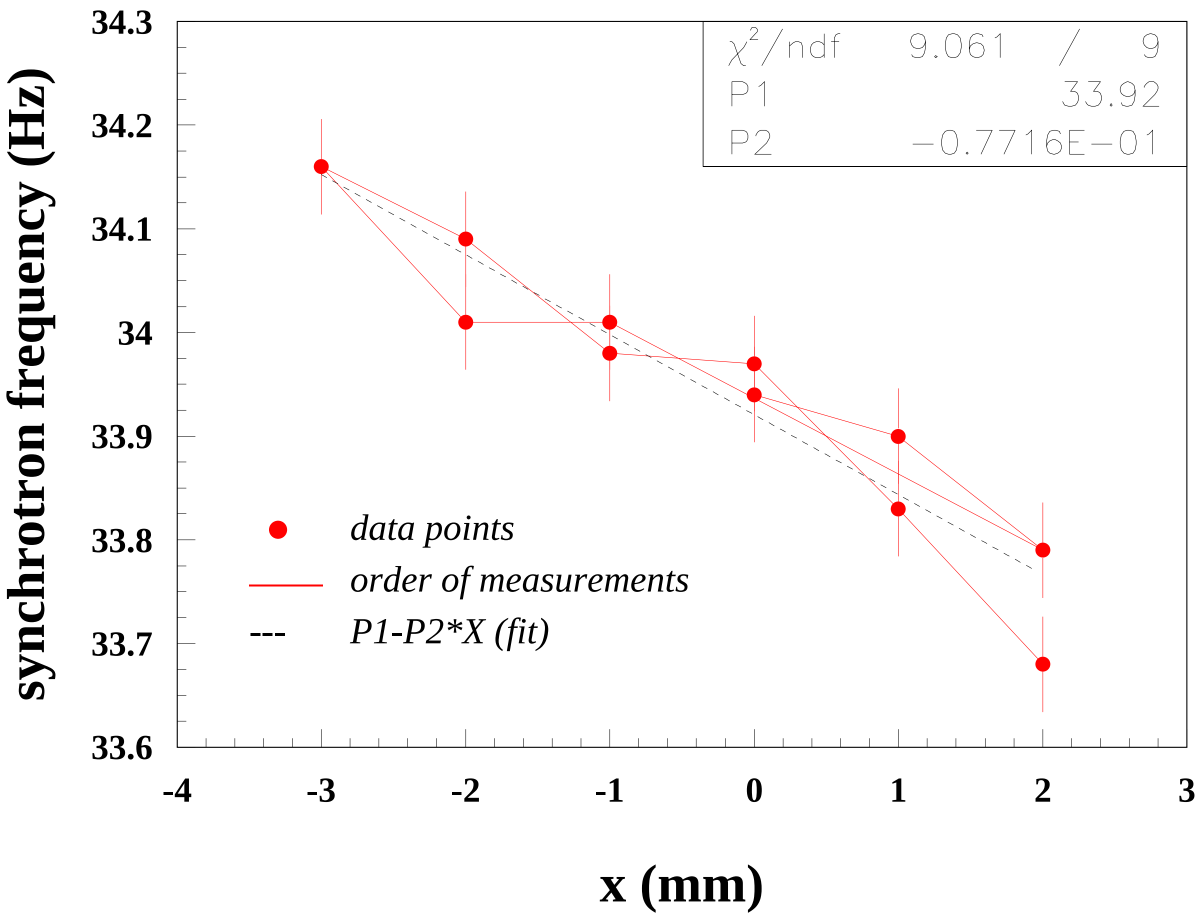

Figure 1: Measured synchrotron frequency versus radial steering setpoint. The first measurement was at $x = 0$ and the second at $x = 1$ mm. An ideal χ^2 requires 1σ errors of 0.046 Hz. The resolution bandwidth of the spectrum analyzer was 1 Hz and measurements were made using 11 synchrotron lines (10 f_s) yielding an expected error $\sim \sqrt{2}/(10\sqrt{12}) = 0.04$ Hz, where the $\sqrt{12}$ comes from a boxcar distribution and the $\sqrt{2}$ for the two independent measurements at the ends of the "picket fence".

The data for the yellow ring and a least squares fit are shown in Figure 1. The reference value for the energy was $\gamma_0 = 20$ and both the horizontal and vertical chromaticities were $\lesssim$1. Assume a bare value of $\gamma_t \equiv 1/\sqrt{\alpha_0} = 22.76$. Assuming the frequency steering is accurate at this value of gamma

$$\frac{df_s}{d\delta} = R_0 \alpha_0 \frac{df_s}{dx}, \tag{13}$$

where $R_0 = 610.2$ m is the reference radius and x is the horizontal position. From the fit to the data

$$\frac{1}{f_s}\frac{df_s}{d\delta_s} = -2.68 \pm 0.23 \tag{14}$$

Now since

$$1 - \frac{{\gamma_t}^2}{\gamma_0^2} = -0.295, \qquad \frac{3{\gamma_t}^2}{2\gamma_0^2} = 1.94,$$

one obtains the final result

$$\alpha_1 = -1.15 \pm 0.10$$

As an additional result one may obtain the values of γ_s and δ_s for which the synchrotron frequency vanishes. To leading order these are related via:

$$\frac{\gamma_s - \gamma_t}{\gamma_t} = -\delta_s(\alpha_1 + 1/2) \tag{15}$$

where $\gamma_t \equiv 1/\sqrt{\alpha_0}$ as above.

References

[1] Handbook of Physics and Accelerator Engineering, *Eds.* Chao & Tigner, World Scientific, (1998), p 92. (beware of definitions!)