Stability Issues of Low Energy Intense Beams

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(June 28, 1999)

— Longitudinal Space charge
- Inductor insertion
- power lost to inductor (ferrite)
- Transient beam loading

— Transverse space charge
- coherent and incoherent tune shifts
- mode coupling instability with space charge

— Short bunch production near transition
- microwave instability near transition

Longitudinal space charge

Fermilab future 1st ring booster (old design)

K.E.	1 GEV
γ	2.0658
Cycling rate	15 Hz
Circumference	180.649 m
rf harmonic	2
# of bunches	2
# per bunch	$0.5 \times 10^{+14}$
Bucket bunching factor	0.25
$\eta = -0.2139 \leftarrow$ Transition γ_t	7
95% bunch area	1.0 eV-s.
95% normalized emittance	200×10^{-6}

Laslett timeshift

~ -0.393

$\langle \beta \rangle = 25\,m \qquad \langle D \rangle = 1.8\,m \qquad \left.\begin{array}{l} \hat{\delta} = 3.32 \times 10^{-3} \\ \hat{\tau} = 64.56\ ns \end{array}\right\}$ parabolic distribution

av. beam radius ~ 5.29 cm.

beam pipe radius ~ 8 cm

$$\left.\frac{Z''}{n}\right|_{sp\,ch} = i\,\frac{Z_0}{2\gamma^2\beta}\left(1 + 2\ln\frac{b}{a}\right) = 92.1\,\Omega$$

This is not so bad for microwave instability because Keil-Schnell limit is

$$\left|\frac{Z^{\parallel}}{n}\right| < F_{\parallel} \frac{E|\eta|}{e\beta^2 I_{pk}} \left(\frac{\Delta E}{E}\right)_{FWHM} = 75.3\ \Omega$$

$I_{AV} = 23.27$ amp $\quad I_{pk} = 93.06$ amp.

<u>Potential-Well distortion</u>

Ignoring space charge

$$\nu_s = \frac{|\eta|\hat{\delta}}{\omega_0 \hat{\tau}} = 0.001207$$

$$V_{rf} = \frac{2\pi\beta^2 E \nu_s^2}{|\eta| h} = \boxed{31.73\ kV}$$

A particle at time advance τ sees

$$V_{rf}\sin(-h\omega_0\tau) \approx V_{rf}\left(\frac{3\pi B}{2}\right)\frac{\tau}{\hat{\tau}} = -37.38\ \frac{\tau}{\hat{\tau}}\ kV$$

But space charge field $E_z^{spch} = -\frac{e Z_0}{4\pi\beta^2\gamma^2 c}\left(1 + 2\ln\frac{b}{a}\right)\frac{d\lambda}{d\tau}$

$\therefore$ Voltage seen by same particle per turn is

$$V_{spch} = E_z^{spch} C = \frac{3\pi I_b}{(\omega_0\hat{\tau})^2}\left|\frac{Z_0''}{h}\right|\frac{\tau}{\hat{\tau}} = +29.1\ \frac{\tau}{\hat{\tau}}\ kV$$

$\therefore$ To maintain the bunch V_{rf} needs to increase to

$$V_{rf} = \frac{37.38 + 29.1}{37.38} \times 31.73\ kV = \boxed{56.42\ kV}$$

If we do not want such a large V_{rf}, we need to cancel the sp-ch by, e.g., inductor insertion.

1. Fermilab-Los Alamos Collaboration

   Fermilab report FN-661 (1997)

   Phys. Rev. ST Accel Beam (accepted)

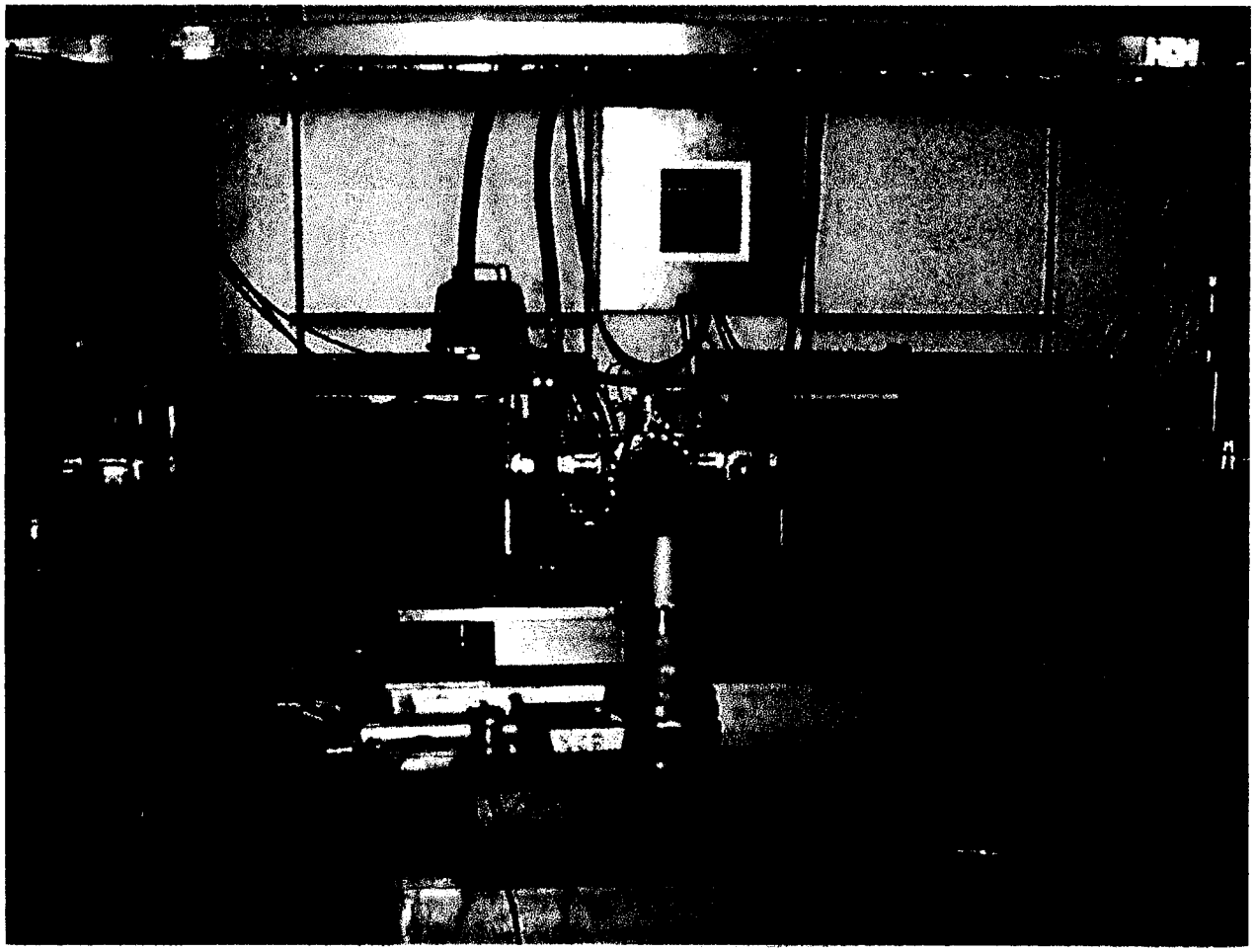
   1. bunch lengthened when ferrite was biased
   2. V_{rf} required for bunching reduced by $\frac{1}{3}$
   3. cleanest gap ever seen.

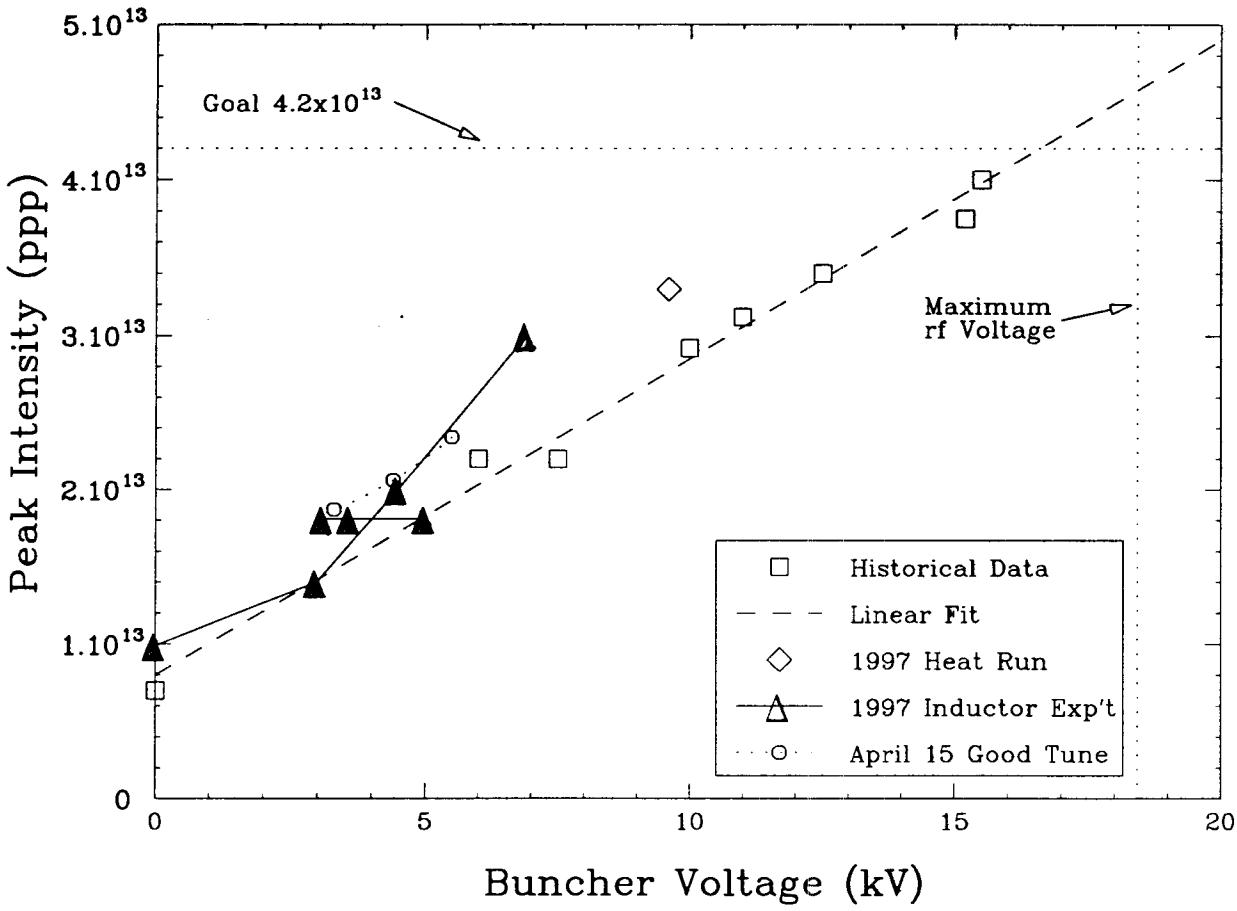
2. KEK experiment by Kiyomi Koba (Finemet)

   measure coherent synchrotron quadrupole frequency shift vs. bunch intensity.

   Result: with inductor (Finemet), the shift was smaller

Beam Loading Studies
1997 FNAL-LANL Inductor Experiment (cont.)




5.10^13
4.10^13
3.10^13
2.10^13
1.10^13
0
Peak Intensity (ppp)
Goal 4.2x10^13
Maximum rf Voltage
Historical Data
Linear Fit
1997 Heat Run
1997 Inductor Exp't
April 15 Good Tune
0
5
10
15
20
Buncher Voltage (kV)

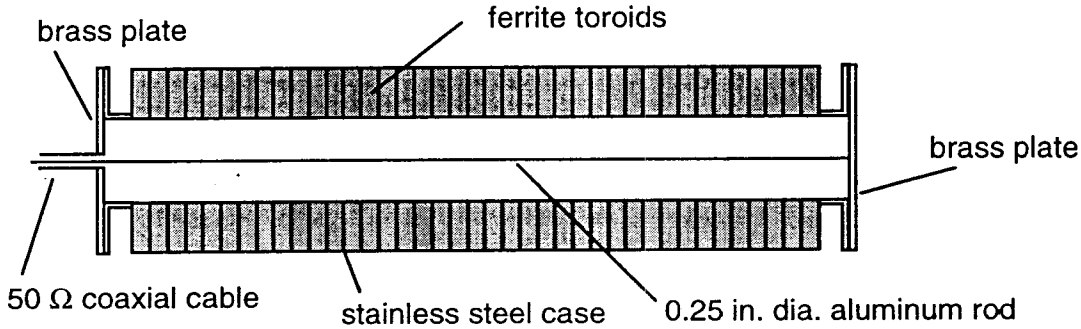
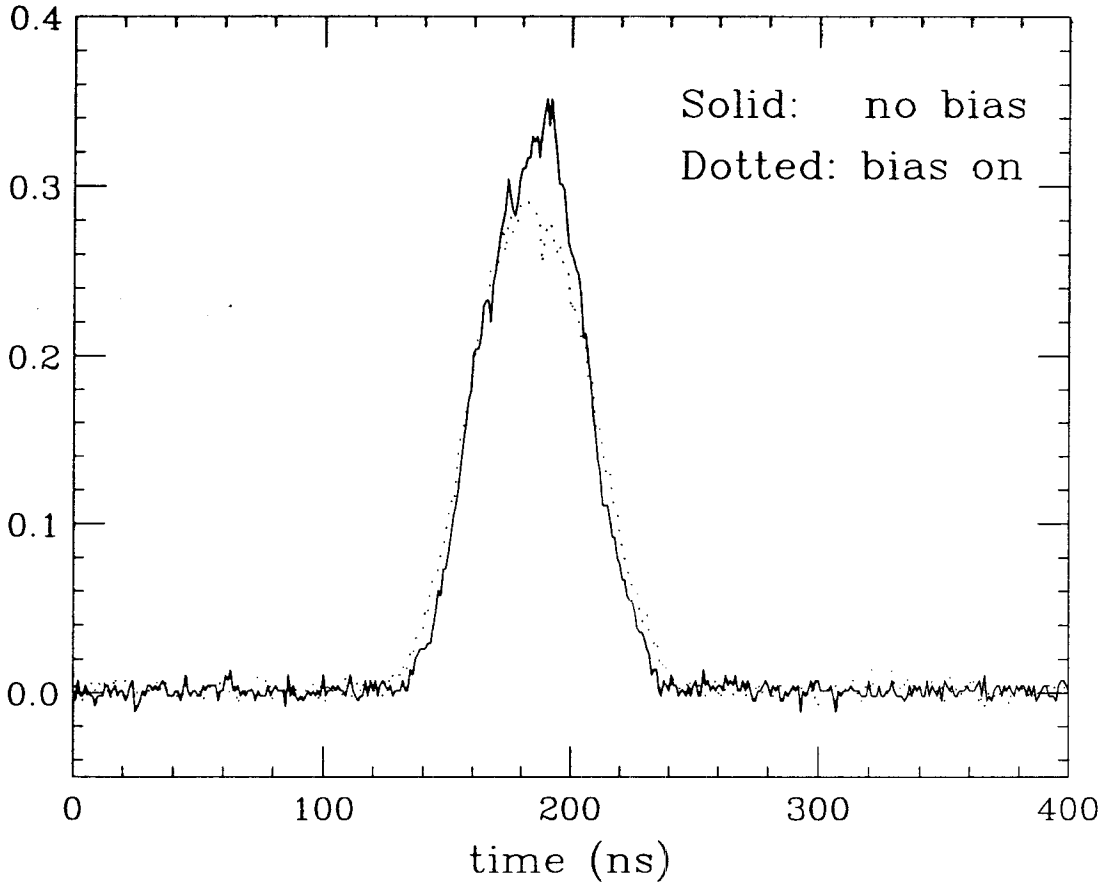
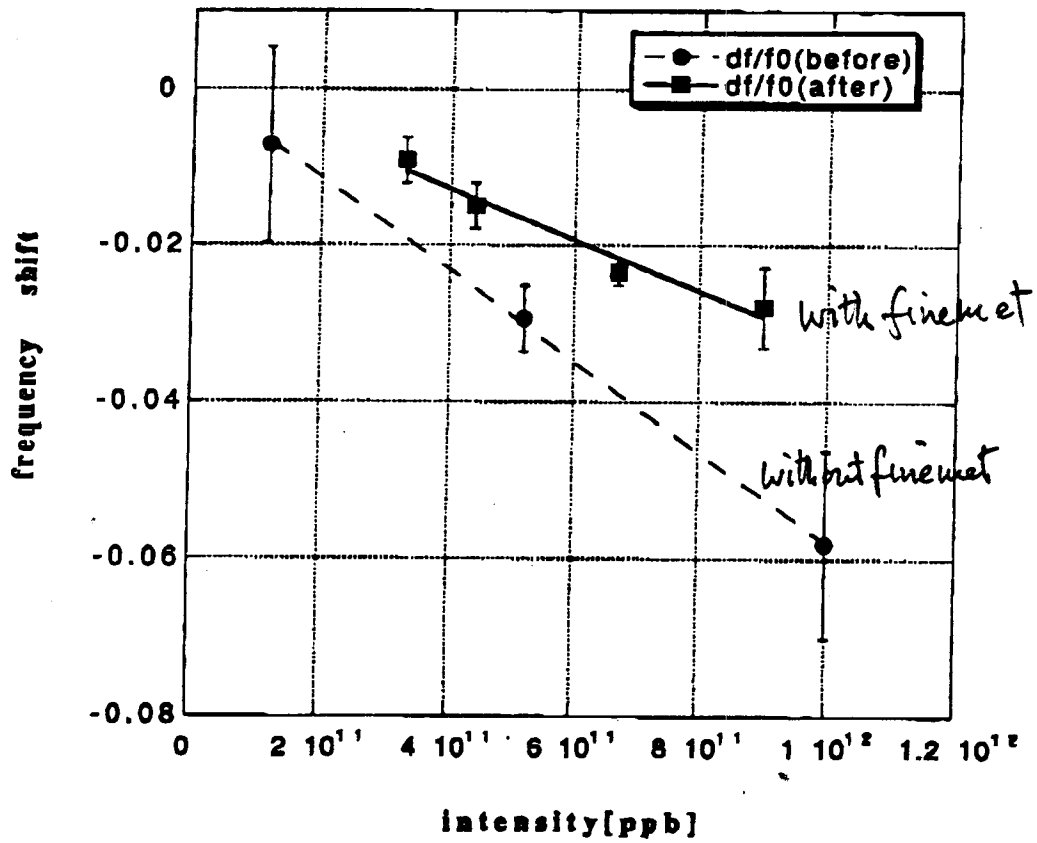


FIG. 6. Schematic cross section of the measurement setup with test fixtures.


0.4
0.3
0.2
0.1
0.0
0
100
200
300
400
time (ns)
Solid: no bias
Dotted: bias on

5. Results

The frequency shift as a function of intensity is plotted in Fig.5. The dashed line is a fitted line of a data before installation and the solid line is that after the installation. The slope of dashed line can be solely explained by space charge impedance, that is calculated as 440 ohms. On the other hand, the slope of solid lines is halved. Therefore, the inductive impedance made by the FINEMET cancels the half of the space charge impedance.



@ 500 MeV

Kiyomi Koba

Fig.5 The measured frequency shifts of the quadrupole oscillations as a function of the beam intensity are plotted. The closed circles and squares are the frequency shift before the installation of the impedance tuner and after the installation, respectively.

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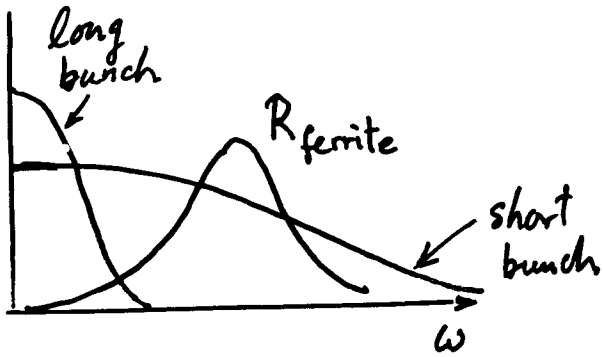
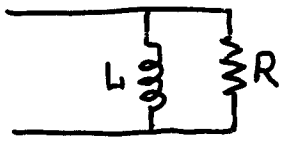
Power lost to ferrite can be severe for short bunches.



Simple model for ferrite



$\omega_r = \frac{R}{L}$

$$Z(\omega) = \frac{j\omega L}{1 + j\omega/\omega_r} \propto j\omega(\mu_s' - j\mu_s'') \Longrightarrow W(t) = R\left[\delta(t) - \omega_r e^{-\omega_r t}\right]$$

For parabolic bunch, energy lost per turn

$$\mathcal{E} = \frac{3\pi e I_b}{(\omega_0 \hat{\tau})^2}\left[\frac{\tau}{\hat{\tau}} + \frac{1}{\omega_r \hat{\tau}}\right]\left|\frac{Z_\parallel}{n}\right|_{ind}$$

assuming $\omega_r \hat{\tau} \gg 1$
or $\hat{\tau} \gg$ wake length

linear loss to compensate sp.ch

actual loss to ferrite

For n_b bunches, total power lost to ferrite is

$$P = \frac{3\pi n_b I_b^2}{\omega_0^2 \omega_r \hat{\tau}^3}\left|\frac{Z_\parallel}{n}\right|_{ind}$$

Example: new Fermilab Booster (newer design)

$n_b = 4$, $N_b = \frac{1}{4} \times 10^{14}$, $C = 158.0676$ m

$\hat{\tau} = 28.25$ ns $\quad \left|\frac{Z}{n}\right|_{sp.ch} \sim 89.5\,\Omega$

$\frac{\omega_r}{2\pi} = 60$ MHz $\Longrightarrow$ $\boxed{P = 0.16\,\text{MW}}$

At Los Alamos PSR $\frac{I_b}{\omega_0} \sim N_b$ is similar, but $\hat{\tau} \sim 100$ ns, $n_b = 1$

$\Longrightarrow P = 0.90$ kV only

But what happens if $\hat{\tau}$ is 2.825 ns $P \to 160$ MW

Sometimes $\omega_r/2\pi \sim 1$ MHz only

Another model that take into account of the width of the resonance gives

$$P = \frac{3\pi n_b I_b^2}{Q\,\omega_0^2 \omega_r \hat{\tau}^3} \left| \frac{Z_u}{n} \right|_{ind}$$

assuming $\frac{\omega_r \hat{\tau}}{2Q} \gg 1$

or $\hat{\tau} \gg$ wake length

Note: this Q is <u>NOT</u> the usual quoted Q of the ferrite which is a fcn. of frequency.

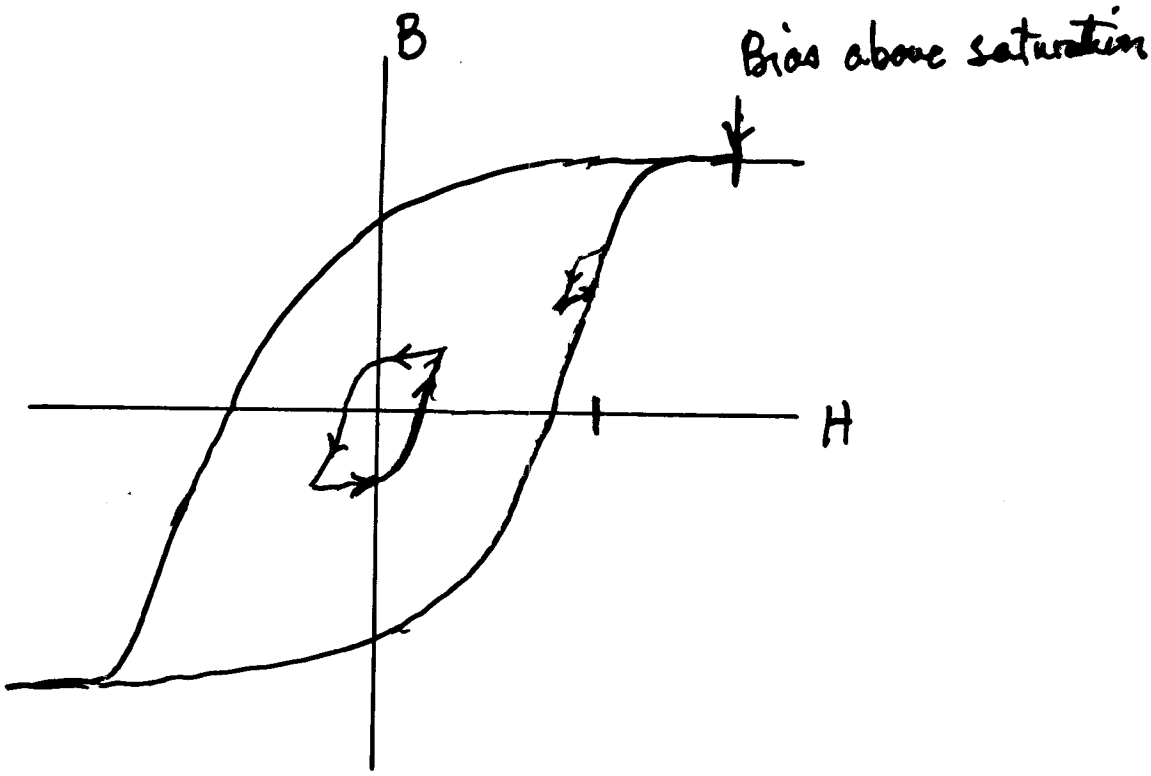
This Q is just a number in the model describing the width of the ferrite's resonant peak, e.g.

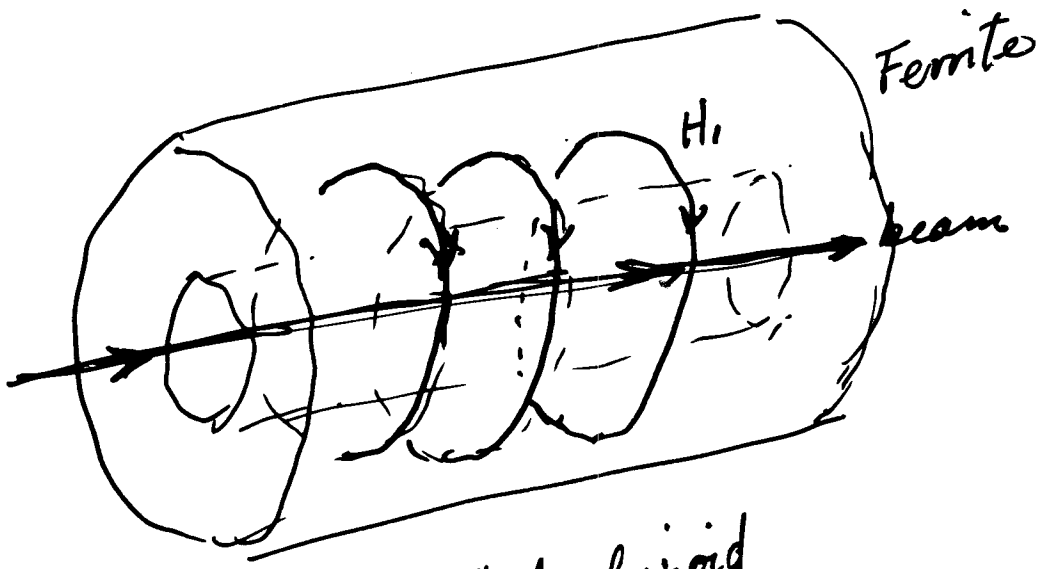
$$\Delta\omega = \frac{\omega_r}{2Q}$$

$\therefore$ to reduce loss, 1. increase ω_r

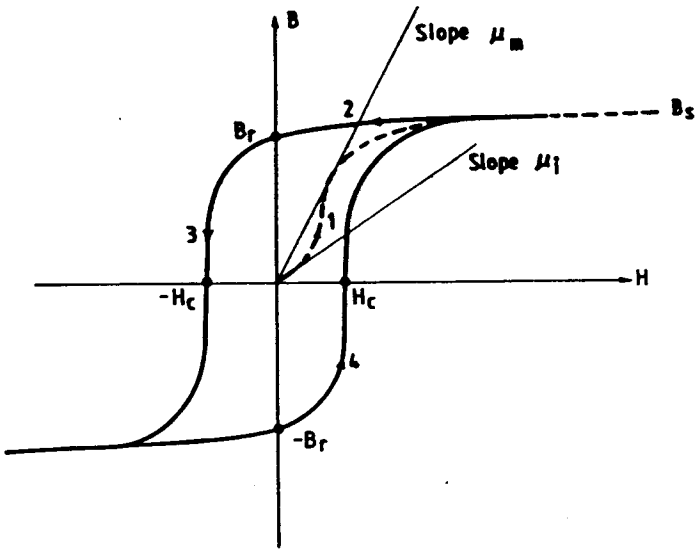
2. increase Q

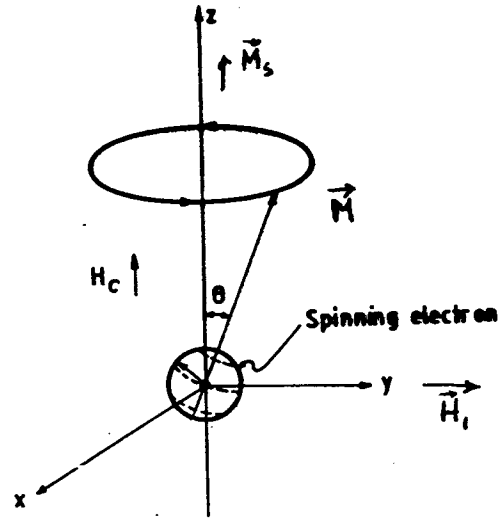
Loss in ferrite is mostly hysteresis effect

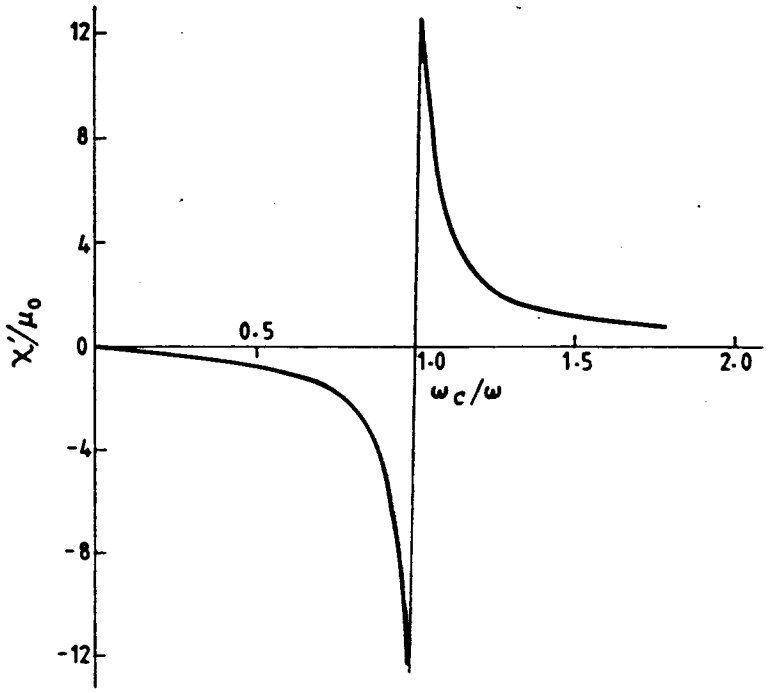


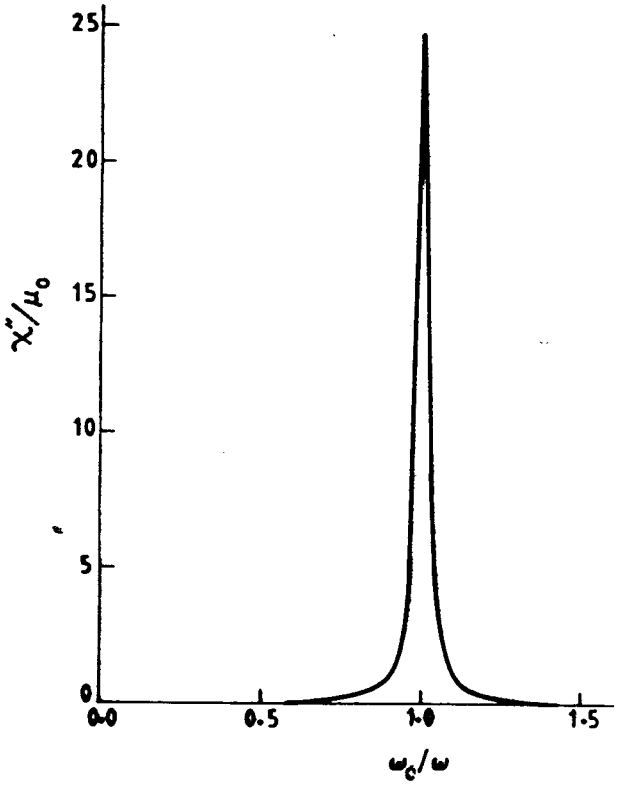


Add solenoid


B
Slope μ_m
2
B_s
B_r
Slope μ_i
1
3
H
$-H_c$
H_c
4
$-B_r$


z
$\vec{M}_s$
$\vec{M}$
H_c
θ
Spinning electron
y
$\vec{H}_i$
x


12
8
4
0
-4
-8
-12
χ'/μ_o
0.5
1.0
1.5
2.0
ω_c/ω


25
20
15
10
5
0
χ''/μ_o
0.0
0.5
1.0
1.5
ω_c/ω

- When $|\vec{H}_1| \ll H_c$, the equation of motion is

$$\frac{d\vec{M}}{dt} = \gamma(\hat{z}M_s \times \vec{H}_1 + \hat{z}\vec{M} \times H_c)$$

where $\gamma = 2.80 \times 2\pi$ MHz/Oersted is the gyromagnetic ratio of the electron. Defining the magnetic susceptibility tensor $\overleftrightarrow{\chi}$ as $\vec{M}_1 = \overleftrightarrow{\chi}\,\vec{H}_1$, the solution is

$$\overleftrightarrow{\chi} = \begin{pmatrix} \chi & -j\kappa & 0 \\ j\kappa & \chi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where

$$\frac{\chi}{\mu_0} = \frac{\omega_c\omega_m}{\omega_c^2 - \omega^2}, \qquad \frac{\kappa}{\mu_0} = \frac{\omega\omega_m}{\omega_c^2 - \omega^2}$$

and

resonance freq. high if bias field H_c is high

$$\boxed{\omega_c = \gamma H_c\,,} \qquad \omega_m = \gamma\frac{M_s}{\mu_0}$$

- There is a resonance at the gyromagnetic resonant frequency $\omega_c = \gamma H_c$, which is proportional to the dc H_c. This explains why we want H_c to be large so that the resonance effect can be avoided.

- Loss can be included by letting $\omega_c \longrightarrow \omega_c - i\omega\alpha$, giving

$$\frac{\chi'}{\mu_0} = \frac{\left(\frac{\omega_m}{\omega}\right)\left(\frac{\omega_c}{\omega}\right)\left[\left(\frac{\omega_c}{\omega}\right)^2 - 1 + \alpha^2\right]}{\left[\left(\frac{\omega_c}{\omega}\right)^2 - 1 - \alpha^2\right]^2 + 4\left(\frac{\omega_c}{\omega}\right)^2 \alpha^2}$$

$$\frac{\chi''}{\mu_0} = \frac{\left(\frac{\omega_m}{\omega}\right)\alpha\left[\left(\frac{\omega_c}{\omega}\right)^2 + 1 + \alpha^2\right]}{\left[\left(\frac{\omega_c}{\omega}\right)^2 - 1 - \alpha^2\right]^2 + 4\left(\frac{\omega_c}{\omega}\right)^2 \alpha^2}$$

Note that, actually the above depend on only M_s and α.

- Usually the ac field comes from a cavity. Then, ω will not be changed by very much and can be considered fixed except very near to the resonance. Therefore, χ is plotted as a function of H_c This explains why the formulas have been written as a function of ω_c/ω.

- In our application, the ac field comes from the beam particles. So ω has the range of the bunch spectrum. Therfore, $\omega/(2\pi)$ is about ~ 2.8 MHz.

- The merit of this application is the low loss, because the ferrite is saturated, there will not be hysteresis loss. The only loss is due to spin wave which is small. The disadvantage is μ' is usually small.

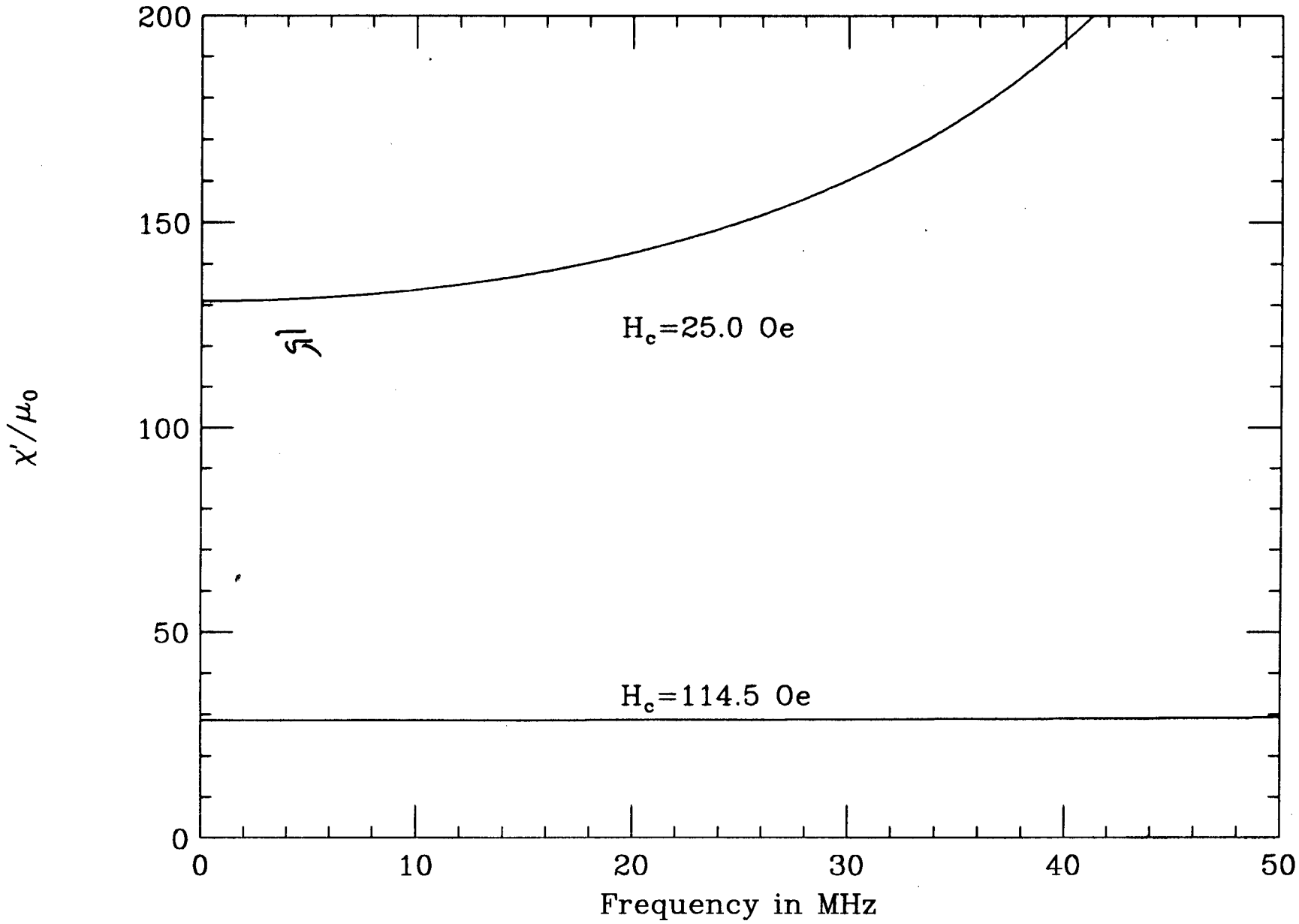
The present situation

- Choose Toshiba M_4C_{21A}, which has saturated flux density of ~ 32 kG at ~ 1 kOersted (??).
- Thus, $M_s = 32 - 1 = 31$ kG.
- Choose $H_c = 1$ kOe.

  This gives resonant frequency $\omega_c/(2\pi) = \gamma H_c = 2.8$ GHz.

  Up to 10 MHz, $\mu' \sim M_s/H_c = 31$.
- With ferrite of inner and outer diameters 5 in and 8 in to cancel $|Z_{\|}/n|_{\text{sp ch}} = 100\ \Omega$,

  we need a length of $\ell = 4$ m of ferrite is required.
- For a machine that ramps, the dc bias field H should be increased during ramping.
- At low frequencies, the loss is $\mu'' \longrightarrow \frac{\alpha\omega\omega_m}{\omega_c^2}$. $Q \sim \frac{1}{2\alpha} \sim 10$
- Take a typical value of $\alpha = 0.05$, we find μ'' varies linearly at lows frequencies and is extremely small.


200
150
100
50
0
0
10
20
30
40
50
Frequency in MHz
χ'/μ_0
H_c=25.0 Oe
H_c=114.5 Oe

Transient beam loading

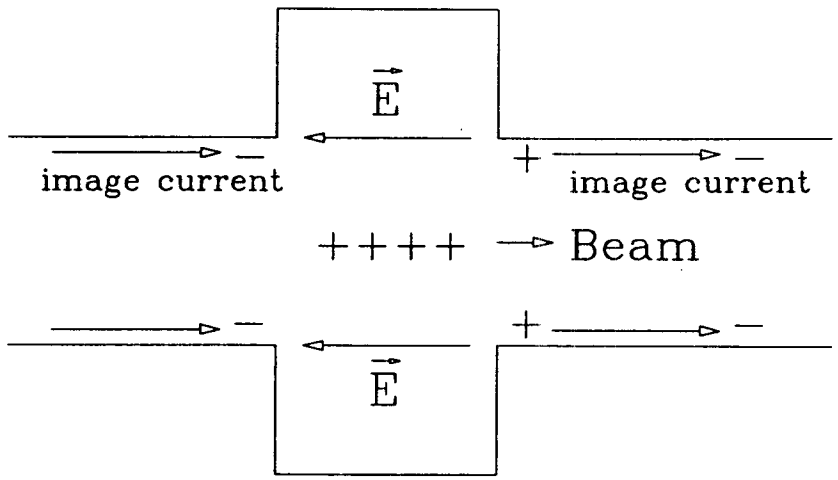


FIG. 11. As a positively charged bunch passes through a cavity, the image current leaves a negative charge at the upstream end of the cavity gap. As the image current resumes at the downstream side of the cavity, a positive charge is created at the downstream end of the gap because of charge conservation, thus setting up an electric field $\vec{E}$ and therefore the transient beamloading voltage.

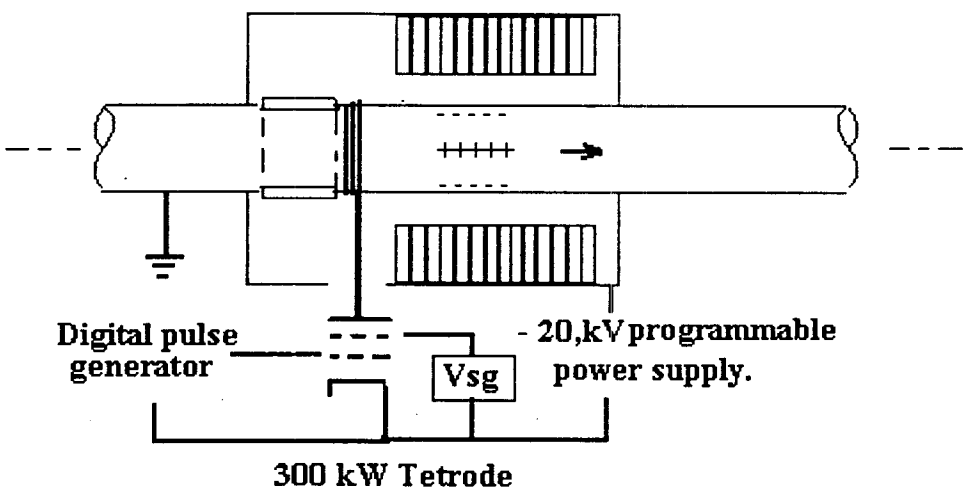


FIG. 12. In the design of the future Fermilab proton driver, the beam linear charge density is monitored and fed into a tetrode, which supplies the same charge density to the downstream end of the cavity gap. Thus the transient beamloading voltage is cancelled.

<u>Example</u> New design of Fermilab 1st booster ring

No. per bunch $\frac{1}{4}\times 10^{+14}$ or total charge $q = 4.0\,\mu C$

Need $V_{rf} \sim 185\,kV$ in total

split into 10 cavity, each with $\boxed{V_{rf} = 18.5\,kV}$

Each cavity : 30 cm. of ferrite ring inner/outer radii 20/35 cm.

$\mu_r = 21$

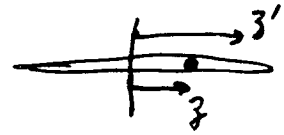
This gives $L \sim 0.61\,\mu H$ $C \sim 820\,\mu F$

$\therefore$ for a point bunch, transient beam loading voltage is $\boxed{V_{t0} = \frac{q}{C} = 5.4\,kV}$

For a longer bunch $V_t(z)$ will be less.

In fact, this is just the wake potential seen by a particle inside the bunch due to the wake of the cavity gap, or for a Gaussian bunch

$$V_t(z) = e\int_z^\infty dz'\,\rho(z')\,W(z'-z)$$



$$= -\frac{eN_b\omega_r R_{\parallel}}{2Q\cos\phi_0}\,\mathrm{Re}\,e^{j\phi_0 - z^2/2\sigma_z^2}\,w\left[\frac{\sigma_z\omega_r e^{j\phi_0}}{c\sqrt{2}} - \frac{jz}{\sqrt{2}\sigma_z}\right]$$

where $\phi_0 = \sin^{-1}\frac{1}{2Q}$

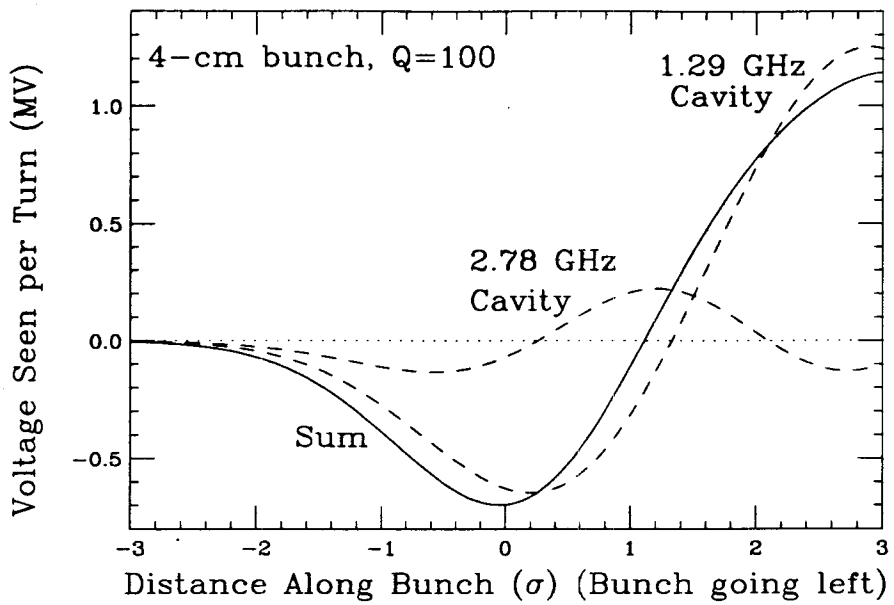


FIG. 9. Beamloading voltages seen by particles in the 4-cm bunch as they traverse the wake-potential compensating cavities. The particles are at distance behind the bunch center in unit of rms bunch length.

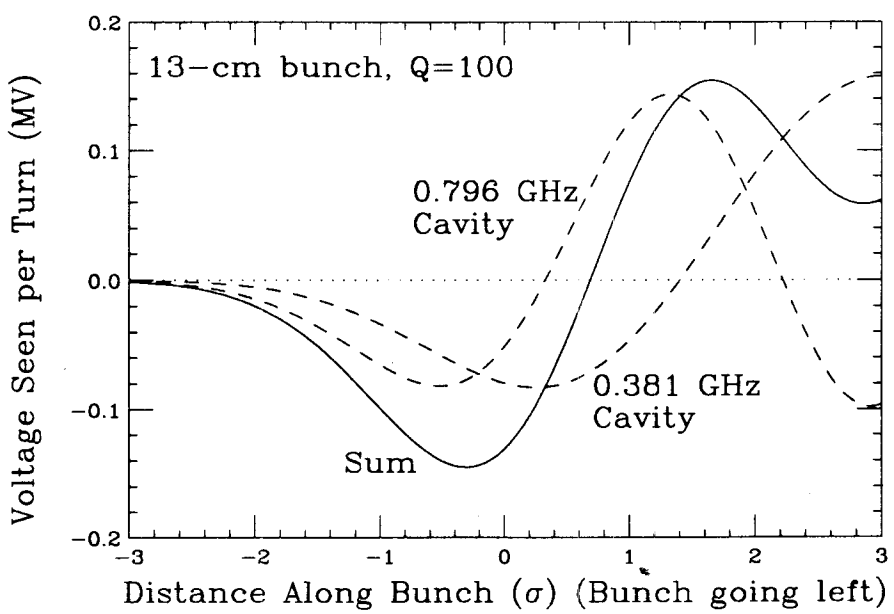


FIG. 10. Beamloading voltages seen by particles in the 13-cm bunch as they traverse the wake-potential compensating cavities. The particles are at distance behind the bunch center in unit of rms bunch length.

Transverse Space-charge effect

K.Y. Ng

We say that a large Laslett tune spread due to space charge will encompass a lot of resonances and lead to instability.

A rule of thumb is $\Delta\nu_{Laslett} \lesssim 0.4$ and we design most low-energy ring according to this rule.

However, I was told that $\Delta\nu_{spch} \gtrsim 1$ for the present Fermilab booster and it is running okay.

Machida & Ikegami pointed out at Shelter Island that it is the coherent tuneshifts that determine the instability of a ring and not the **incoherent**

We know that when $\nu_{coh.}$ = integer, we have integer resonance.

But we also have instabilities or emittance growth when the coherent quadrupole and sextupole tunes hit an integer.

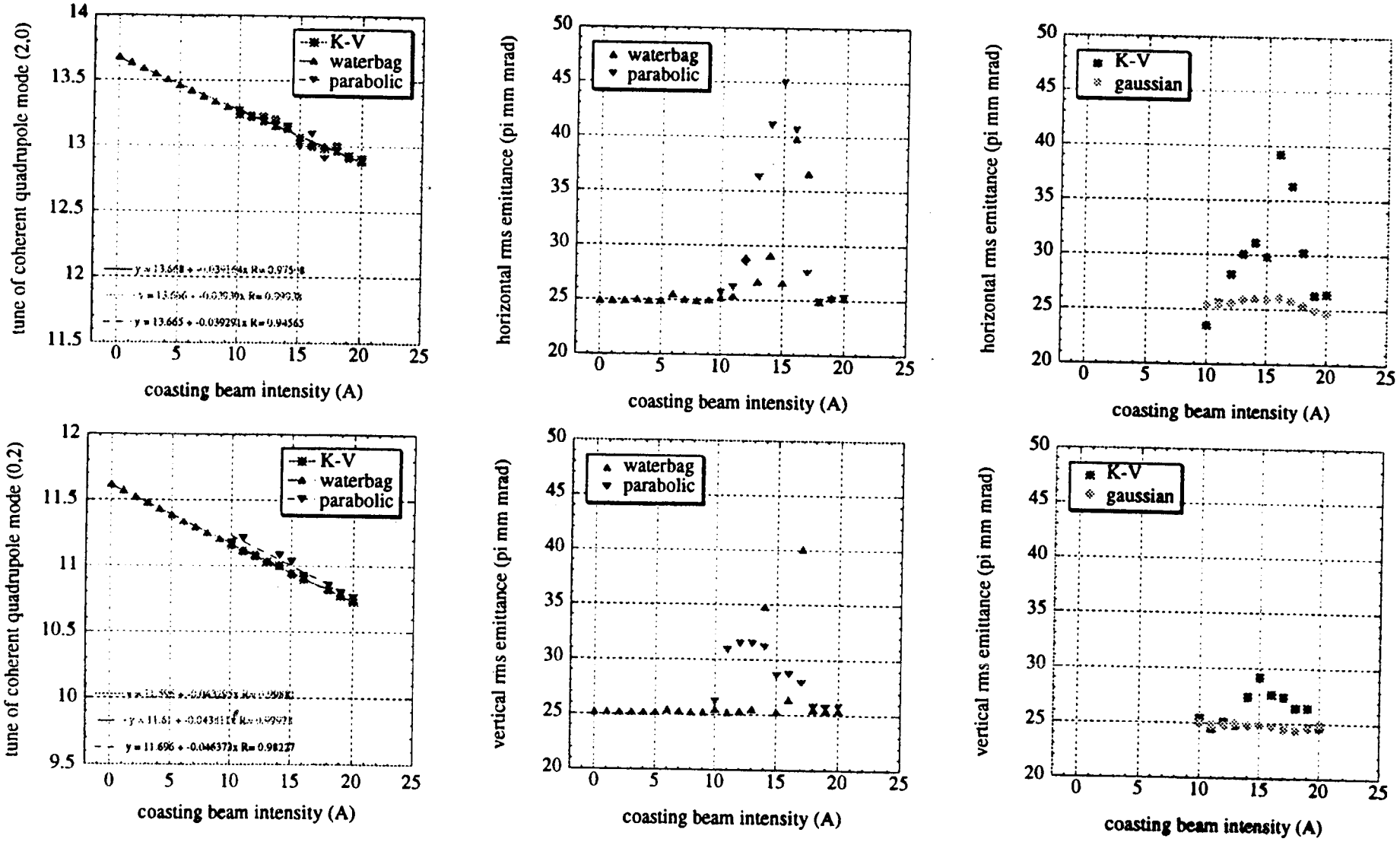


FIGURE 6. Tune of coherent quadrupole mode (left) and rms emittance at 512 turns after injection (center and right). Upper figures show horizontal results and lower ones vertical. In both planes, rms emittance growth is observed when the coherent quadrupole tune becomes integer.

Space charge is not necessarily bad.

It can be good also.

Mike Blaskiewicz wrote a very nice paper on

"Fast head-tail instability with space charge" Phys Review ST Accel Beam 044201 (1998)

These are mostly simulation results.

He found that "Fast head tail" or "Transverse Mode Coupling" is damped in the presence of space-charge.

One simple explanation is collective transverse instabilities are damped by incoherent sp. ch. tune spread

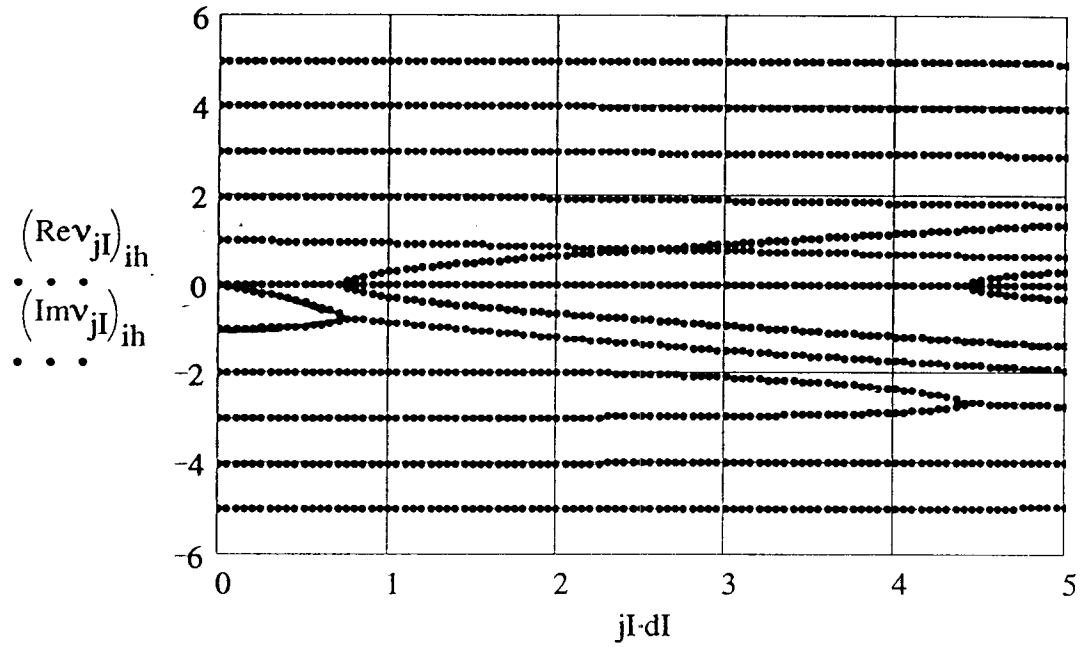
This may explain why Fast head tail instabilities are observed, up to now, only in electron machines but not hadron machines.

When this is discussed at Fermilab, Alexy Burov gave a very nice explanation.

Alexey Burov
Fermilab

space charge parameter TMC with SC

$\xi = 0$



$\mathrm{Im}\nu_{kI}^{\ T} =$ [redacted]
0 0 0 0 0 0.169 -0.169 0 0.349 -0.349 0

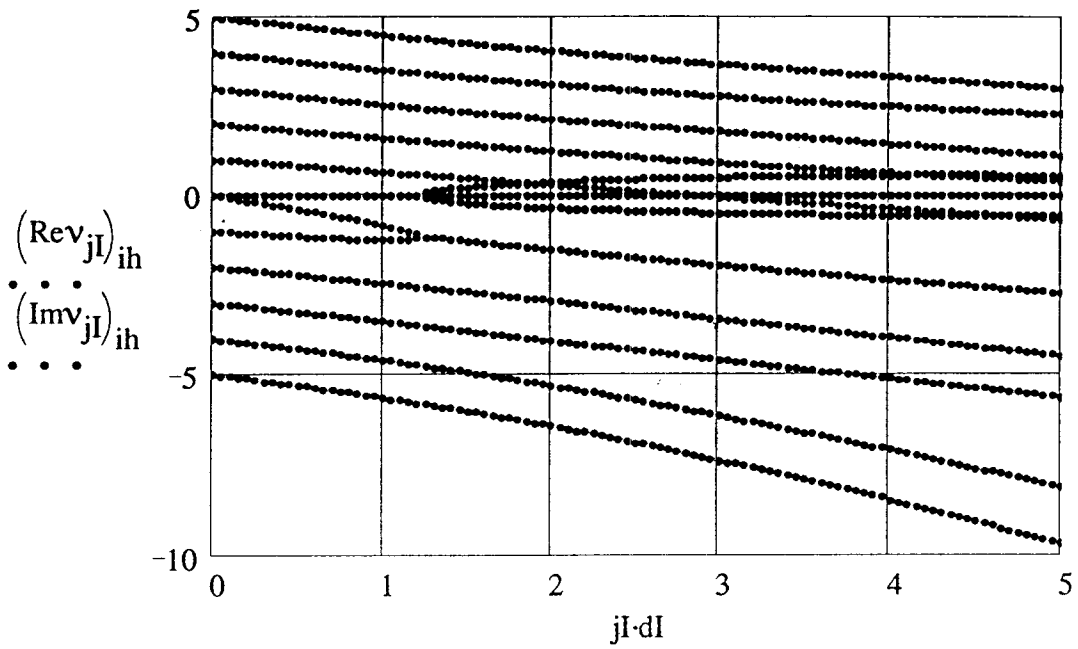
Transverse wake moves m=0 mode down by much

but moves m=-1 mode up only slightly

TMC with SC

with space charge

$\xi = 2$



$\mathrm{Im}\nu_{kI}^{T} =$ [redacted]

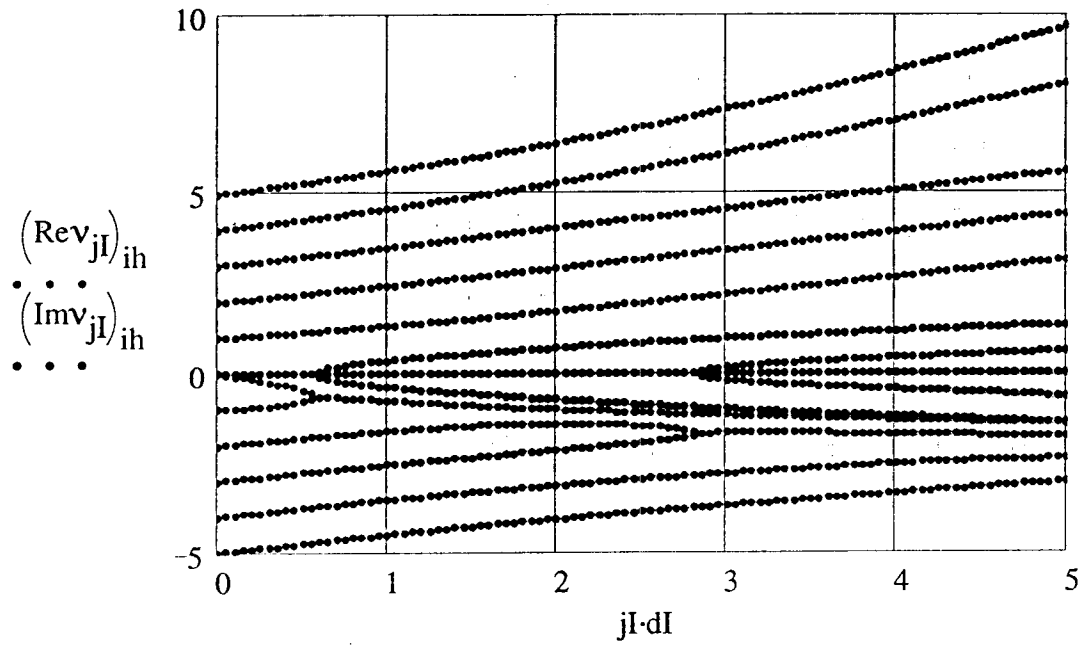
[redacted] 0 0 0 0 0 0.169 -0.169 0 0.349 -0.349 0

Space-charge shifts all modes downward except for the $m=0$ mode

$\therefore$ threshold increases with space charge.

TMC with SC

$\xi = -2$ (artificial negative space charge)



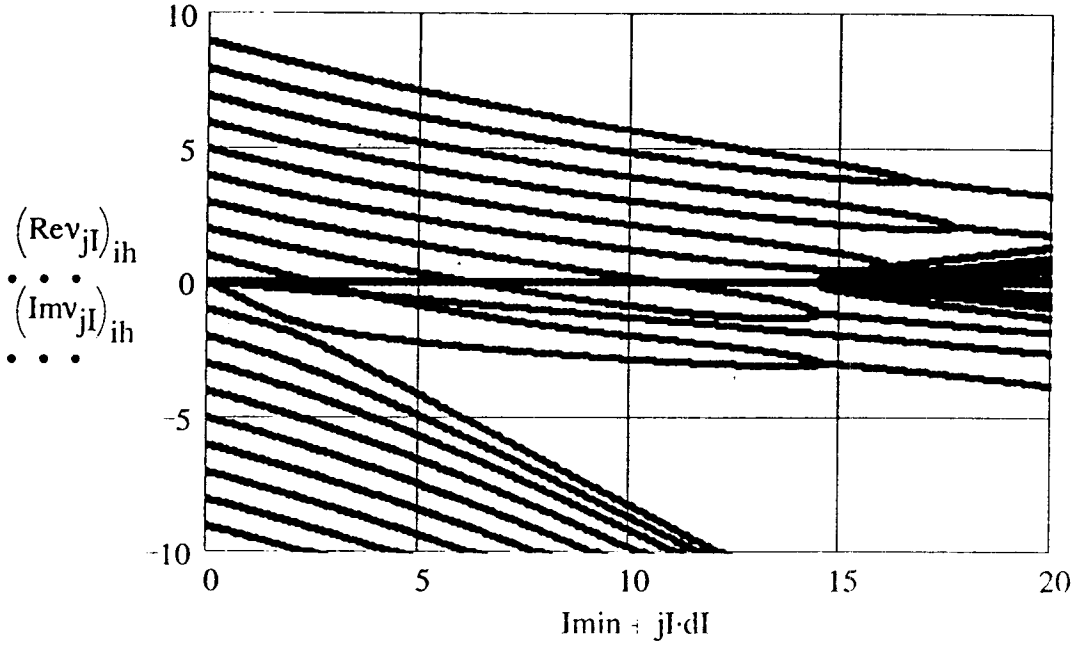
$\mathrm{Im}\,\nu_{kI}^{\,T} =$

	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0.169	-0.169	0	0.349	-0.349	0

Threshold decreases with artificial negative space charge.

TMC with SC

$\xi = 3.3$ $\alpha = 0.01$ $\delta c = 0$



But when space charge is large enough m=1 mode meets with m=0 mode

$Im\nu_{kI}^T =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	0	0.479	-0.479	0.507	-0.507	0.91	-0.91	1.318	-1.318	0.734	-0.734	0	0	0

jj = 1 Imin + jj·dI = 0.05

$Im\nu_{jj}^T =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$Re\nu_{jj}^T =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	-0.04	-9.126	-8.126	-7.126	8.876	-6.126	-5.127	7.876	-4.127	-3.128	6.876	-2.129	5.876	-1.1

ν_{th} eigenvals(Mm(jj·dI))$_{14}$ $\nu_{th} =$

So there is a jump in threshold at a certain sp. ch. after that threshold decreases again

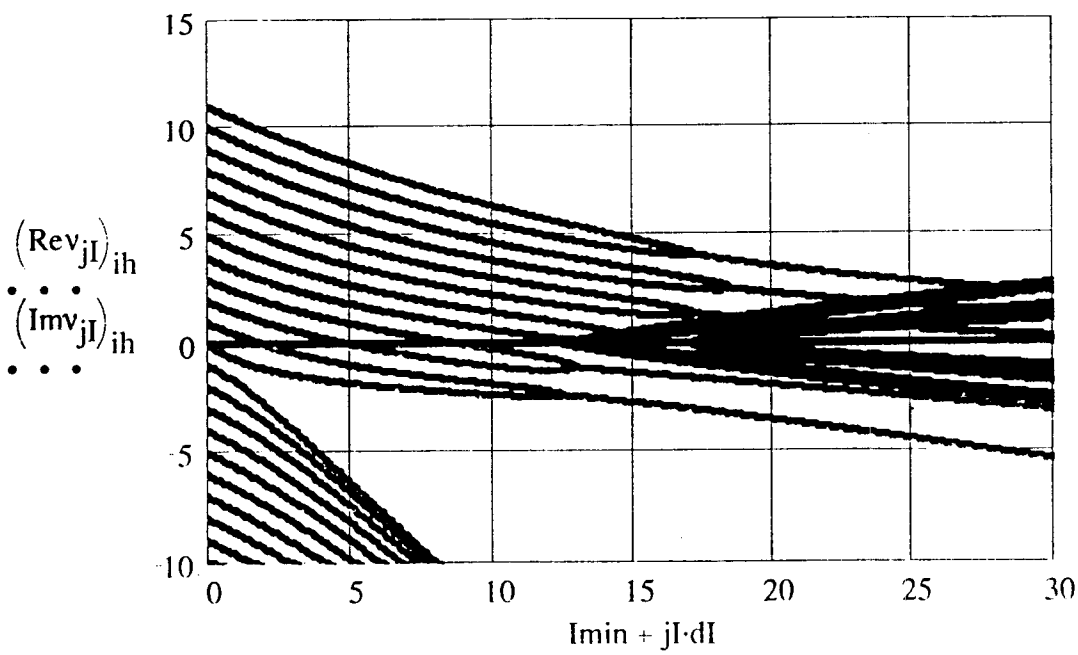
$dI := (Imax - Imin)\cdot kI^{-1}$ $\qquad jI := 0..kI$

$Rev_{jI} := Re(eigenvals(Mm(jI\cdot dI + Imin)))$ $\qquad Imv_{jI} := Im(eigenvals(Mm(jI\cdot dI + Imin)))$

$ih := 0..Nd - 1$

TMC with SC

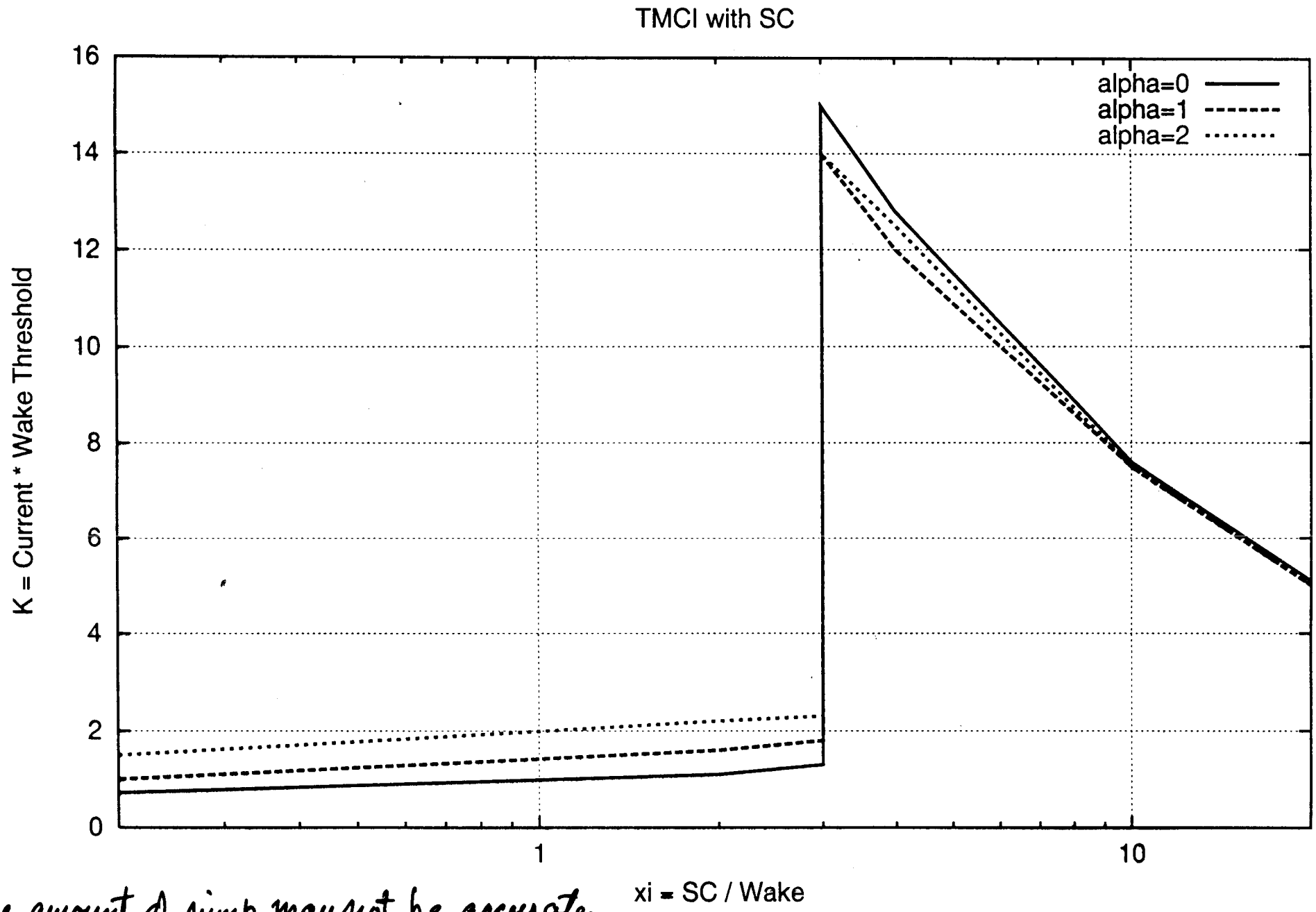
$\xi = 5$ $\qquad \alpha = 0.01$



$Imv_{kI}^{T} =$

	0	1	2	3	4	5
0	1.399	-1.399	2.78	-2.78	2.443	-2.443

jj 64 After m=1 & m=0 meet, threshold decreases with space charge.



The amount of jump may not be accurate because for large space charge, need a very large dimension coupling matrix