Coherent Beam-Beam Effects, Theory & Observations

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Contents:

- Coherent beam-beam modes
- Coherent beam-beam resonances
- Effect of finite bunch length on coherent modes
- Interplay between impedance driven instabilities and beam-beam effect
- Schottky noise in interacting beams

Coherent beam-beam modes:

- *Vlasov perturbation theory, discreet & continuum modes*
- *Transition from weak-strong to strong-strong regime*
- *Numerical simulations*
- *Experimental observations*
- *Methods of suppression of discreet modes*
- *Multi-bunch modes & long-range collisions*

A bit of history:

1979, Piwinsky: $\nu^{coh} = 2\xi$ *(rigid uniform bunches)*

1981, Meller & Siemann: $\nu^{coh} = 1.34\xi$ *(Vlasov eq., slab geometry)*

1988, Hirata: $\nu^{coh} = \xi$ *(rigid Gaussian bunches)*

1989, Yokoya et al.: $\nu^{coh} = (1.21 \div 1.33)\xi$ *(Vlasov eq., general aspect ratio)*

Vlasov perturbation theory

Gaussian equilibrium distribution

$$F_0 = \frac{1}{(2\pi)^3 \varepsilon_x \varepsilon_y \varepsilon_s} \exp\left(-\underline{\varepsilon}^{-1} \cdot \underline{I}\right)$$

$k = 1, 2$ - beam number

$$\frac{\partial}{\partial \theta} F_1^{(k)} + \underline{\nu}^{(k)}(\underline{I}) \cdot \frac{\partial}{\partial \underline{\psi}} F_1^{(k)} = -F_0 \underline{\varepsilon}^{-1} \cdot \frac{\partial}{\partial \underline{\psi}} K_1^{(k)}(\underline{I}, \underline{\psi}; \theta)$$

includes equilibrium tuneshift

action-angle variables

$$K_1^{(k)}(\underline{I}, \underline{\psi}) = \sum_{IP} \frac{r_p N_{3-k}}{\gamma} \delta_{\mathrm{p}}(\theta - \theta_{IP}) \int G^{(k)} F_1^{(3-k)}(\underline{I'}, \underline{\psi'}) d^3 I' d^3 \psi'.$$

result of the "synchro-beam transformation" to nominal IP (Hirata, Moshammer, Ruggiero, 1992)

$$G = -\ln\left\{ [x - x' + \frac{p_x + p'_x}{2}(\sigma - \sigma')]^2 + [y - y' + \frac{p_y + p'_y}{2}(\sigma - \sigma')]^2 \right\}$$

offset

periodic phase advance $\phi_x = \mu_x - \nu_{x0} \cdot \theta$

$- \arctan[(\sigma - \sigma')/2\beta_x^*]$

$$\Delta x = (-1)^{k-1} d_x + \sigma_x \sqrt{2} \{ \sqrt{J_x} \sin[\psi_x + \phi_x^{(k)}(\theta_{IP}) - \chi\sigma - \varphi] -$$

$$- \sqrt{J'_x} \sin[\psi'_x + \phi_x^{(3-k)}(\theta_{IP}) - \chi\sigma' + \varphi] \} + \alpha(\sigma - \sigma') + D_x(\delta_p - \delta'_p),$$

$\nu'_x / (\alpha_M R)$

half crossing angle

Eigenvalue problem

$$\mathbf{f} = e^{\underline{\varepsilon}^{-1}\cdot\underline{I}/2}\begin{pmatrix}\sqrt{r_\xi}\,F_1^{(1)}\\ F_1^{(2)}\end{pmatrix} = \sum_{\underline{m}}\exp(i\underline{m}\cdot\underline{\psi})\,\mathbf{f}_{\underline{m}}(\underline{I},\theta), \qquad r_\xi = \frac{N_1}{N_2}$$

$$i\frac{\partial}{\partial\theta}\mathbf{f}_{\underline{m}} = \sum_{\underline{m}'}\hat{A}_{\underline{m},\underline{m}'}\cdot\mathbf{f}_{\underline{m}'}$$

$$\hat{A}_{\underline{m},\underline{m}'} = \begin{pmatrix}\underline{m}\cdot\underline{\nu}^{(1)} & 0\\ 0 & \underline{m}\cdot\underline{\nu}^{(2)}\end{pmatrix}\delta_{\underline{m},\underline{m}'} + \underline{m}\cdot\underline{\varepsilon}^{-1}\sum_{IP}\frac{r_p}{\gamma}\sqrt{N_1N_2}\,\delta_{\mathrm{p}}(\theta-\theta_{IP})\begin{pmatrix}0 & \hat{G}^{(1)}_{\underline{m}\underline{m}'}\\ \hat{G}^{(2)}_{\underline{m}\underline{m}'} & 0\end{pmatrix}$$

1/2π

Uncoupled modes $(\underline{m}\cdot\underline{\nu}^{(1)} + \underline{m}'\cdot\underline{\nu}^{(2)} \neq n)$

$$\hat{A}_{\underline{m},\underline{m}}\cdot\mathbf{\Psi}_\lambda = \lambda\mathbf{\Psi}_\lambda$$

Orthonormality condition

$$(\mathbf{\Psi}_\lambda, \mathbf{\Psi}_\mu) = \delta_{\lambda\mu}$$

$$(\mathbf{f},\mathbf{g}) \equiv (f^{(1)}, g^{(1)}) + (f^{(2)}, g^{(2)}) = \int(\bar{f}^{(1)}g^{(1)} + \bar{f}^{(2)}g^{(2)})\,d^3J$$

Equal intensities and tunes: Σ- and π-modes (Yokoya et al., 1989)

$$f^{(\pm)} = \frac{1}{2}(f^{(1)} \pm f^{(2)})$$

$$\frac{1}{\xi_x}\hat{A}f^{(\pm)} = Q(\underline{I})f^{(\pm)} \mp \hat{G}f^{(\pm)}$$

alone has continuous spectrum, $\lambda \in (0,1)$:
$Q(J)\delta(J-J_0) = \lambda\delta(J-J_0), \quad Q(J_0)=\lambda$

well-behaved integral operator, by itself has discrete spectrum

discrete Σ-mode (rigid-body):

$$\lambda = 0, \quad \Psi_0 = \sqrt{J_x}\, e^{-(J_x+J_y+J_s)/2}$$

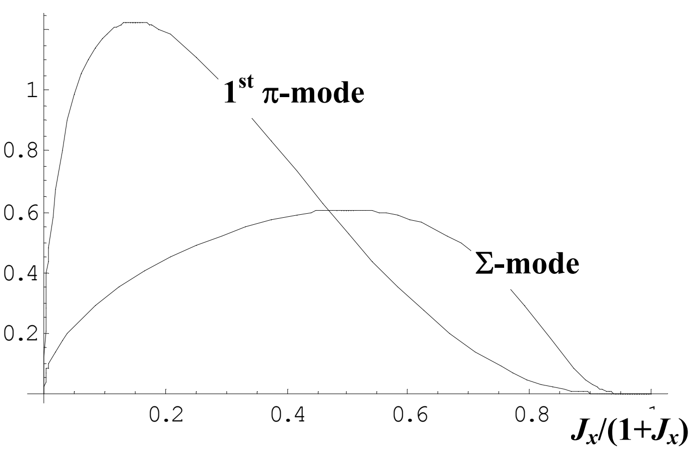
discrete π-modes:

round beams: $\lambda = 1.214$

flat beams (hor.): $\lambda = 1.330,\ 1.026,\ 1.002$

flat beams (ver.): $\lambda = 1.239$

+ *continuum* (0, 1) ***in all cases***



Discrete modes of horizontal oscillations in flat beams

Spectrum of oscillations excited by a dipole kick:

Spectral coefficients: $$c_k(\lambda) = (\Psi_0, \Psi_\lambda^{(k)}), \quad k = 1,2$$

Spectral density of center-of-mass oscillations in beam k after a kick at beam j:

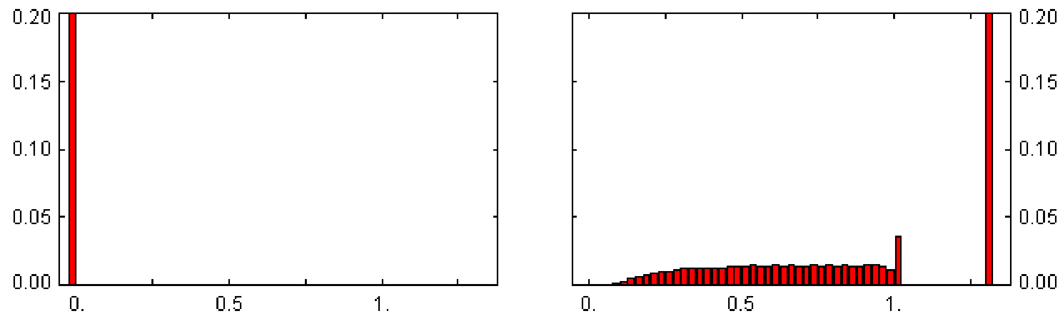
$$s_{kj}(\lambda) = r_\xi^{(k-j)/2} c_k(\lambda) c_j(\lambda) \frac{dw(\lambda)}{d\lambda}$$

Stieltjes integrating function:

$w(\lambda+0) - w(\lambda-0) = 1, \quad \lambda \in \boldsymbol{P},$

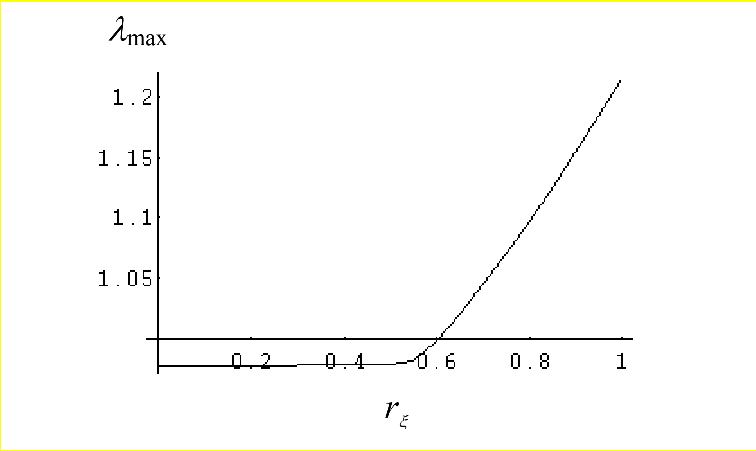
$dw(\lambda)/d\lambda = 1, \quad \lambda \in \boldsymbol{C},$

$dw(\lambda)/d\lambda = 0, \quad \lambda \notin \boldsymbol{C}, \boldsymbol{P}.$



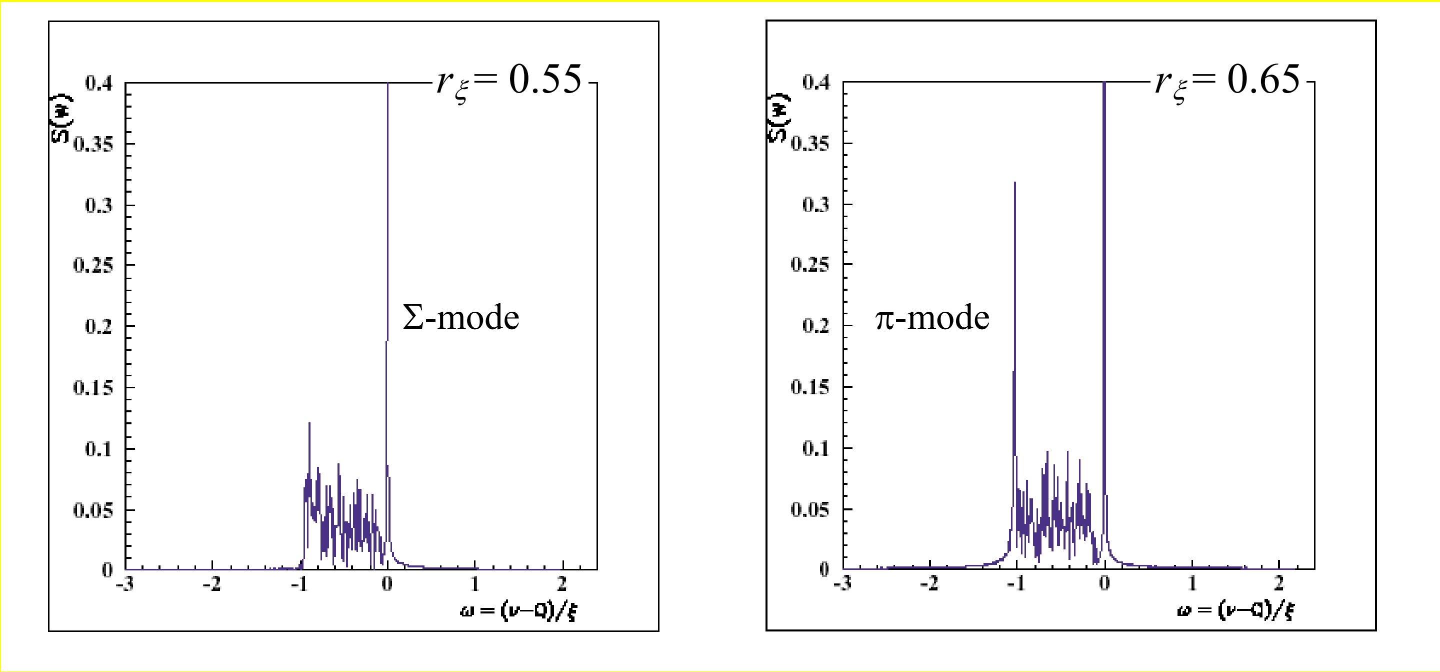
Spectral density of horizontal Σ (left) and π (right) oscillations in flat beams

Transition from weak-strong to strong-strong regime



In round beams the discrete π-mode emerges from the continuum at intensity ratio r_ξ=0.6 (analytics, YA, 1996)

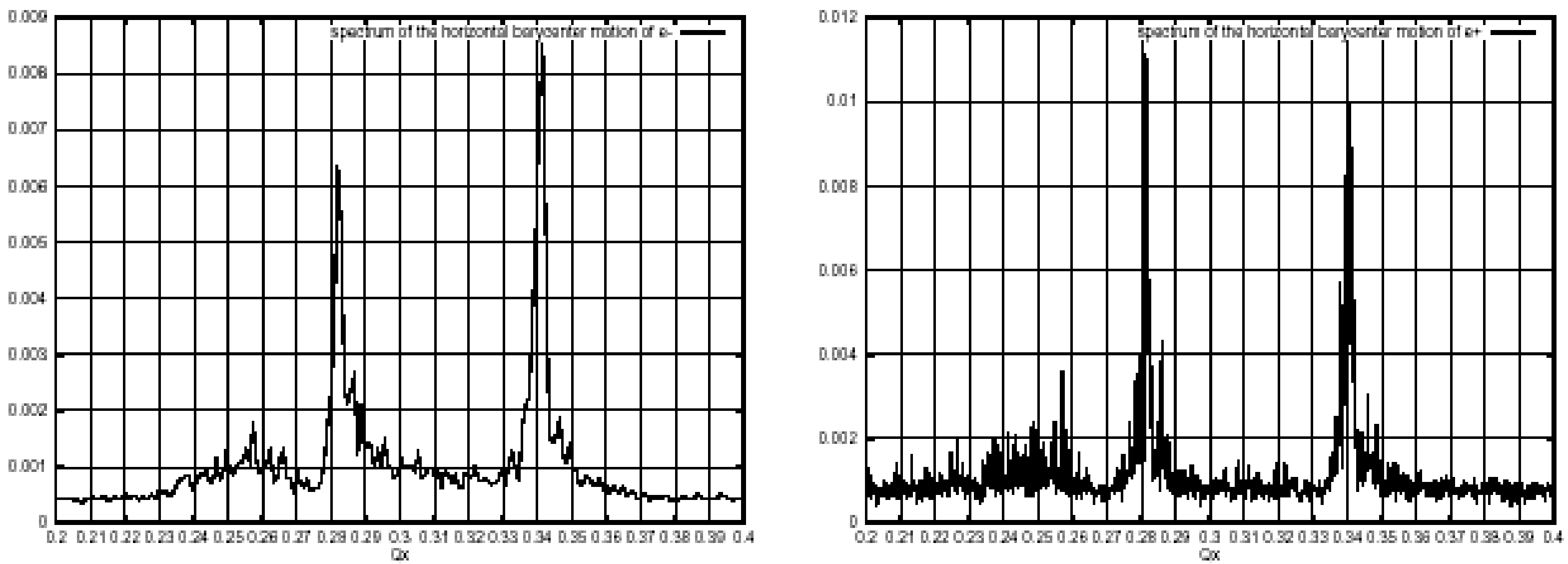
Simulation by the Hybrid Fast Multipole Method (Herr, Jones & Zorzano, 2001) confirms this result



Experimental observations

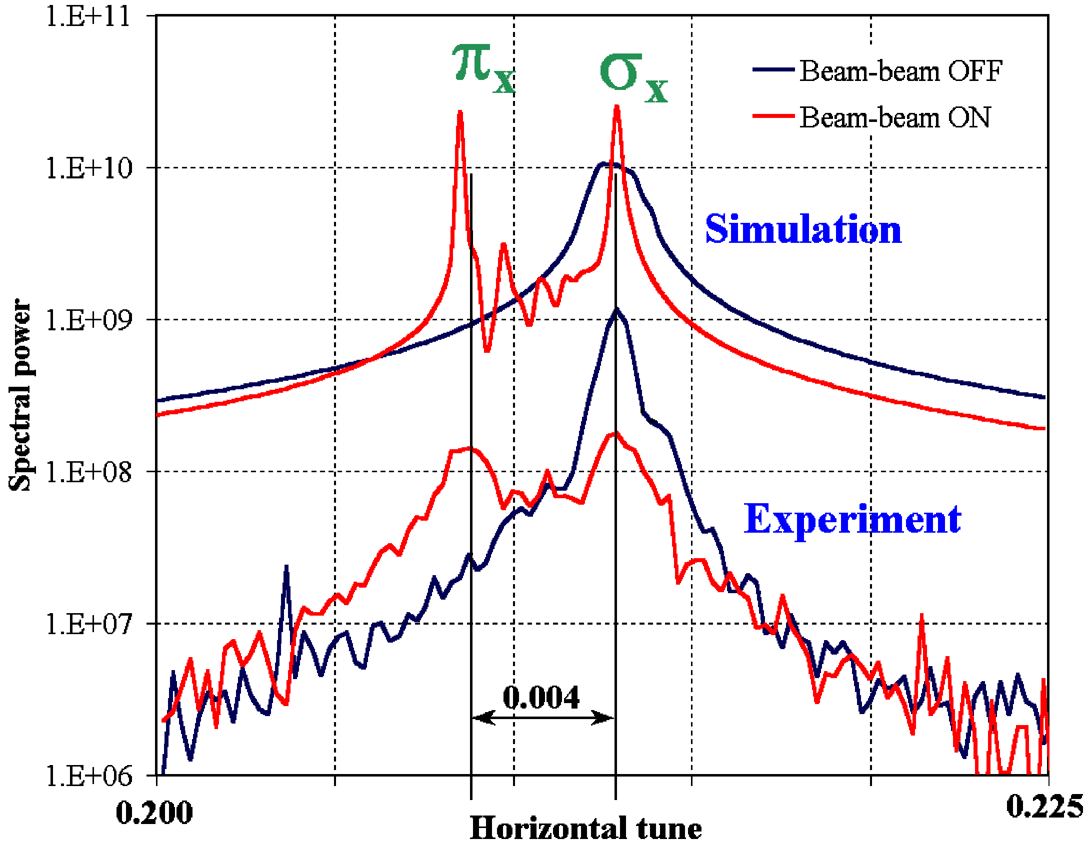
TRISTAN: ***precise measurements of λ (K.Yokoya et al., 1989)***

LEP:



Spectra of horizontal oscillations in LEP of two bunches colliding at two IPs: left – electron beam, right – positron beam (courtesy of G.Morpurgo)

RHIC:



Spectra of two colliding p-bunches in RHIC (courtesy of W.Fischer)
Simulations by M.Vogt et al. (2002)

Measured
Σ-π tunesplit:

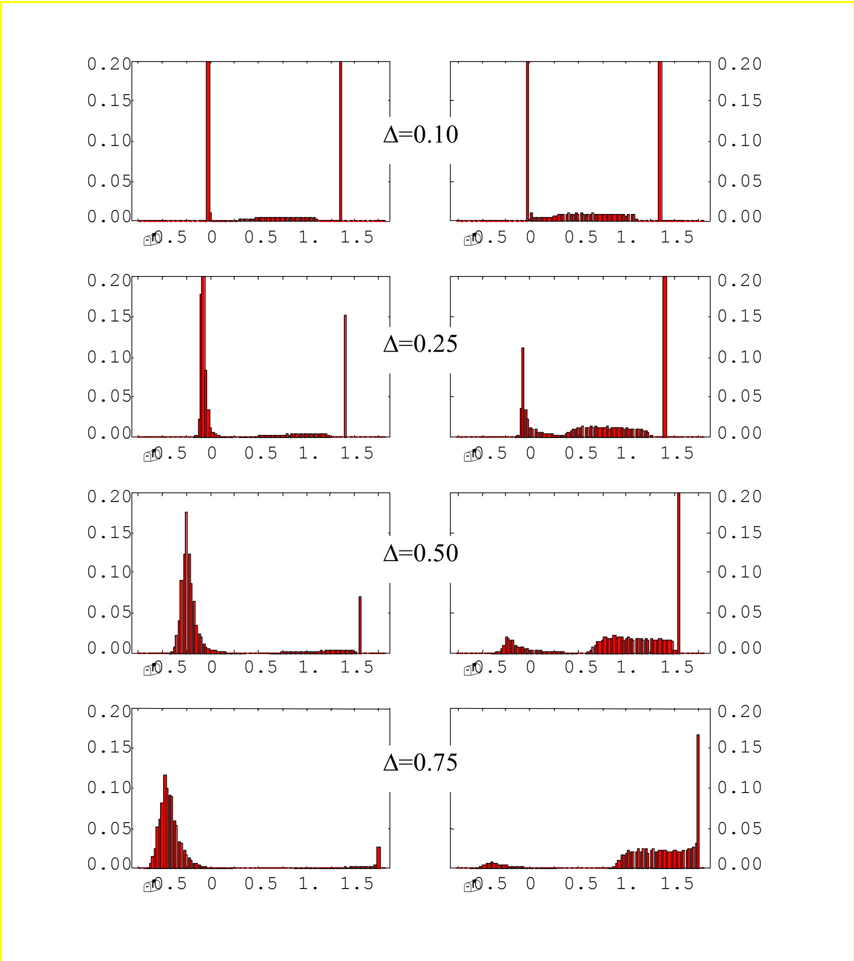
0.004

Expectation:

$1.214\,\xi_x$=0.0036

Methods of suppression of discreet modes

- *Splitting bare lattice tunes (A.Hoffman)*
- *Redistribution of phase advances between IPs (A.Temnykh, J.Welch)*
- *Different parity of integer parts of the tunes in separate rings (W.Herr)*

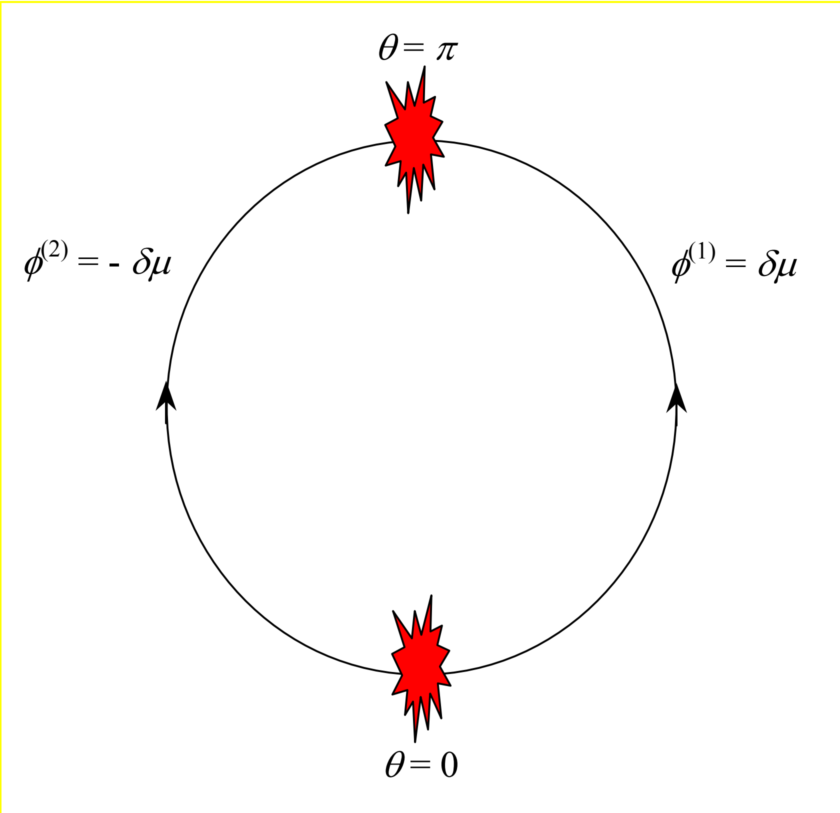


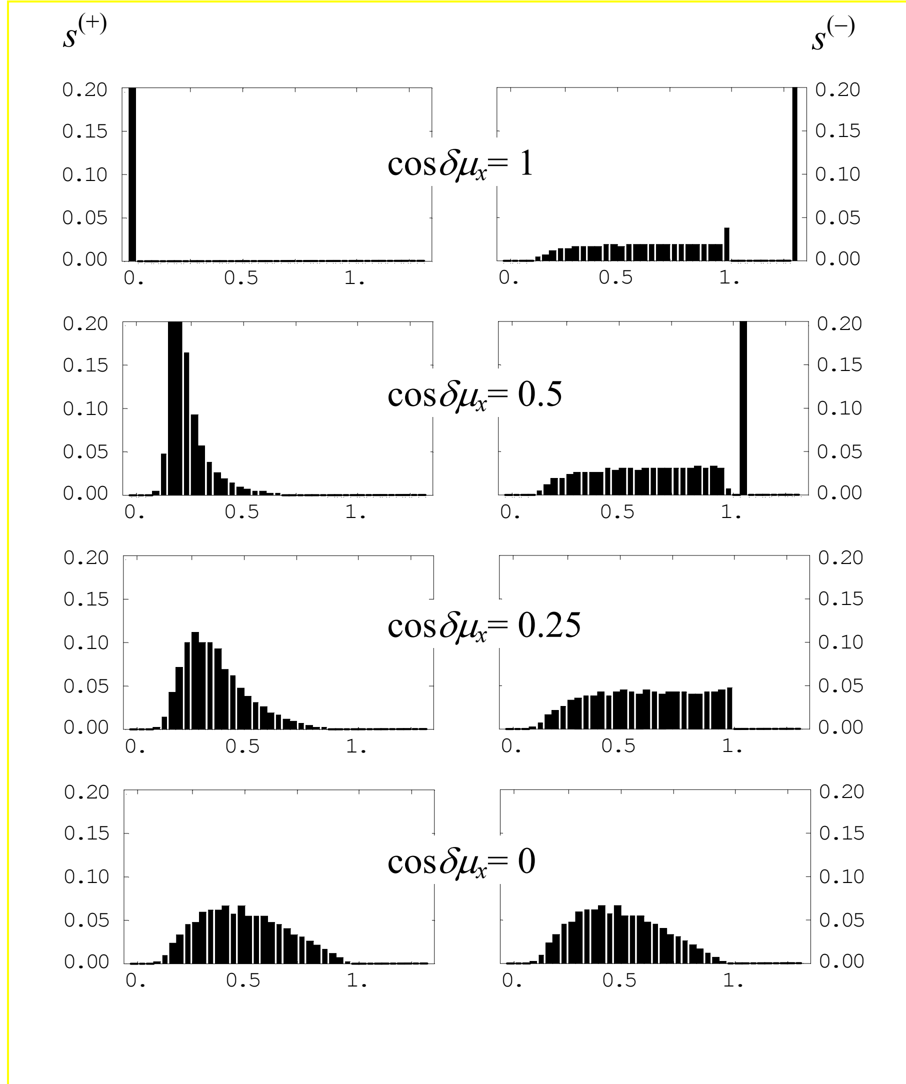
Effect of tunesplit

$$\nu_{x0}^{(2)} - \nu_{x0}^{(1)} = 2\Delta$$

in flat beams

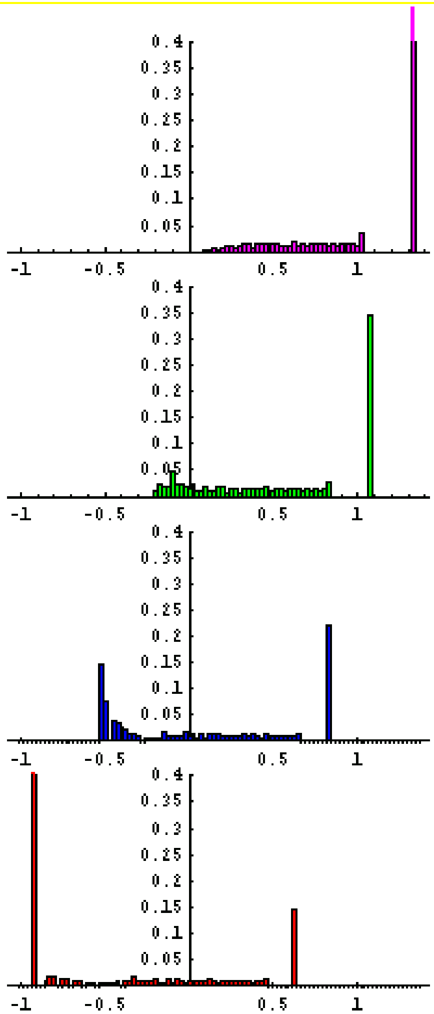
Redistribution of phase advances between IPs



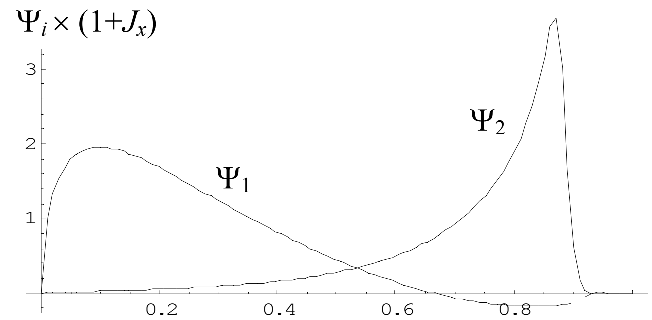


With $\delta\mu_x = \pi/2$ discrete modes are completely surpressed, the same effect would have integer tunesplit

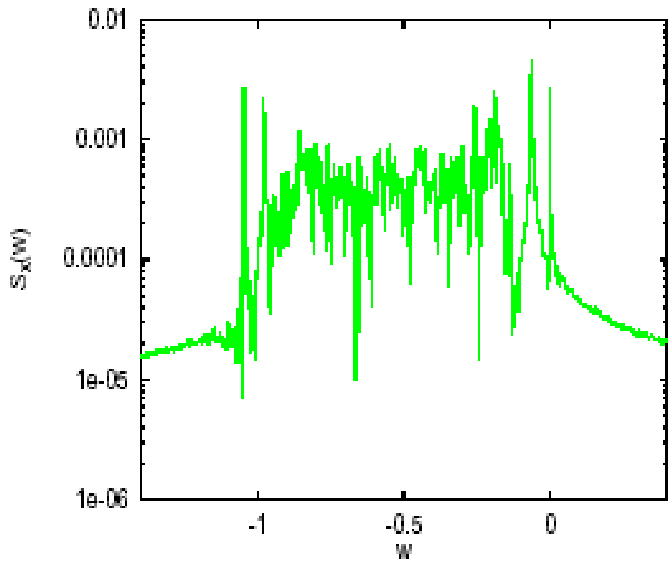
Long-range interactions



π-mode with N_{LR} = 0, 4, 8, 12 lumped long-range interactions at separation $d = 5\sigma_x$

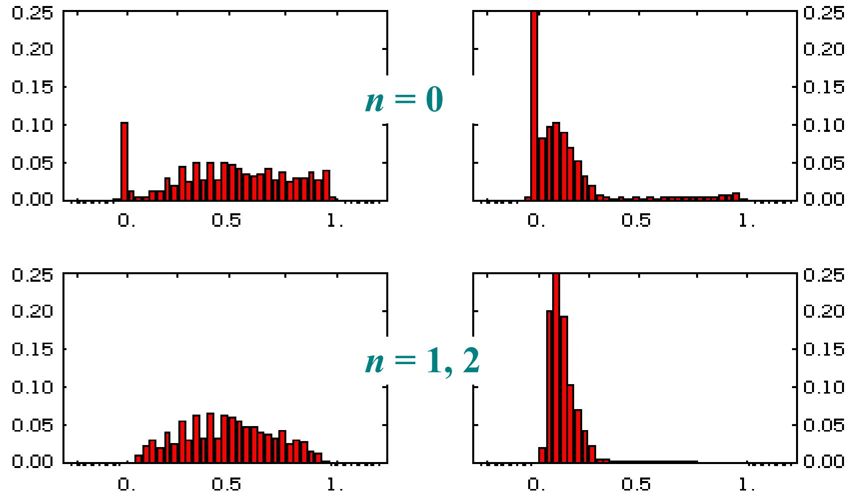


Two eigenfunctions from the left plot at N_{LR} = 12; λ_1 = 0.613, λ_2 = - 0.928.



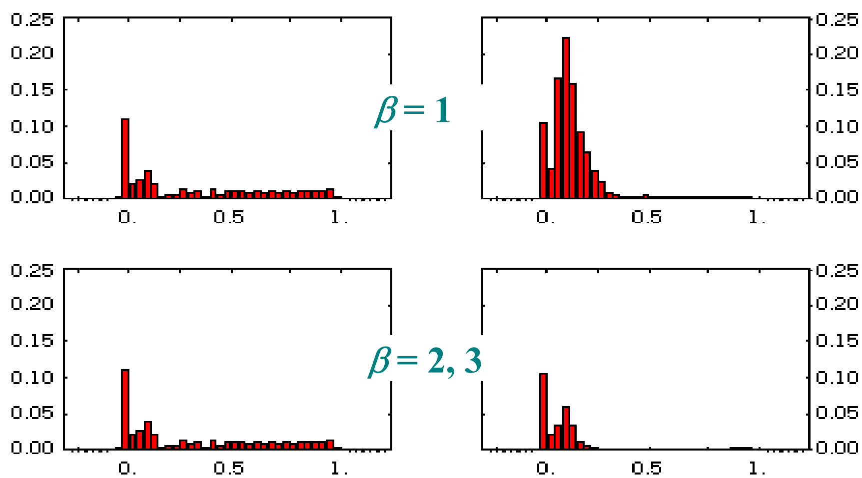
3 head-on and 1 halo ($d = 4\sigma_x$) collisions in LHC (soft-Gaussian model, W.Herr, M.-P.Zorzano, 2001)

Multi-bunch modes in Tevatron (3×3colliding head-on at 2 IPs)



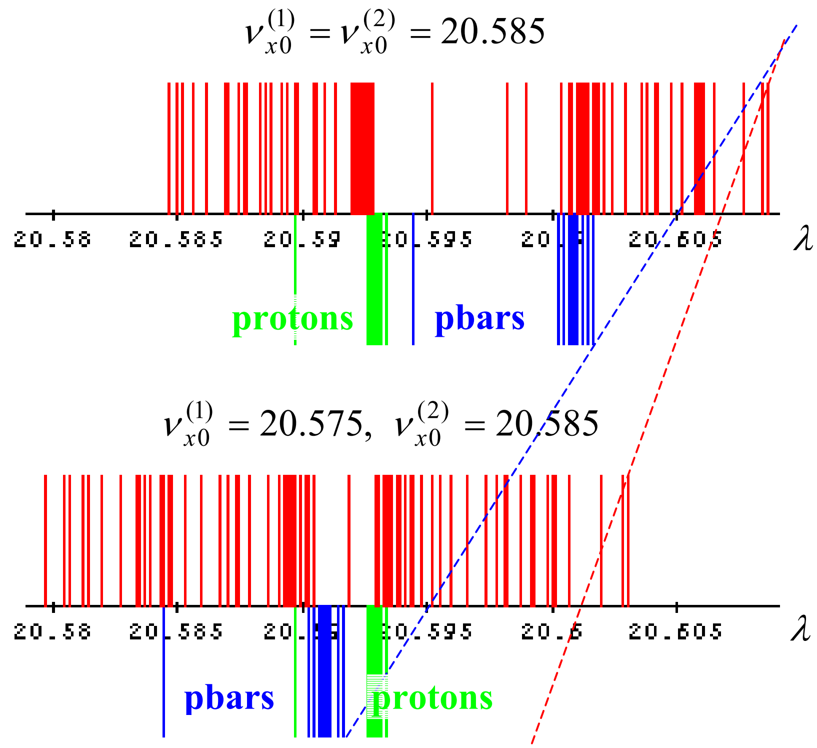
Spectra of the normal modes as seen in the weak (left) and strong (right) flat beams with $r_\xi = 0.3$ and $\nu_s/\xi_x = 0.05$, $\sigma_s/\beta_x^* = 1$, $\chi = 0$.

$n = 0$ – fundamental modes
$n = 1,2$ – intermediate modes



Spectra of oscillations in the bunches of the weak (left) and strong (right) beams after a dipole kick at the first bunch of the strong beam.

Multi-bunch modes in Tevatron 36× 36 bunches



Spectral lines of rigid bunch oscillations (red) and average values of incoherent tunes in proton bunches (green) and pbar bunches (blue) with intensity ratio r_ξ = 0.5

Each bunch collides at 2 head-on and 70 LR IPs:

$$\xi^{(\text{pbar})} = 2\times 0.01 + .005 = 0.025$$

With equal tunes the utmost coherent line lies within the incoherent pbar tunespread.

With pbars tunes shifted down by 0.01 the utmost line shifts only by ~0.006 and gets out of the pbar tunespread.

Coherent beam-beam resonances

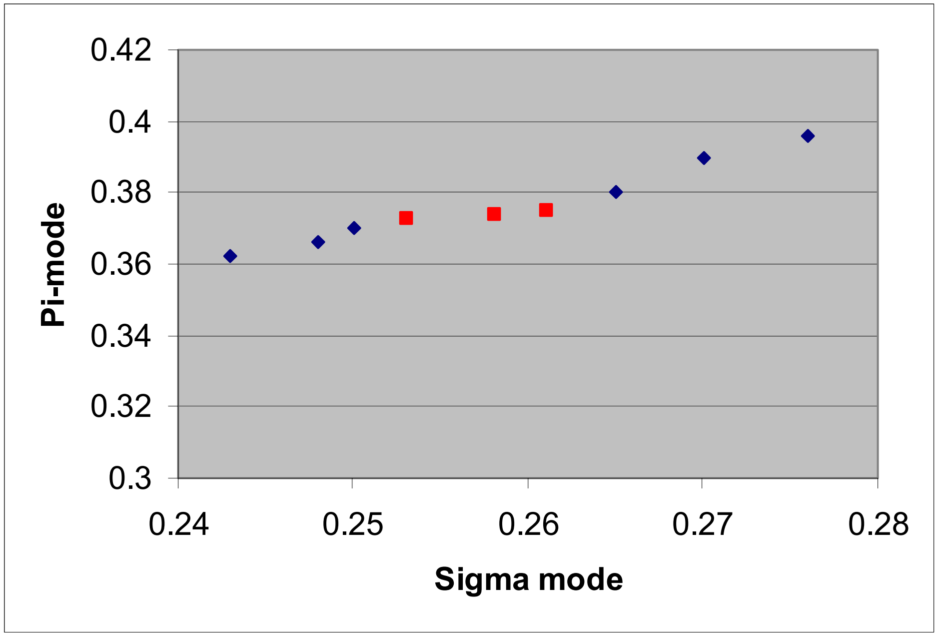
$$\underline{m}\cdot\underline{\nu}^{(1)}+\underline{m}'\cdot\underline{\nu}^{(2)}=n$$

- coupling of coherent oscillations

dipole + dipole $(m=m'=1)$: at $\nu=n/2$

dipole + quadrupole $(m=1,\ m'=2)$: at $\nu=n/3$ ***- due to LR or offsets***

and so on



Spontaneous excitation of π-mode observed in LEP (courtesy of K.Cornelis)

Explained (YA, 1999) by coupling of dipole π-mode

$$\nu=\nu_0+1.33\xi$$

to quadrupole Σ-mode

$$2\nu\approx 2\nu_0+\xi$$

$$\nu_0\approx\frac{n}{3}-\frac{1+1.33}{3}\xi_x\approx .265\quad \text{at}\ \ \xi_x=0.088$$

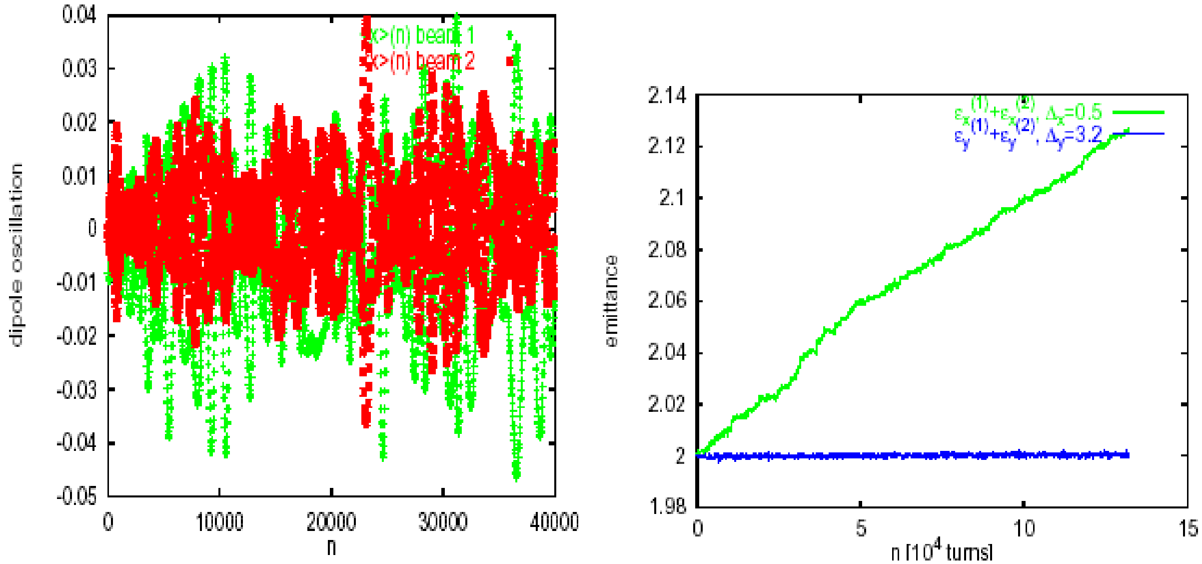
Specific case of different working points

To suppress discrete modes in LHC combinations of tunes considered

$$\nu_{x1} = 0.232,\ \nu_{x2} = 0.310,\ \nu_{x3} = 0.385$$

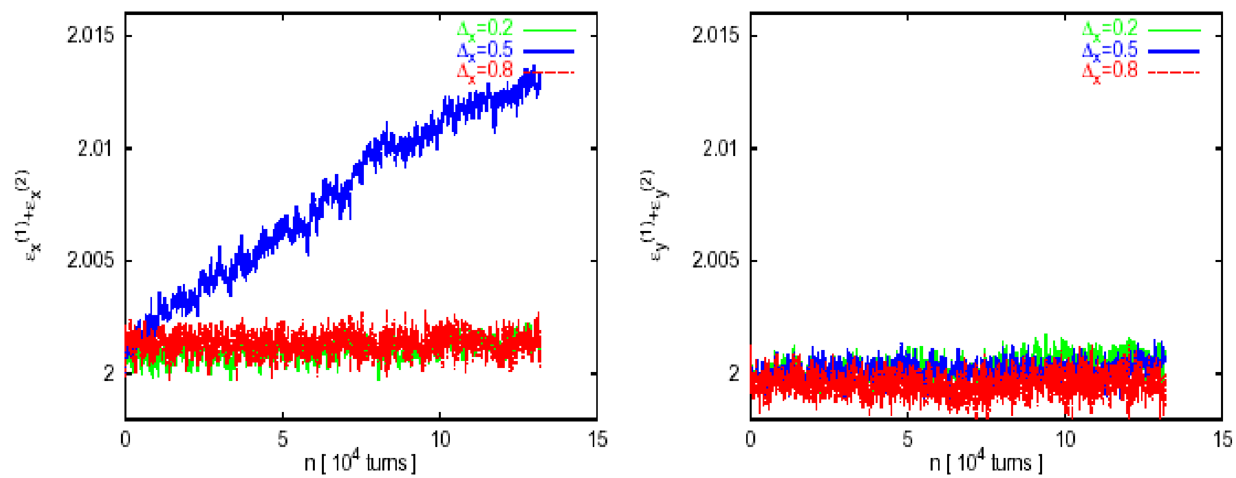
The following resonances can be encountered

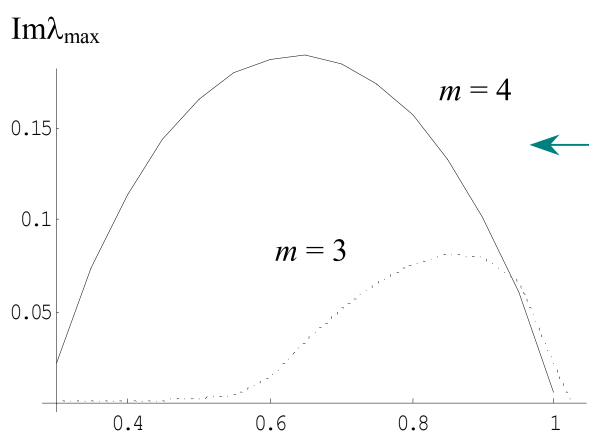
$$3\,\nu_{x1} + \nu_{x2} = 1.006; \quad 2\,\nu_{x2} + \nu_{x3} = 1.005; \quad \nu_{x1} + 2\,\nu_{x3} = 1.002$$



Tracking simulation (soft-Gaussian model) of the dipole-quadrupole resonance at an offset of 0.3σ_x (M.-P. Zorzano, 2000). Growth of dipole oscillations saturates (Landau damped?) at the expense of emittance growth. Such behavior was predicted analytically by S.Heifets (1999).

In absence of offset 6th order resonance (octupole-quadrupole) shows up!

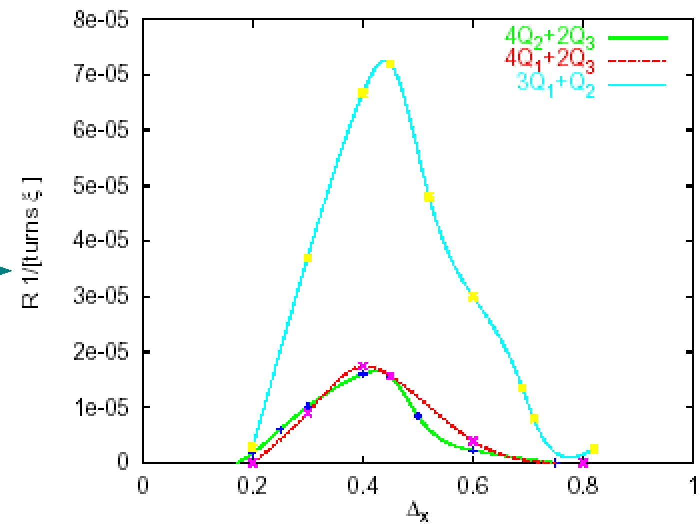




stopbands:

analytics

tracking



Effect of finite bunch length on coherent modes

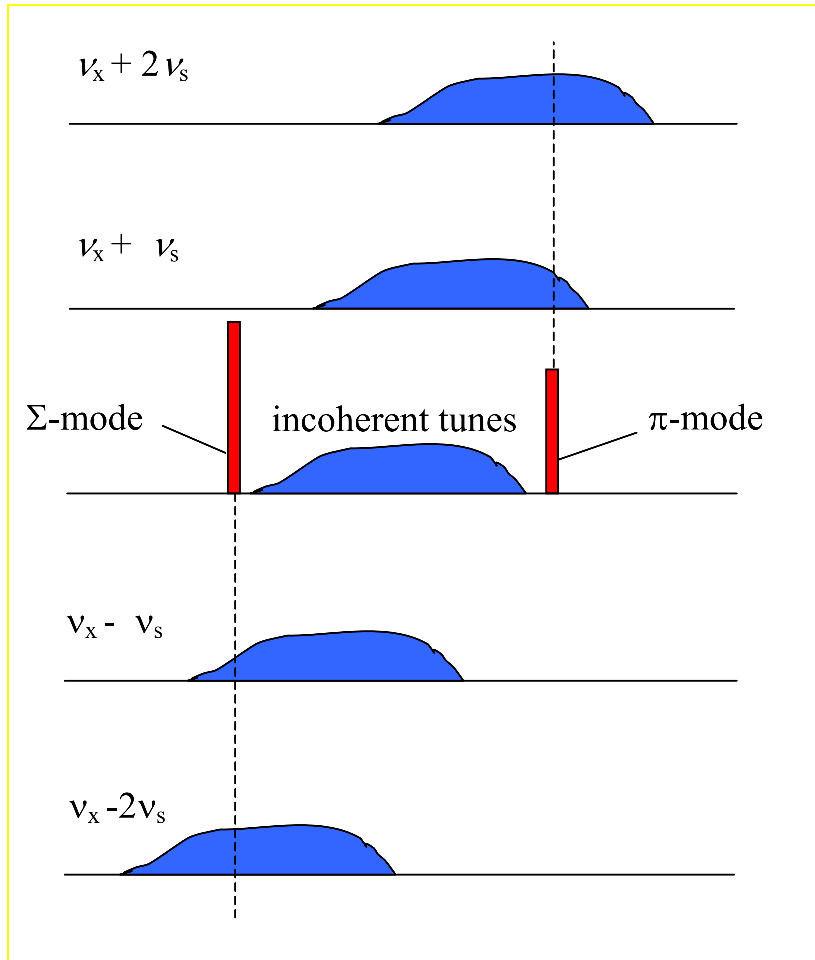
***Sources of synchro-betatron coupling*:**

- ***betatron phase variation along interaction region ("finite length effect")***
- ***chromaticity***
- ***finite crossing angle***
- ***dispersion at IP***

- all reduce coherence of oscillations

(angle $2\alpha \to 1.4\sigma_s / \sigma_{x\beta}$: $\lambda = 1.33 \to 1.21$ ***)***

- introduce coupling to synchrotron sidebands of incoherent tunes – may provide Landau damping.



For short bunches "finite length" and chromaticity combine in parameter

$$\kappa = (\nu_x' / \alpha_M R - 1/\beta^*)^2 \sigma_s^2$$

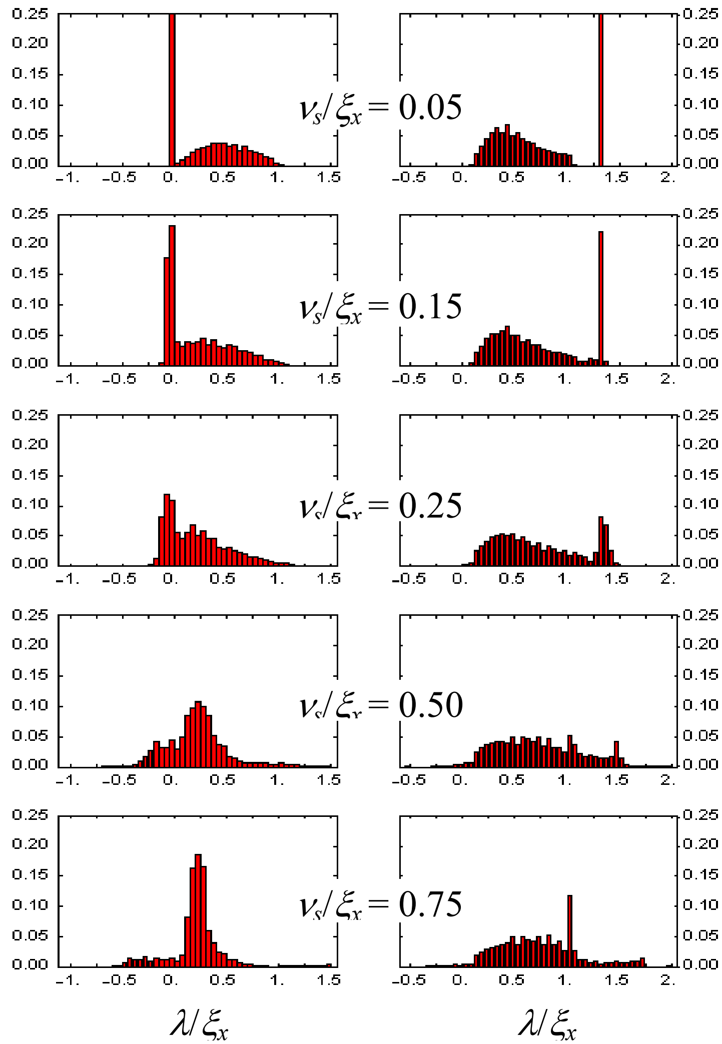
- possibility of cancellation!

Landau damping in long bunches

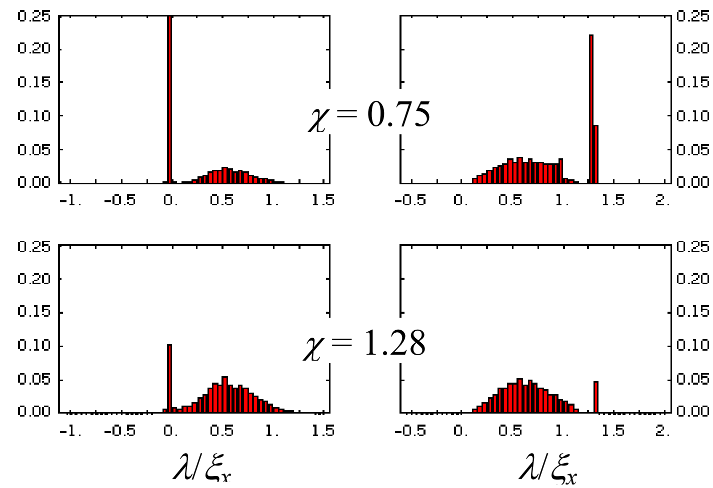
effect of the synchrotron tune

effect of chromaticity

$$\chi = \nu'_x / (\alpha_M R)$$



Spectral density of Σ (left) and π (right) modes in long bunches ($\sigma_s = \beta_x^*$) at $\chi = 0$.

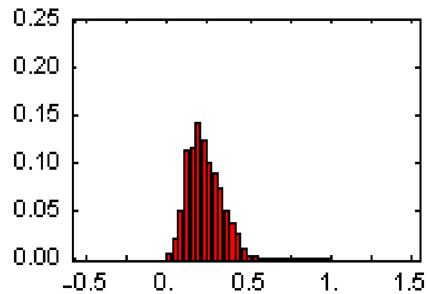


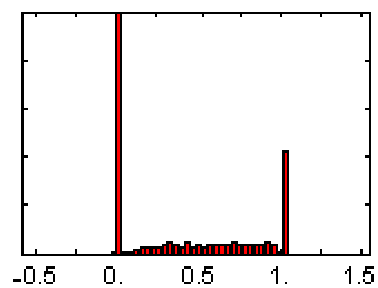
Spectral density of Σ and π modes at $\nu_s/\xi_x = 0.15$, $\sigma_s/\beta_x^* = 1$.

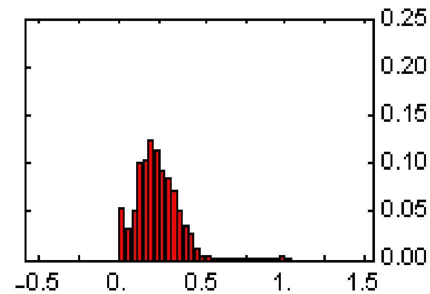
Predictions for Tevatron Run II Upgrade

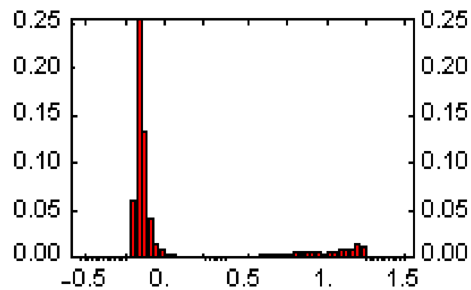
chromaticity:

$\nu_x' = -5$ $\qquad\nu_x' = +5$ $\qquad\nu_x' = +15$









moderate tunesplit

$\nu_x^{\,\mathbf{(pbar)}} - \nu_x^{\,\mathbf{(proton)}} = \xi/2$

restores Landau damping

flat beams, $\sigma_s/\beta^* = 50/35$, $\nu_s/\xi = 0.035$, $\xi = 0.02$ (two IPs), $r_\xi = 0.5$

Interplay between impedance driven instabilities and beam-beam effect

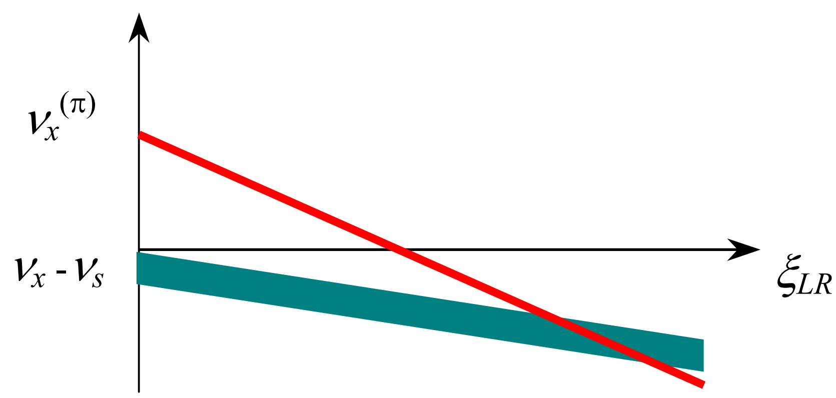
- *Aggravation of TMCI by LR interactions (LEP)*
- *Landau damping by the beam-beam tunespread*
- *Blow-up of the weak beam by coherent oscillations of the strong one (Tevatron)*

TMCI in LEP 8× 8 operation

- coupling of ν_x π*-mode and* $\nu_x - \nu_s$ *synchro-betatron mode*

- threshold ~30% lower for 8× 8 than for 4× 4 bunches

- tentative explanation: twice larger tuneshift of the coherent π*-mode (YA, 1996)*



- another possibility: larger impedance on pretzel orbits

Landau damping by the beam-beam tunespread

Large gap between π-mode and continuum may switch off Landau damping in the strong-strong regime (J.Gareyte, 1989)

What is really the case in the weak-strong regime?

Dispersion relation for arbitrary intensity ratio r_ξ (YA, unpublished):

$$(1-\omega_1 r_\xi \int \frac{c_1^2 dw(\mu)}{\lambda-\mu})(1-\omega_2 \int \frac{c_2^2 dw(\mu)}{\lambda-\mu}) = \omega_1 \omega_2 r_\xi (\int \frac{c_1 c_2 dw(\mu)}{\lambda-\mu})^2$$

$\omega_{1,2}$ – coherent tuneshifts (in units of ξ_{x1}) the strong beam would see alone

$$\lambda = (\nu_x - \nu_{x0})/\xi_{x1}$$

$$\int [c_1^2(\lambda) + c_2^2(\lambda)] dw(\lambda) = 2,$$

$$\int c_1(\lambda) c_2(\lambda) dw(\lambda) = 0$$

= 0 ***for uncoupled beams, e.g. due to large tunesplit***

Special case of equal bare lattice tunes and impedances:

$$c_1(0) = \sqrt{\frac{r_\xi}{1+r_\xi}}, \quad c_2(0) = \frac{1}{\sqrt{1+r_\xi}}$$

- rigid Σ-mode

$$c_1(\lambda) = -\frac{1}{\sqrt{r_\xi}} c_2(\lambda), \quad \lambda \neq 0$$

$$\int_{\lambda \neq 0} c_2^2(\lambda) dw(\lambda) = \frac{r_\xi}{1+r_\xi}$$

$$\boxed{(1 - 2\omega \int_{\mu \neq 0} \frac{c_2^2(\mu) dw(\mu)}{\lambda - \mu})(1 - \frac{1+r_\xi^2}{1+r_\xi}\frac{\omega}{\lambda}) = \frac{(1-r_\xi)^2}{1+r_\xi}\frac{\omega^2}{\lambda}\int_{\mu \neq 0} \frac{c_2^2(\mu) dw(\mu)}{\lambda - \mu}}$$

For $r_\xi = 1$ equations for π- and Σ-modes decouple

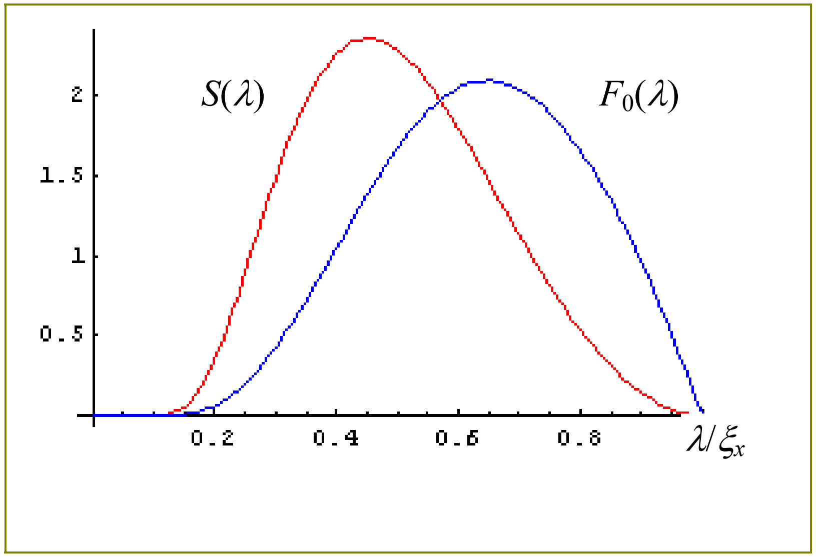
For $r_\xi << 1$ the rigid Σ-mode $\lambda \approx \omega$ is undamped, something else is needed for stability (tunesplit, overlapping sidebands, etc.)

Landau damping in the case of weak coupling (due to tunesplit)

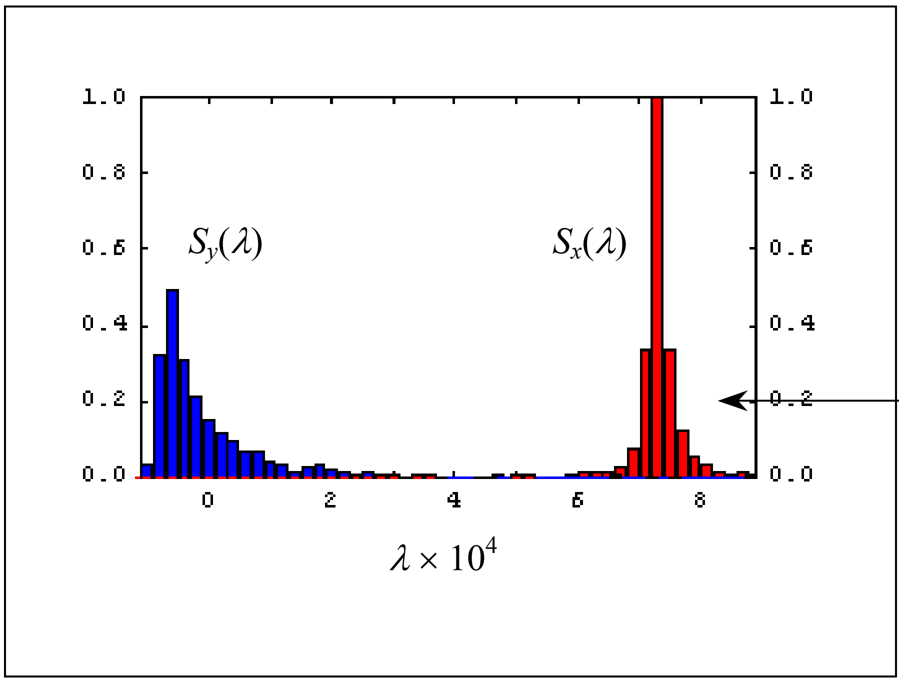
Flat beams:

$$S(\lambda) \equiv c_2^2(\lambda) = \frac{I_\lambda F_0(I_\lambda)}{|\nu_x'(I_\lambda)|}, \quad \nu_x(I_\lambda) = \lambda$$

Round beams:



Tevatron flattop tunespreads



horizontal tunespread is not sufficient