

Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, L.I., N.Y.

AGS DIVISION TECHNICAL NOTE

No. 65

G. Fiske
August 26, 1969

PHASE SPACE ELLIPSES FOR A MODEL AGS

The accompanying graphs are plots of the AGS phase space ellipses for the horizontal (H) and vertical (V) directions, obtained on the CDC-6600 and the Calcomp 565 plotter.

The calculation assumes a model horizontal $\Psi(s)$ function¹ of the following form:

$$\Psi(s) = \Psi_L \left[\frac{S}{L} + \frac{\tau}{2\pi} (\cos \frac{2\pi}{L} S - 1) + \frac{\sigma}{2\pi} (\cos \frac{6\pi}{L} S - 1) \right] \text{ where:}$$

$$L = \frac{2\pi R}{60}; R = \text{radius of machine} = 5057.266 \text{ inches}$$

$$\Psi_L = \frac{2\pi v}{60}; \text{ and for a form factor } \frac{\beta_{\max}}{\beta_{av}} = 1.5$$

$$\tau = 3/8$$

$$\sigma = 1/72$$

This phase function has a periodicity of four magnets (1/60 of the circumference), and agrees well with the results obtained from the BEAM program at intermediate field. The corresponding β function is of the form:

$$\beta(s) = \frac{R/v}{1 - \tau \sin \frac{2\pi}{L} s - 3\sigma \sin \frac{6\pi}{L} s}$$

Each set of eight graphs gives the phase space ellipses in either the horizontal or vertical direction for one particle at eight points within the super-superperiod (1/60 of the circumference). The following table lists the graphs shown:

<u>Graph Numbers</u>	<u>v</u>	<u>Y_{max} (inches)</u>
1-8	8.0 (H)	2.00
9-16	8.0 (V)	1.00
17-24	8.5 (H)	2.00
25-32	8.5 (V)	1.00
33-40	9.0 (H)	2.00
41-48	9.0 (V)	1.00

In general, the program is designed to plot graphs for any v value and any value of Y_{\max} , where Y_{\max} is the maximum value of the transverse excursion of the particle taken at the point where β itself is maximum. Thus, at no point in the machine is the greatest transverse excursion of the particle any larger than Y_{\max} .

The phase invariant, W , is set at Y_{\max}^2/β_{\max} , and the equation of each ellipse is:

$$\gamma y^2 + 2\alpha yy' + \beta y'^2 = W$$

A diagram of a super-superperiod with the horizontal β function superimposed is given in Fig. 1.

The phase space ellipses are computed and plotted at the indicated points, A through H.

The graphs contain all parameters pertinent to the calculation, as well as the area (emittance) of the ellipses; that is:

$$A = \pi W \text{ rad-in.}$$

References

1. J.C. Herrera, BNL Accel. Dept. Int. Rept. AGS DIV 69-3 (1969).

Distr:

Department Administration
AGS Division Physicists

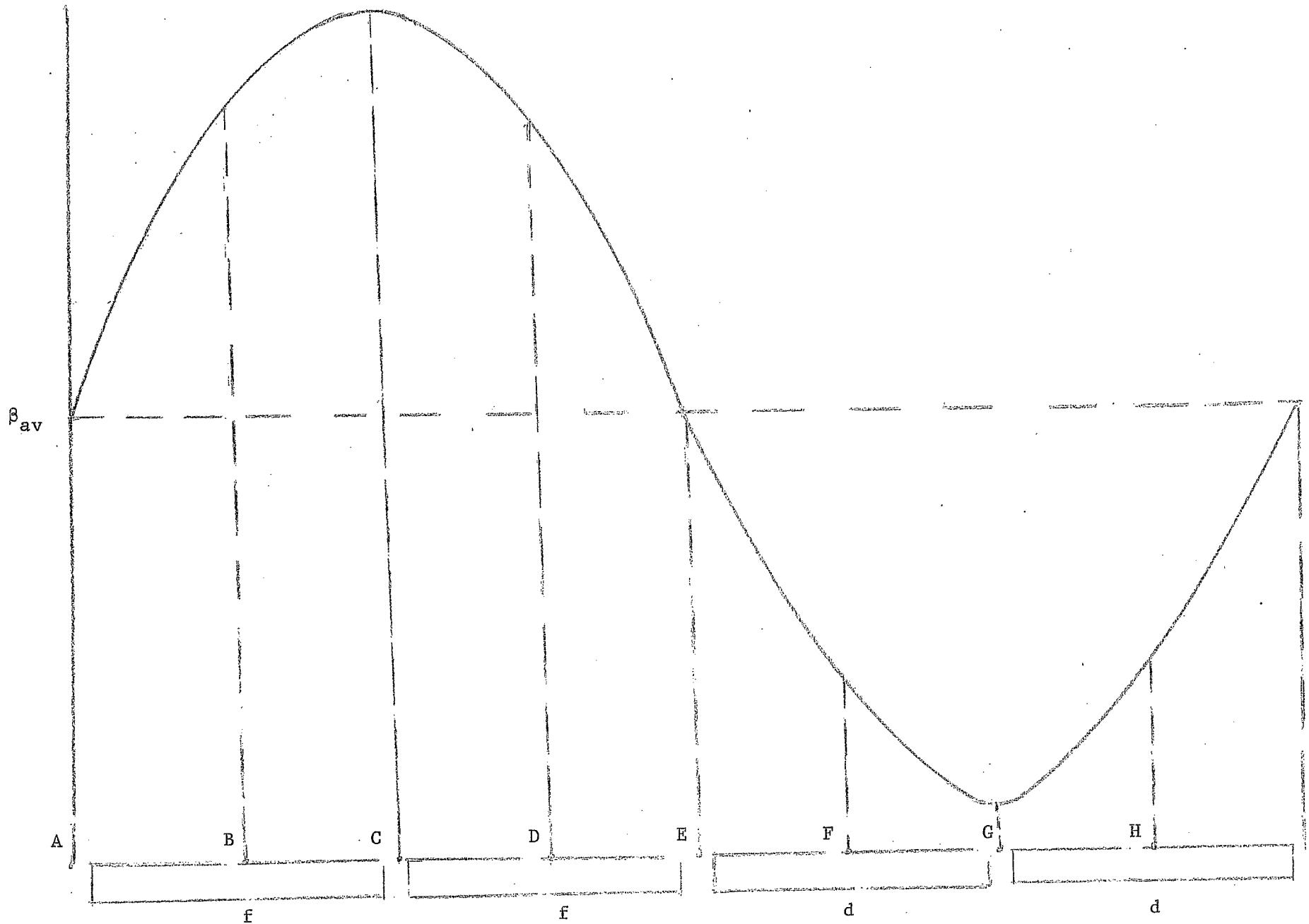
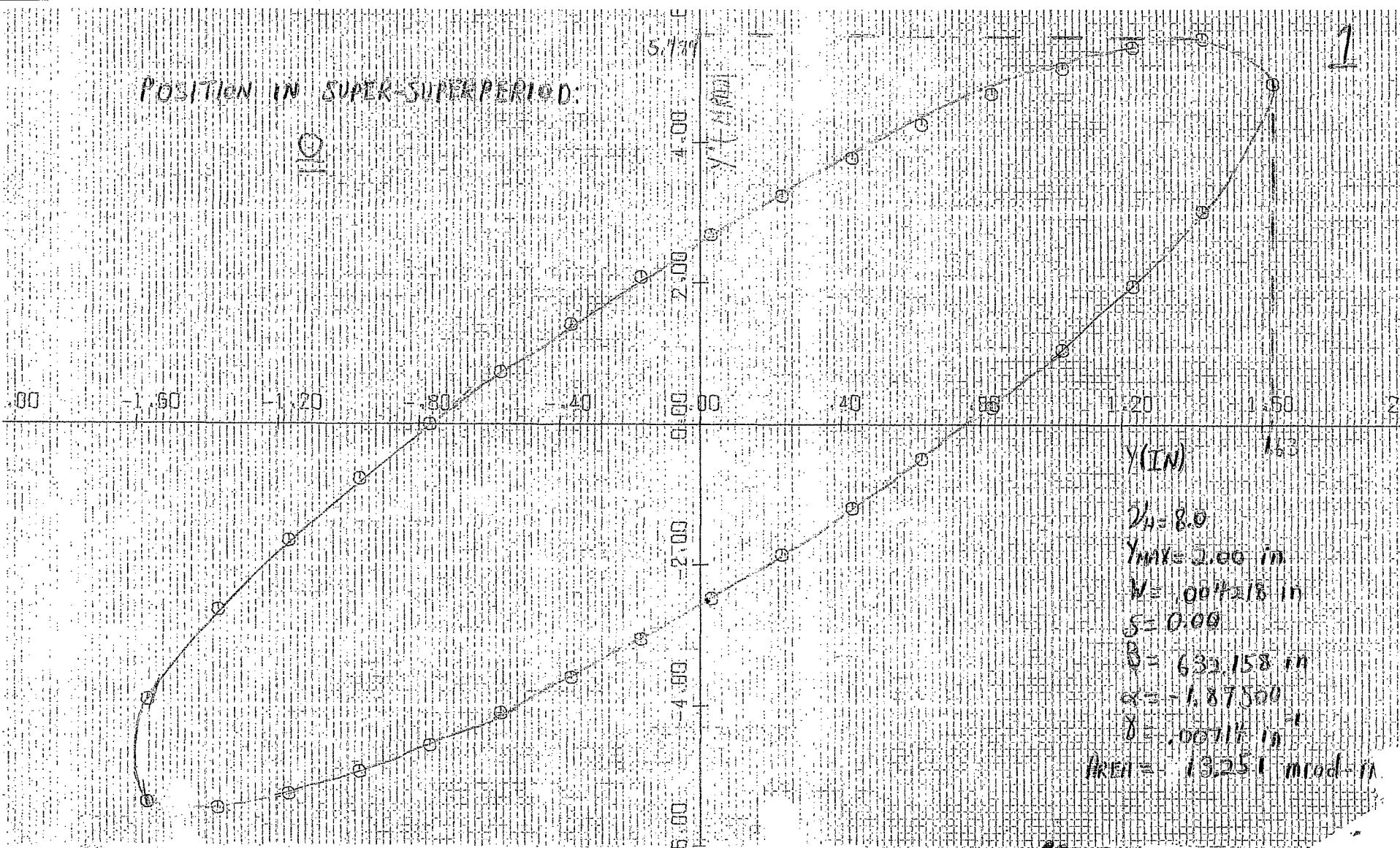


Fig. 1

POSITION IN SUPER-SUPER PERIOD



$$D_4 = 8.0$$

$$Y_{MAX} = 2.00 \text{ in}$$

$$S = 0.00$$

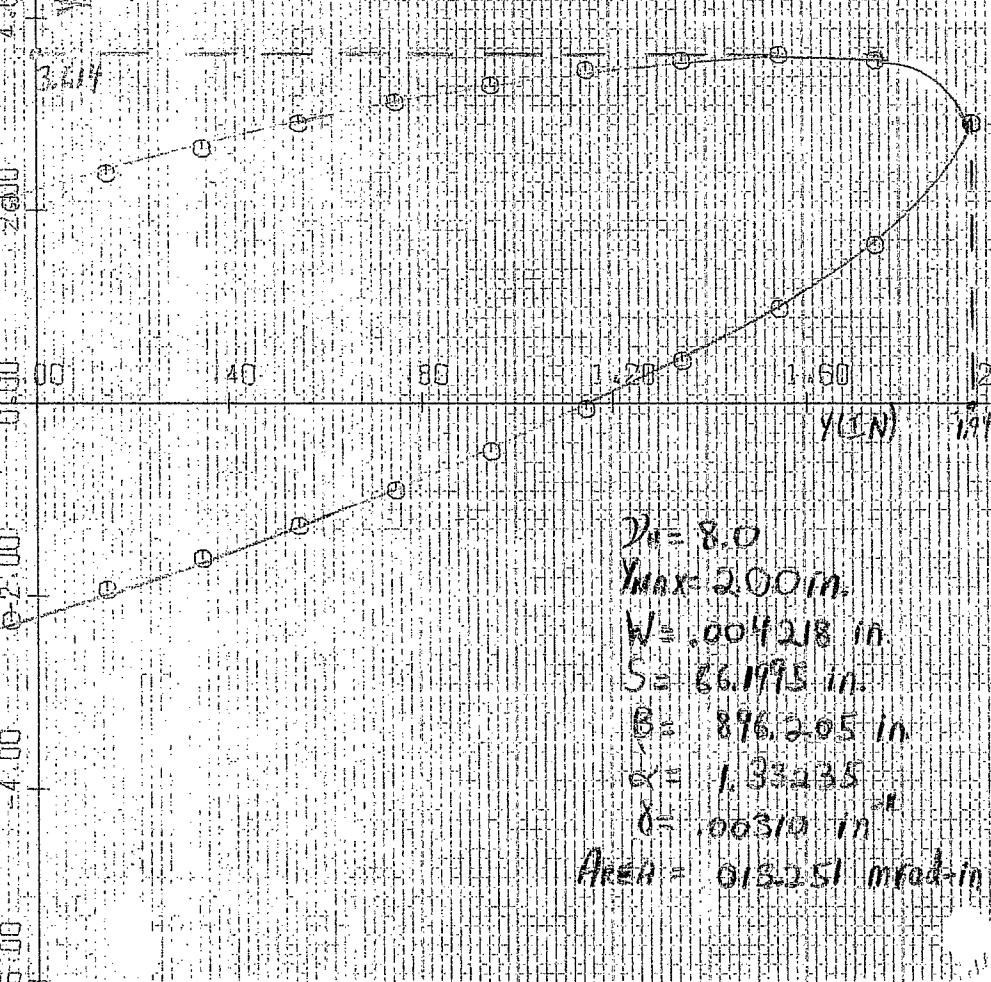
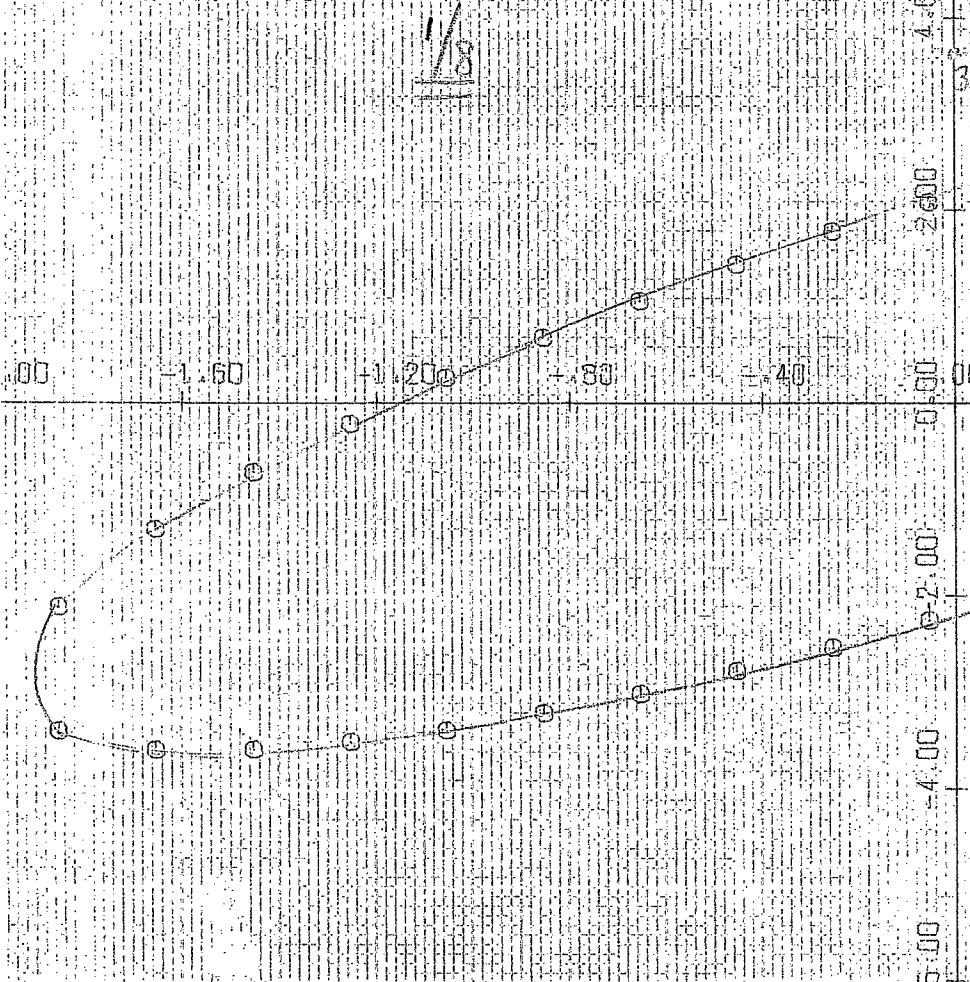
$$B = 631158 \text{ in}$$

$$\alpha = -1.87500$$

$$Y = 0.007114 \text{ in}$$

$$R_{EN} = 13.251 \text{ inrad/in}$$

POSITION IN SUPER-SUPERPERIOD



2 = 8.0

$y_{max} = 2.00 \text{ m}$

W-1004218 in

5-6-1991

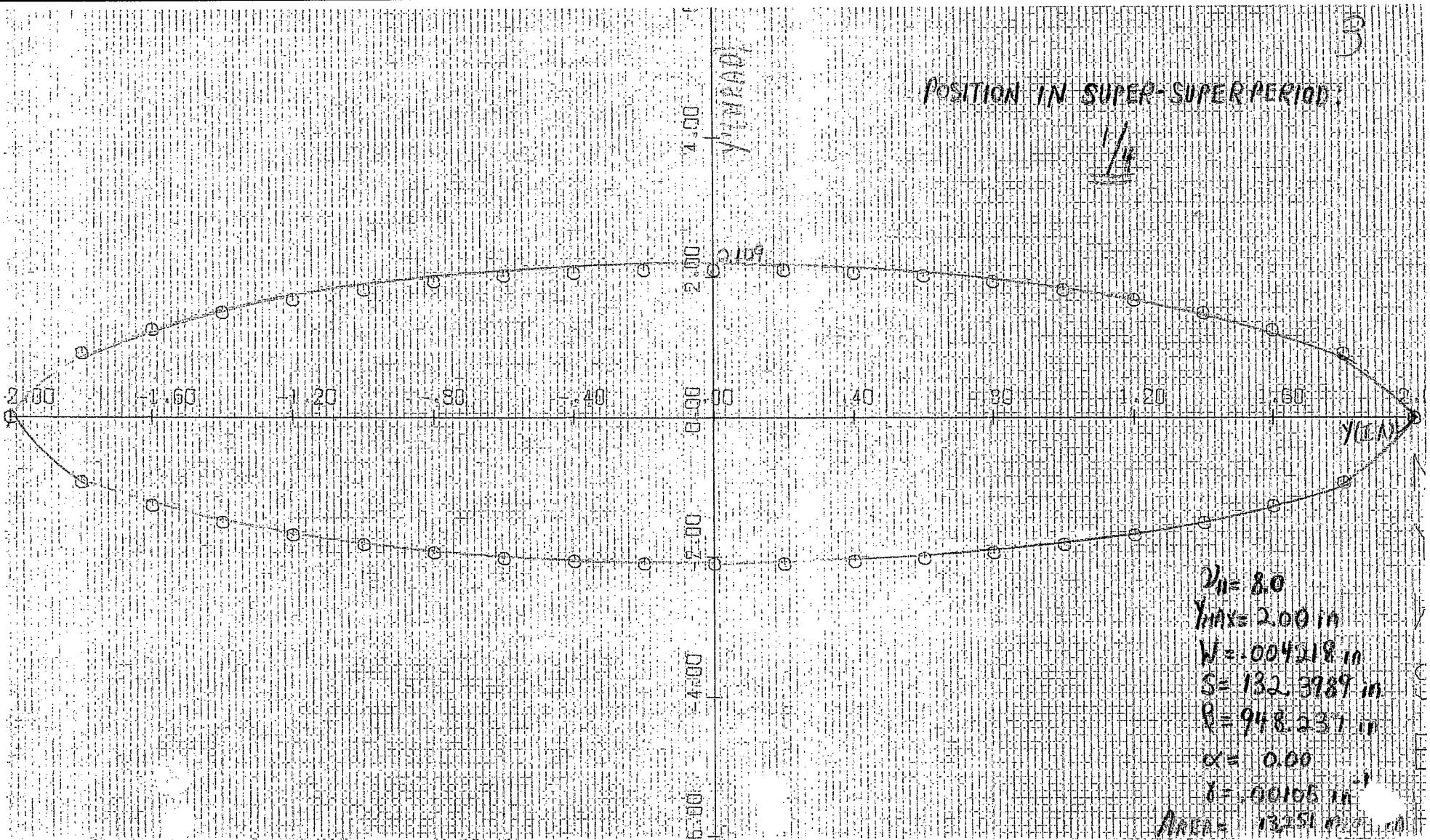
3-148 876-20510

Q. 1. - 1. 2. 3. 4. 5.

卷之三

1937-1

Area = 013-251 mrad-in



POSITION IN SUPER-SUPERPERIOD:

3/8

3.602

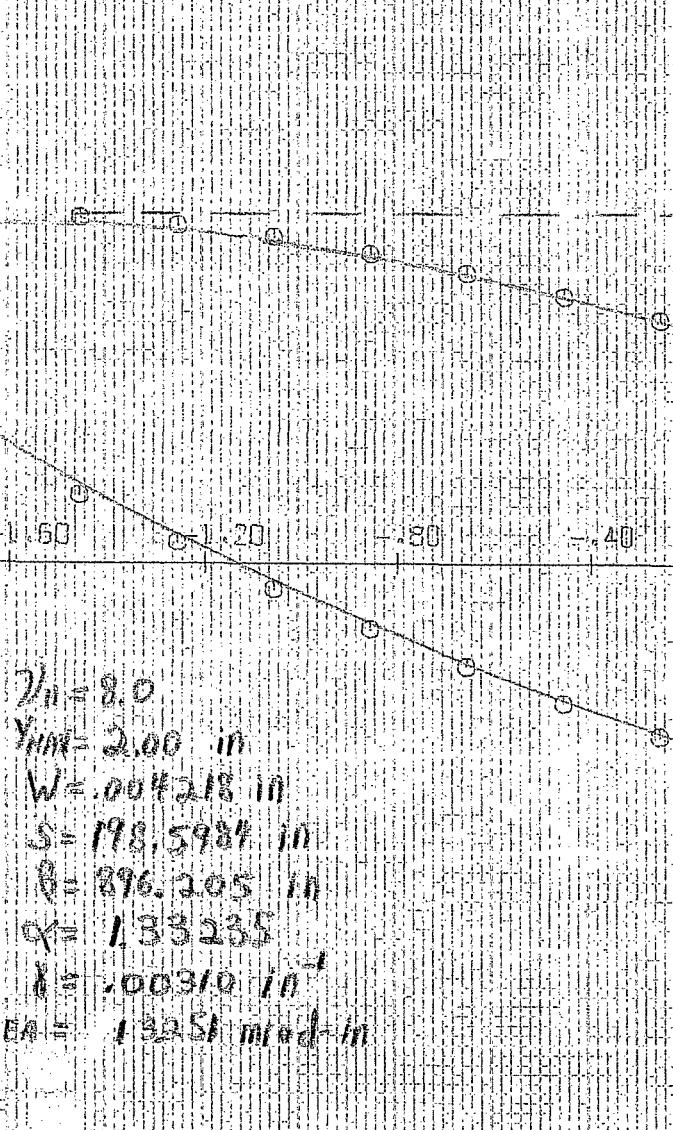
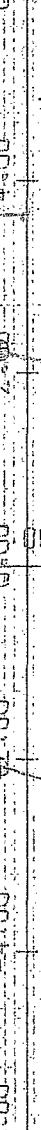
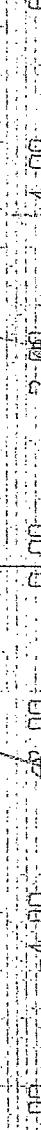
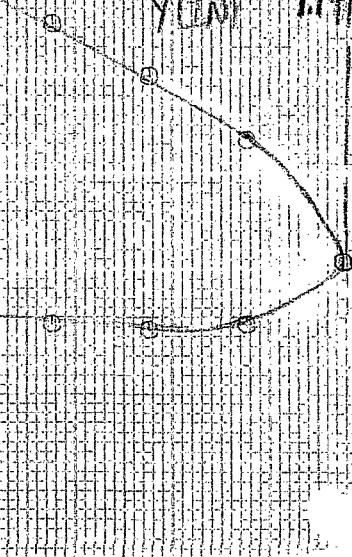
Y IN

4

2

1

0



$$2h = 8.0$$

$$Y_{min} = 2.00 \text{ m}$$

$$W = .004218 \text{ m}$$

$$S = 193.5984 \text{ m}$$

$$S = 896.305 \text{ m}$$

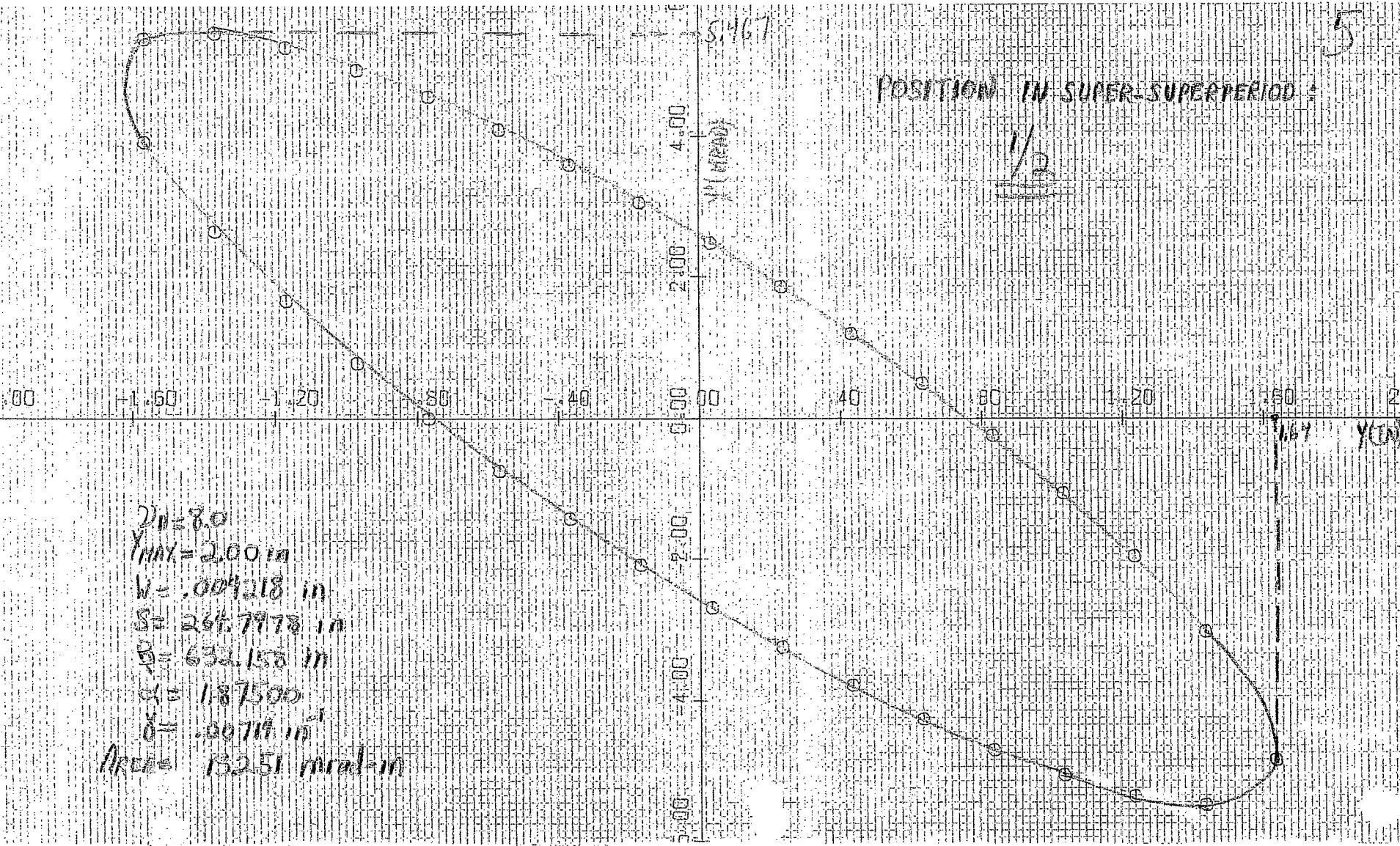
$$S = 1.33233 \text{ m}$$

$$S = .00310 \text{ m}$$

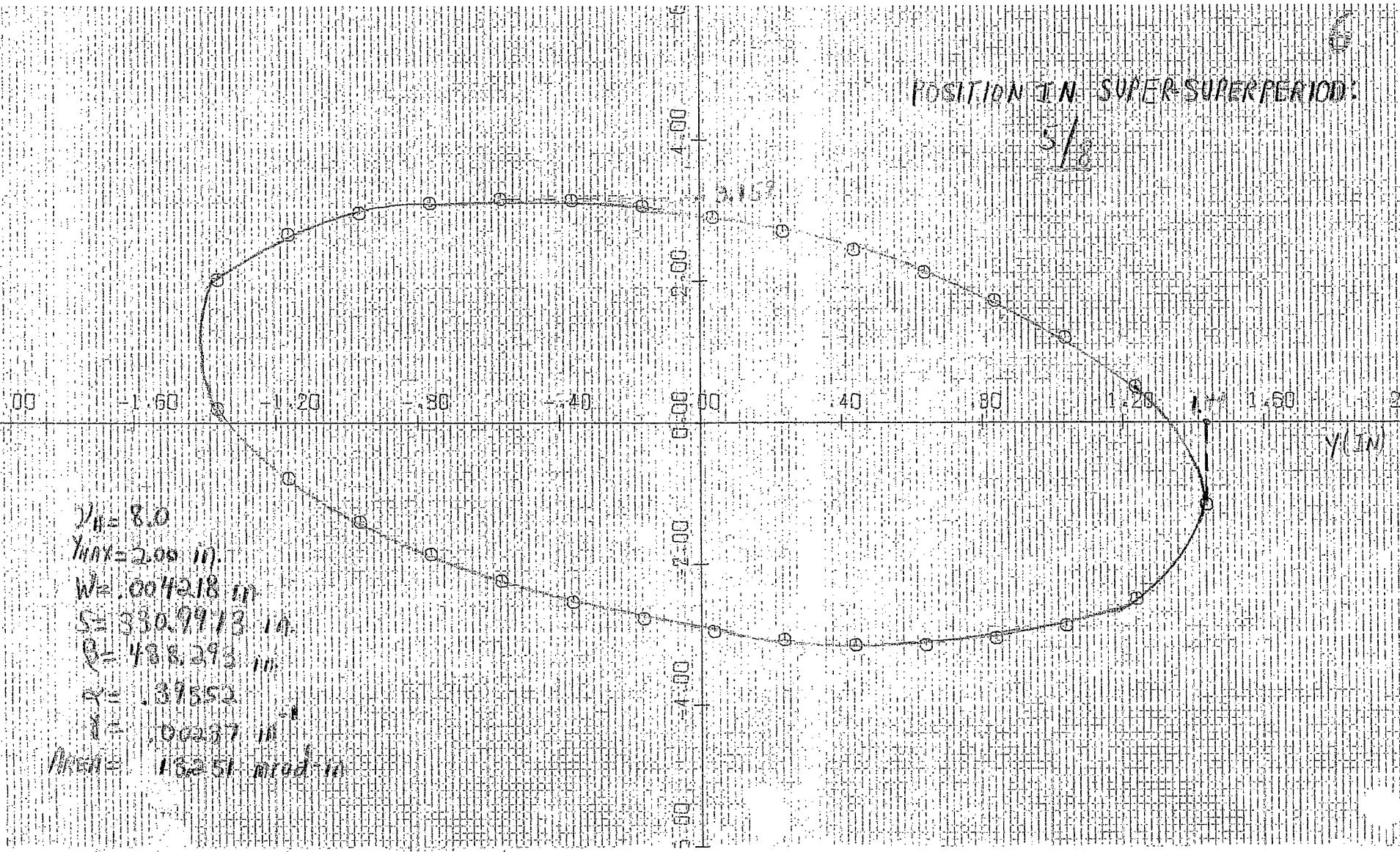
$$\text{Area} = 1.3251 \text{ m}^2 \cdot \text{m}$$

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POSITION IN SUPER-SUPERPERIOD

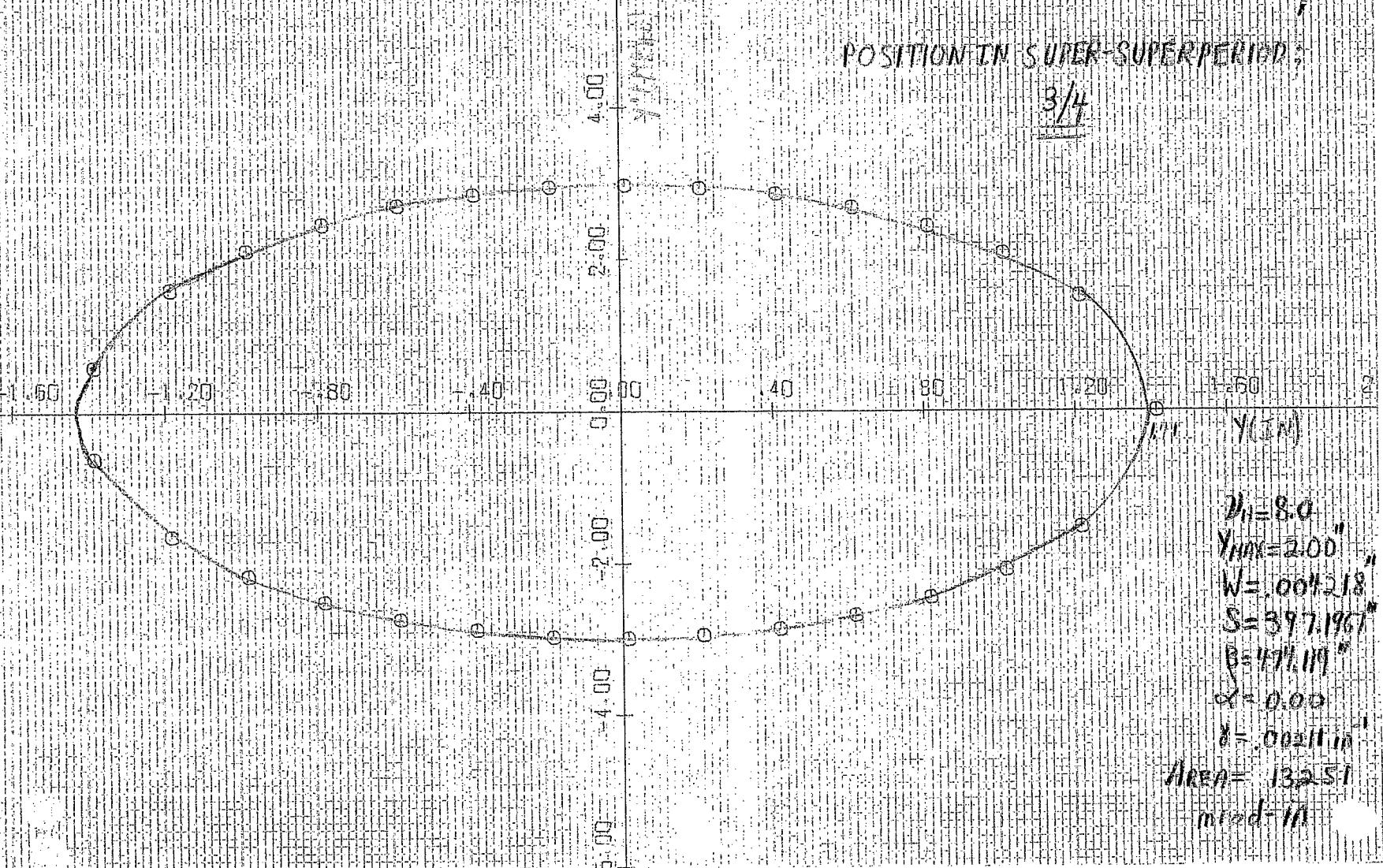


POSITION IN SUPER-SUPER PERIOD:



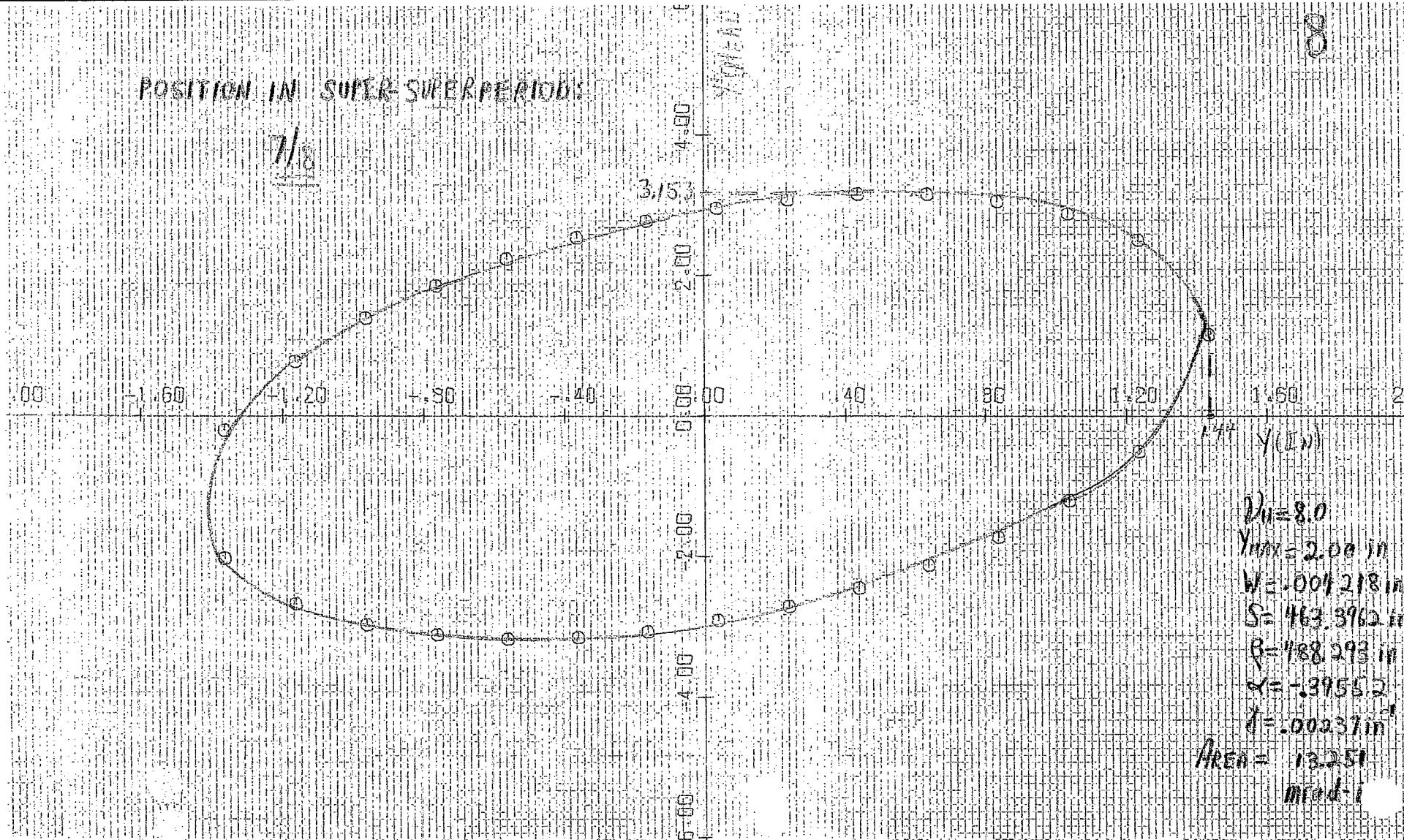
POSITION IN SUPER-SUPER PEND.

3/4



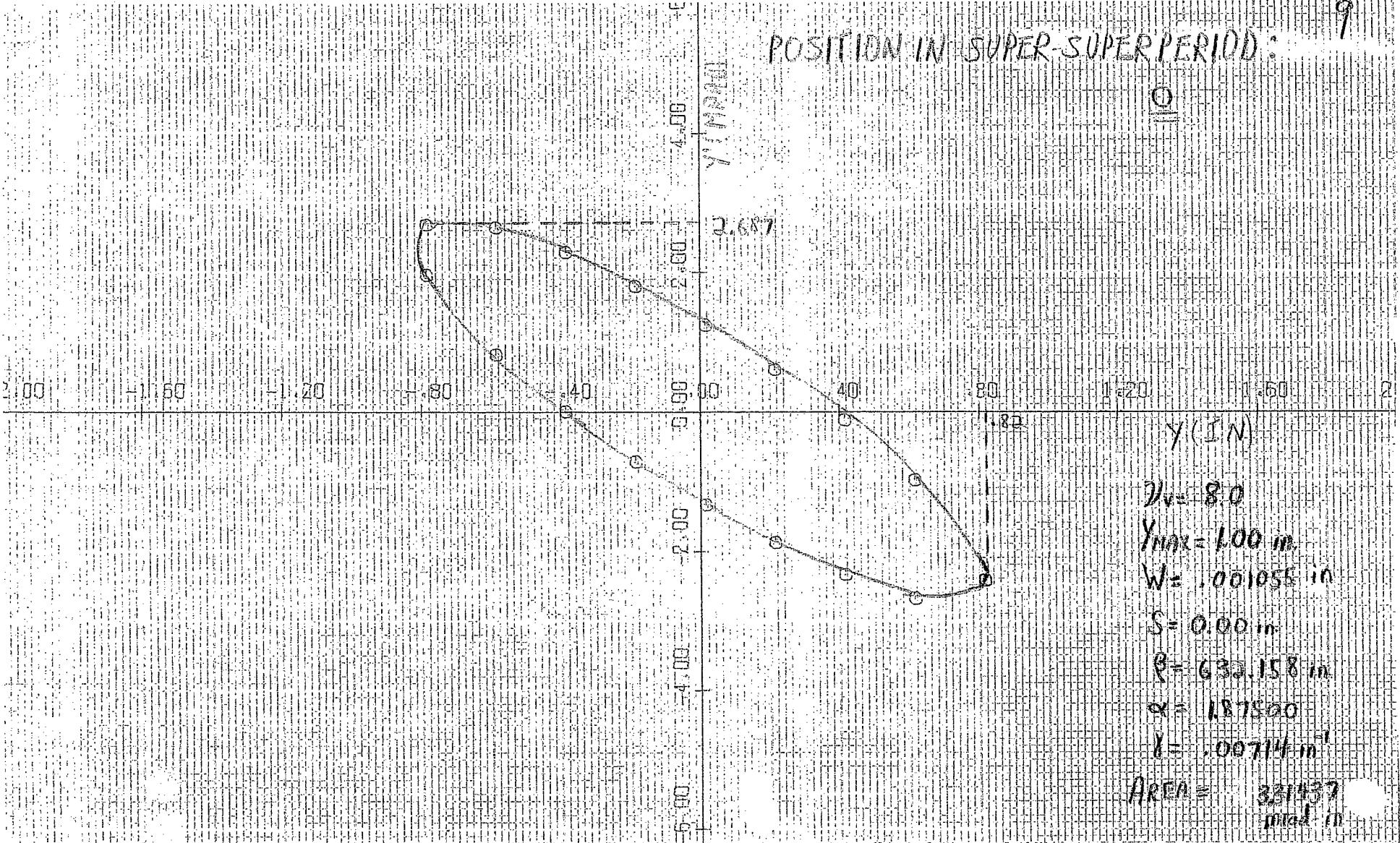
$D_1 = 8.0$ "
 $Y_{MAX} = 2.00$ "
 $W = .004218$ "
 $S = 39.71967$ "
 $B = 474.119$ "
 $\alpha = 0.00$ "
 $\delta = .00211$ in
 $AREA = 132.51$ "
 $m^2 d = 11$

POSITION IN SUPER-SUPER PATTERN



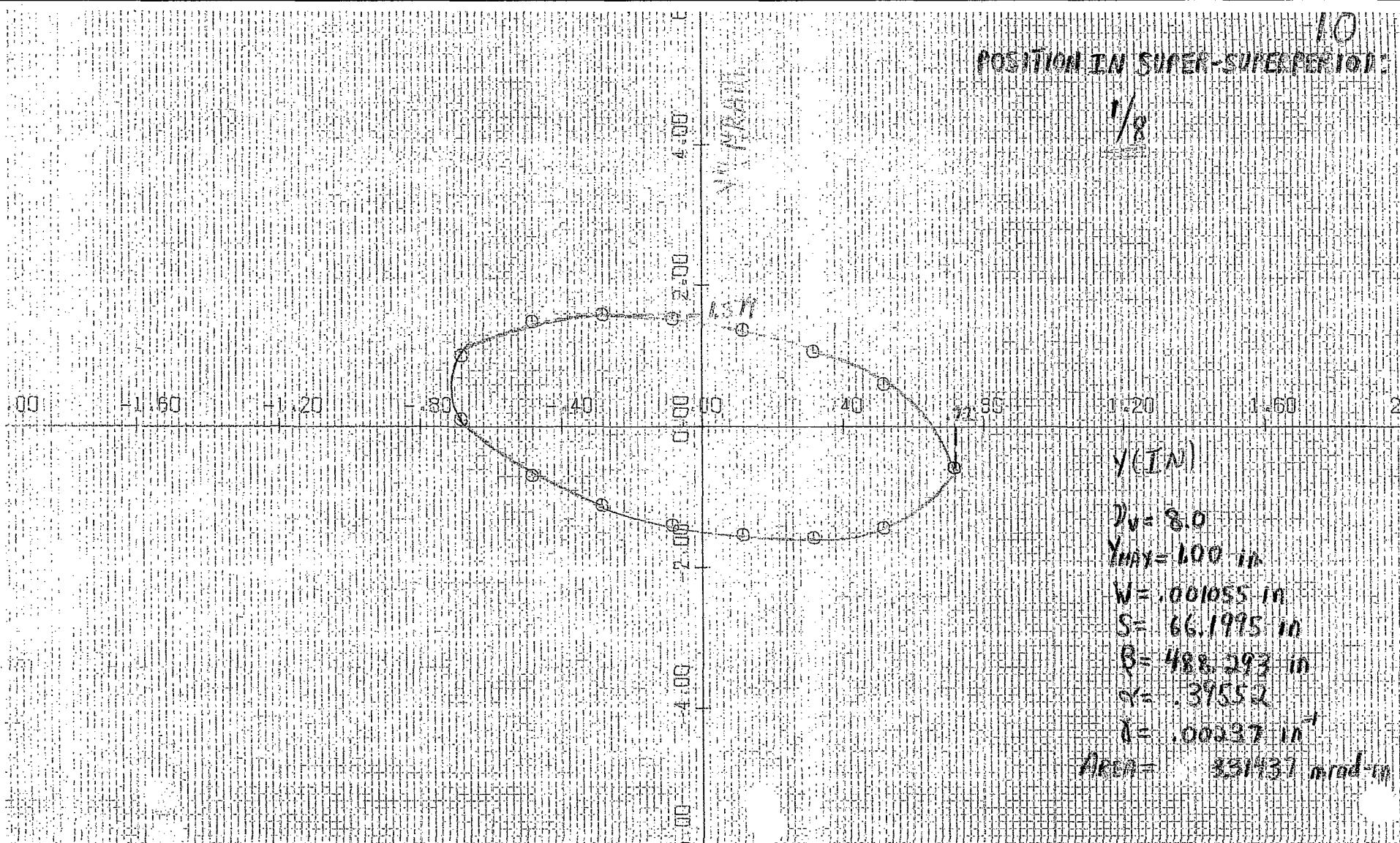
$$\begin{aligned}D_0 &= 8.0 \\Y_{max} &= 2.00 \text{ m} \\W &= 1.004218 \text{ m} \\S &= 462.3962 \text{ m} \\B &= 188.293 \text{ m} \\x &= -3.9555 \text{ m} \\y &= -0.00337 \text{ m} \\Area &= 13.251 \text{ m}^2 \text{ per d-1}\end{aligned}$$

POSITION IN SUPER-SUPER PERIOD:



10
POSITION IN SUPER-SUPERIOR

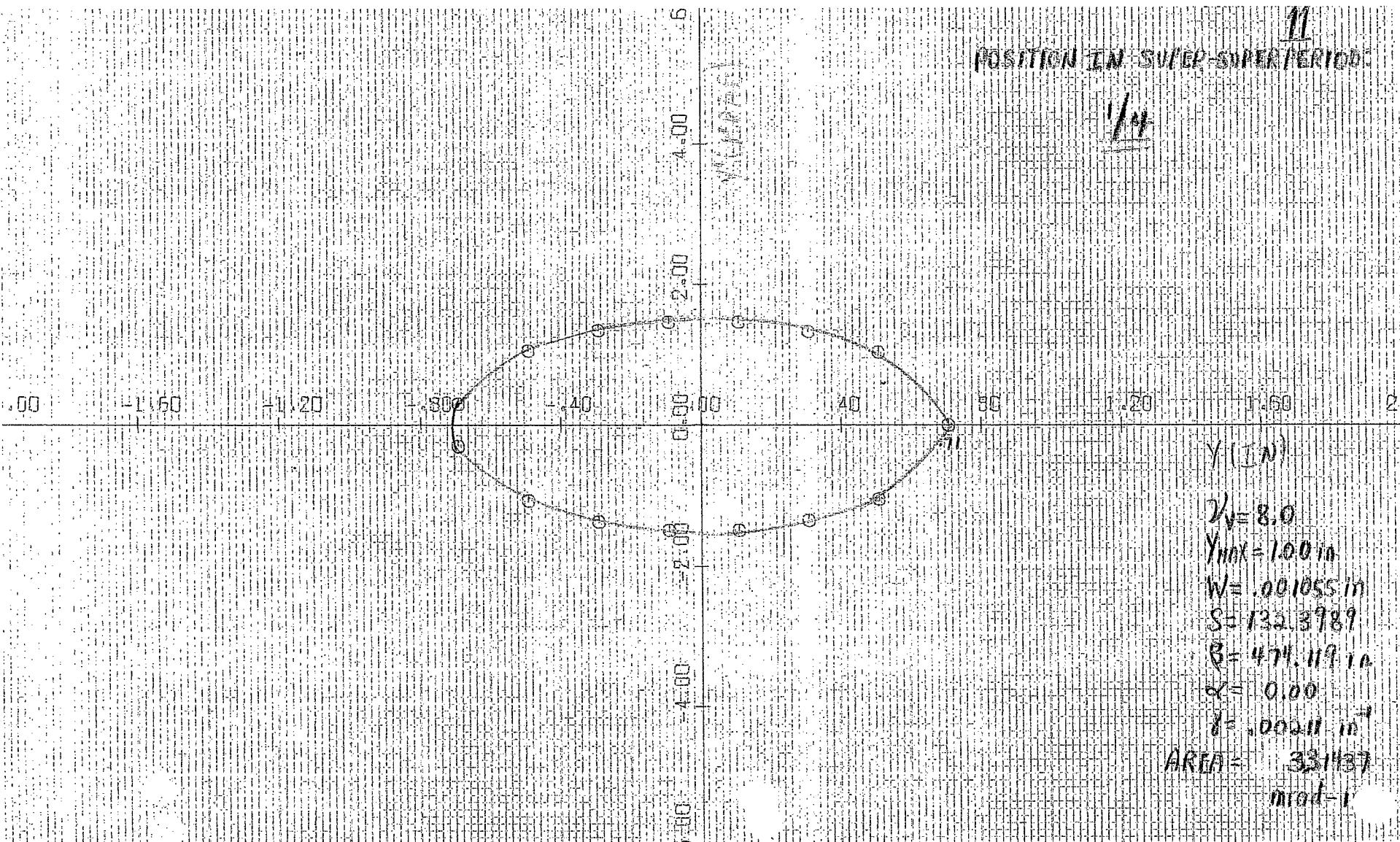
1/8



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POSITION IN SWEEP-SUPER PERIOD

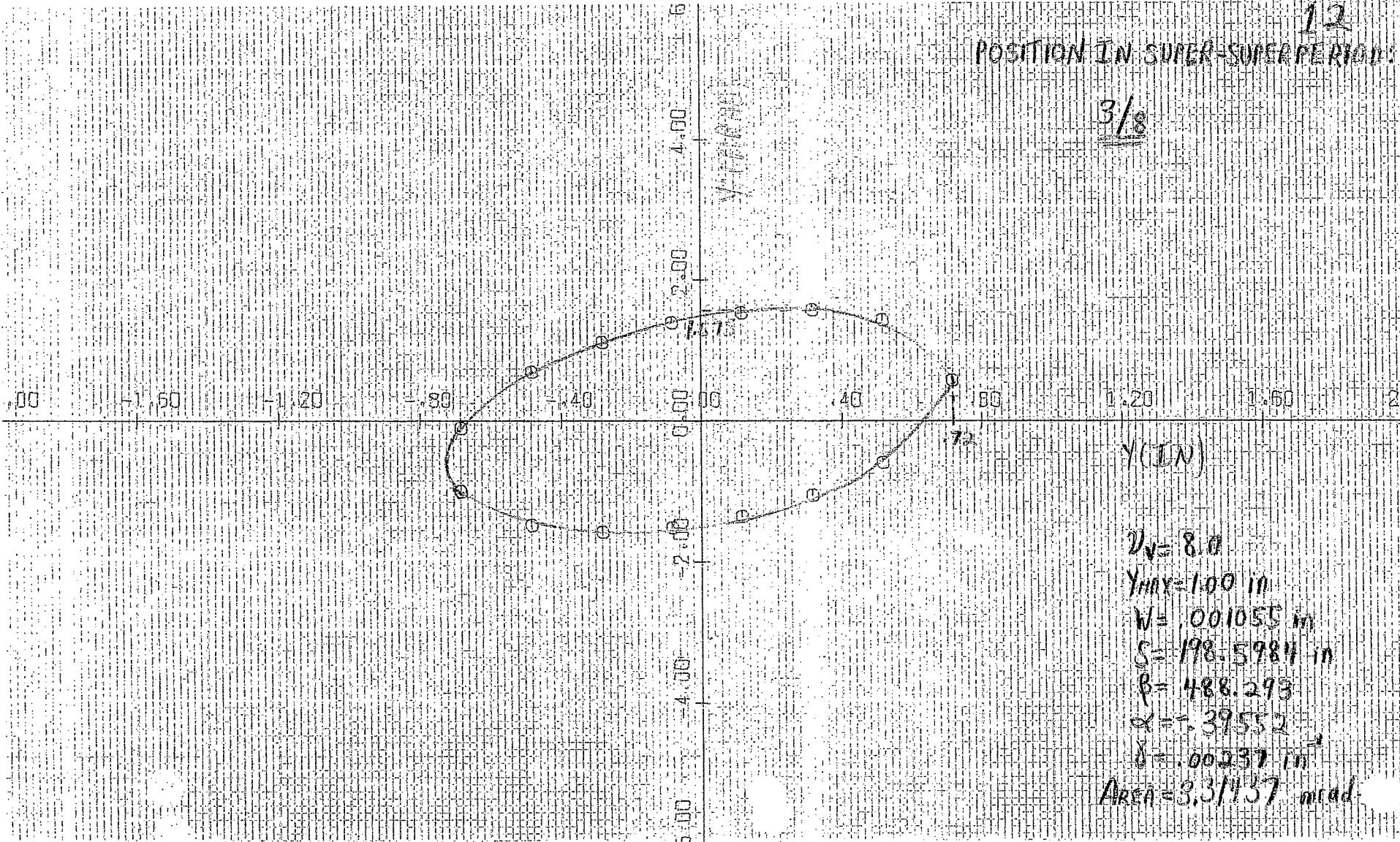
14



12

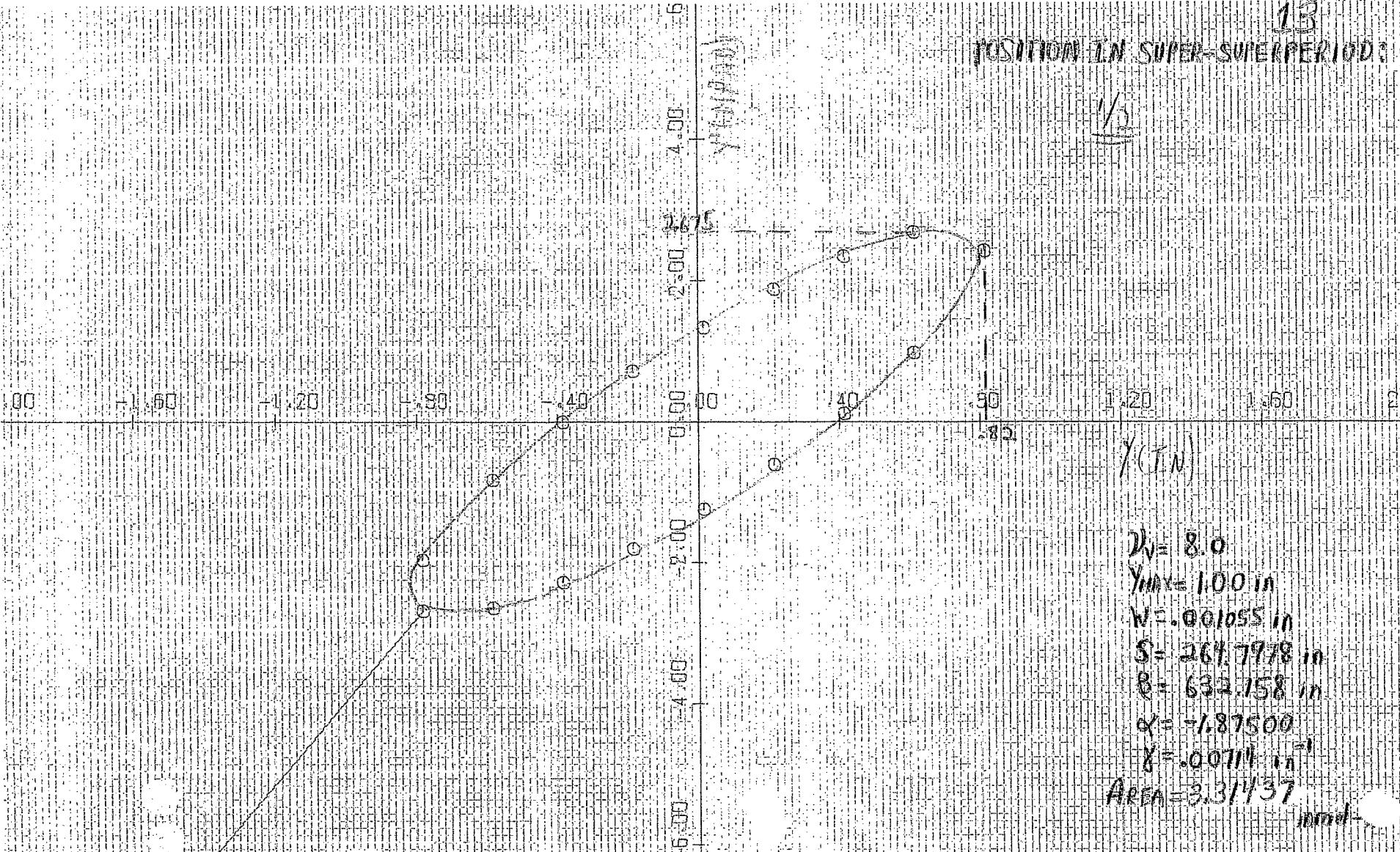
POSITION IN SUPER-SUPERSONIC REGION.

3/8



POSITION IN SUPER-SUPER PERIOD:

1/1



(IN)

$$D_V = 8.0$$

$$Y_{max} = 100 \text{ in}$$

$$N = .001055 \text{ in}$$

$$S = 264.7978 \text{ in}$$

$$\theta = 632.158 \text{ in}$$

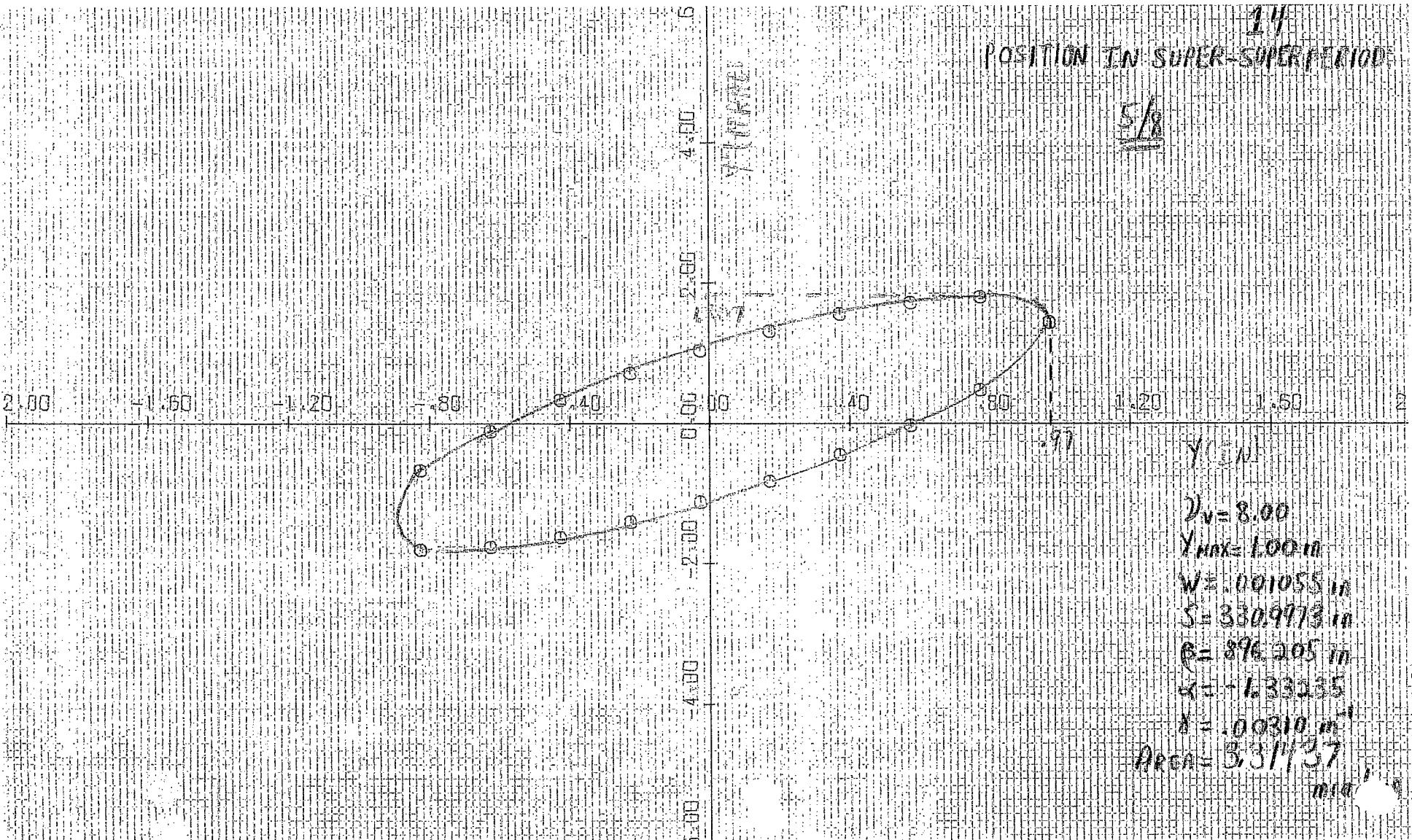
$$\alpha' = -1.87500$$

$$\gamma = .00714 \text{ in}^{-1}$$

$$Area = 3.37137 \text{ in}^2$$

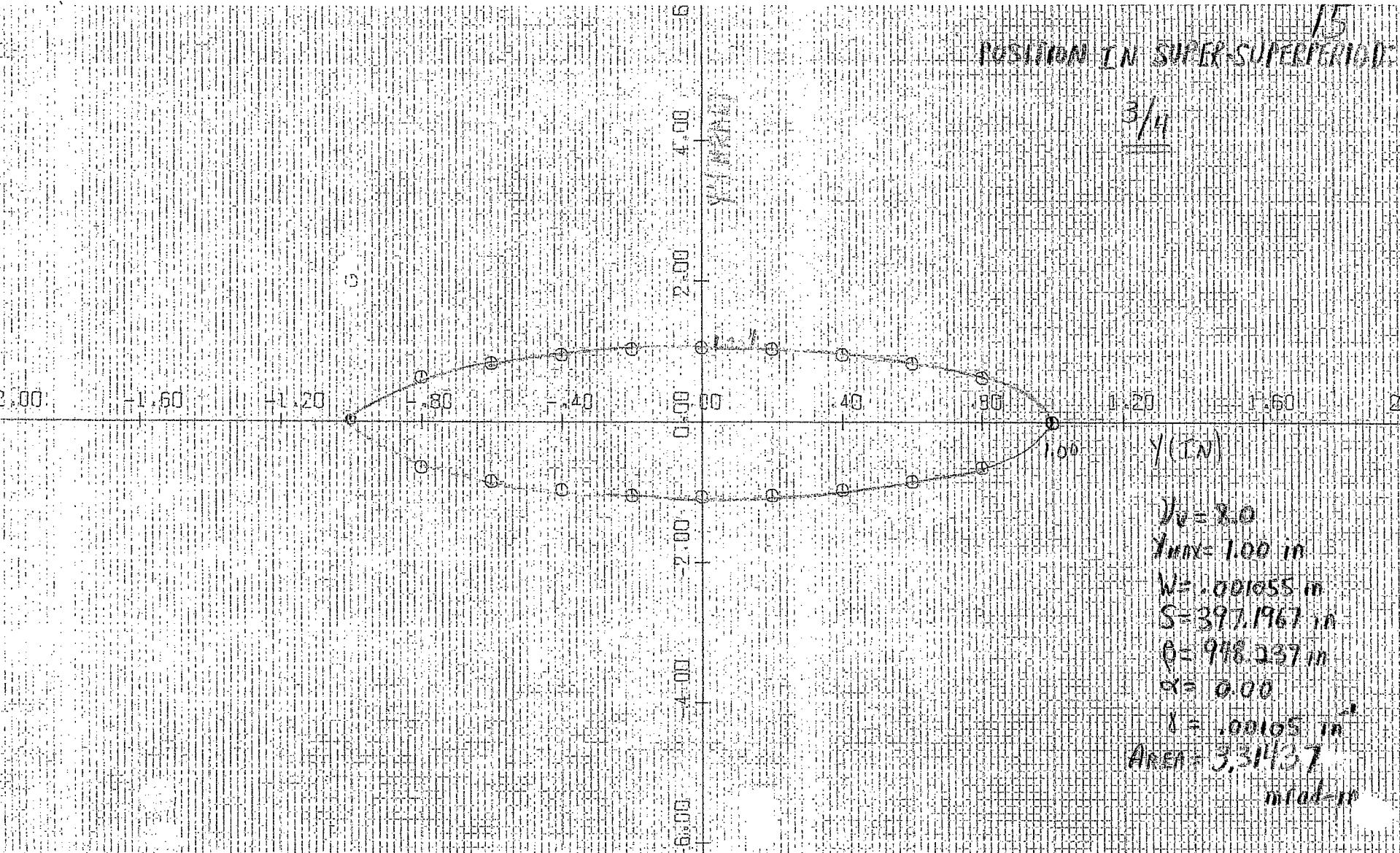
14
POSITION IN SUPER-SUPERIOR

5/8



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POSITION IN SUPER-SUPERPERIOD

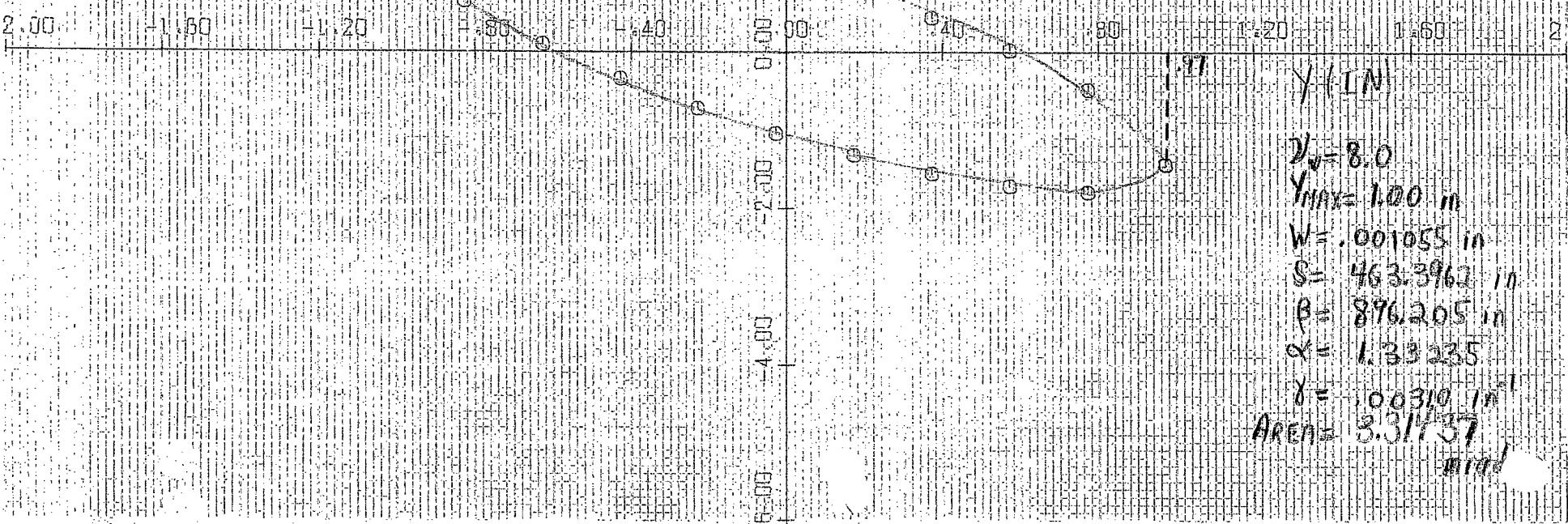
3/4



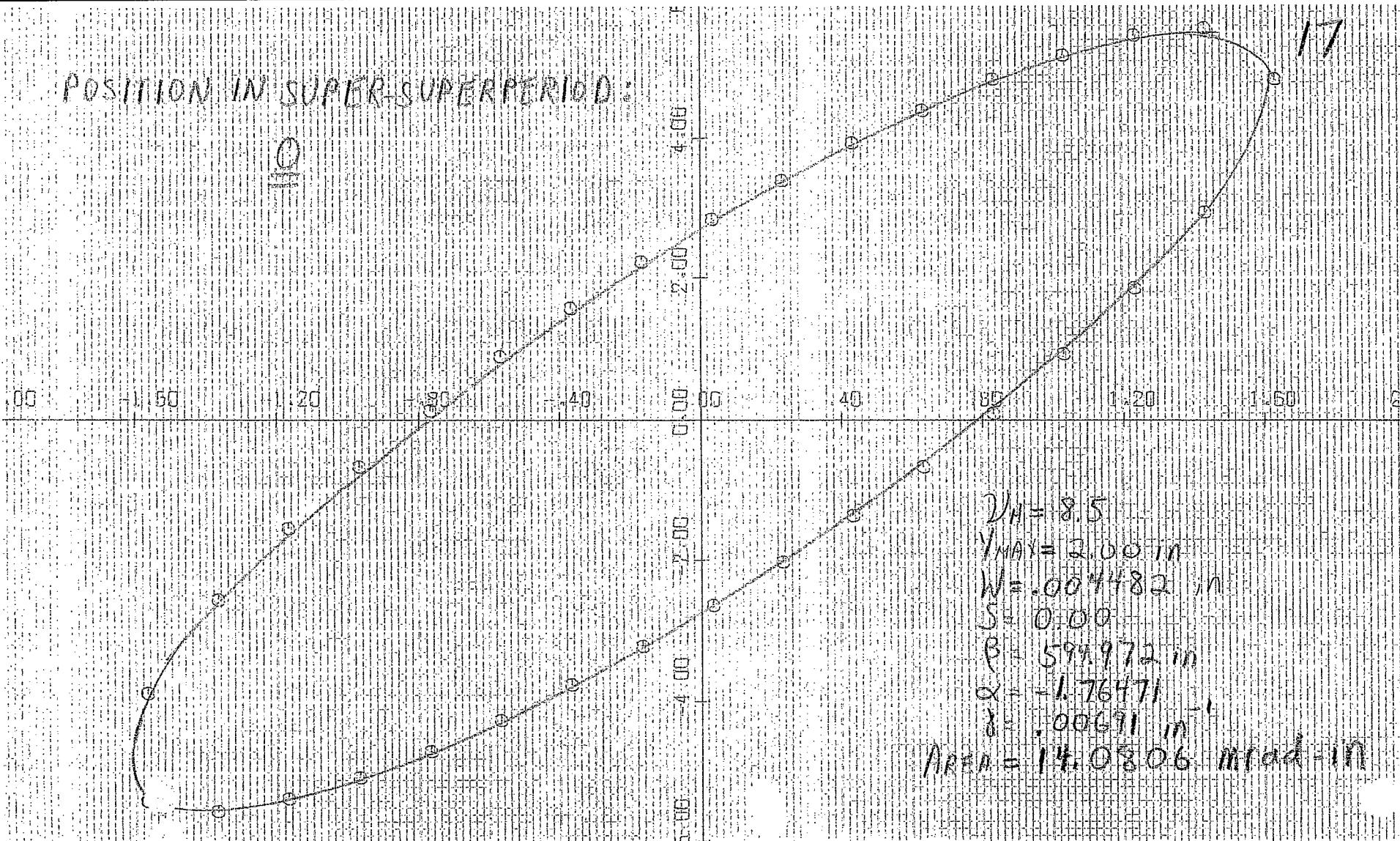
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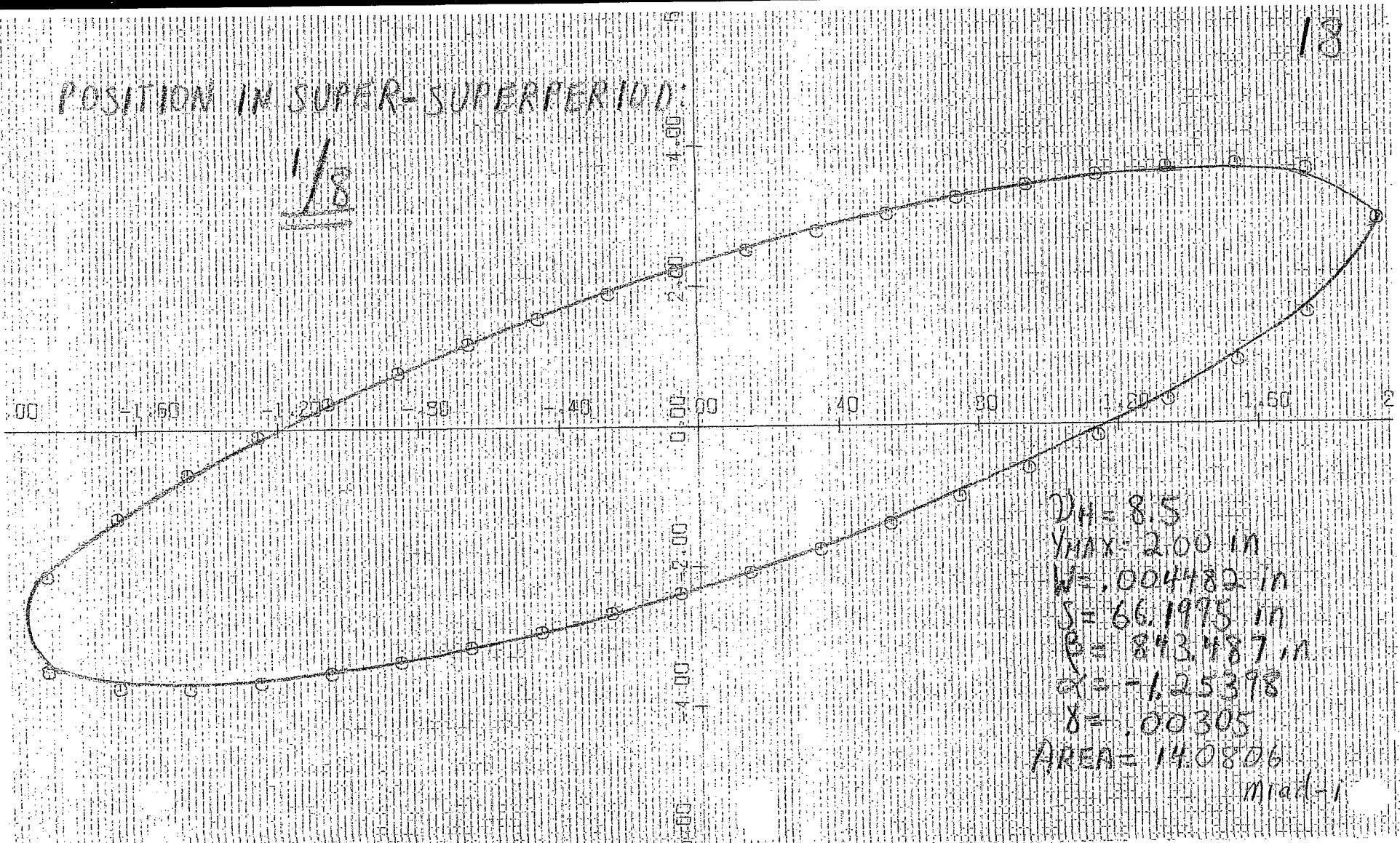
Position IN SUPER-SUPER PERIOD

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POSITION IN SUPER-SUPER PERIOD



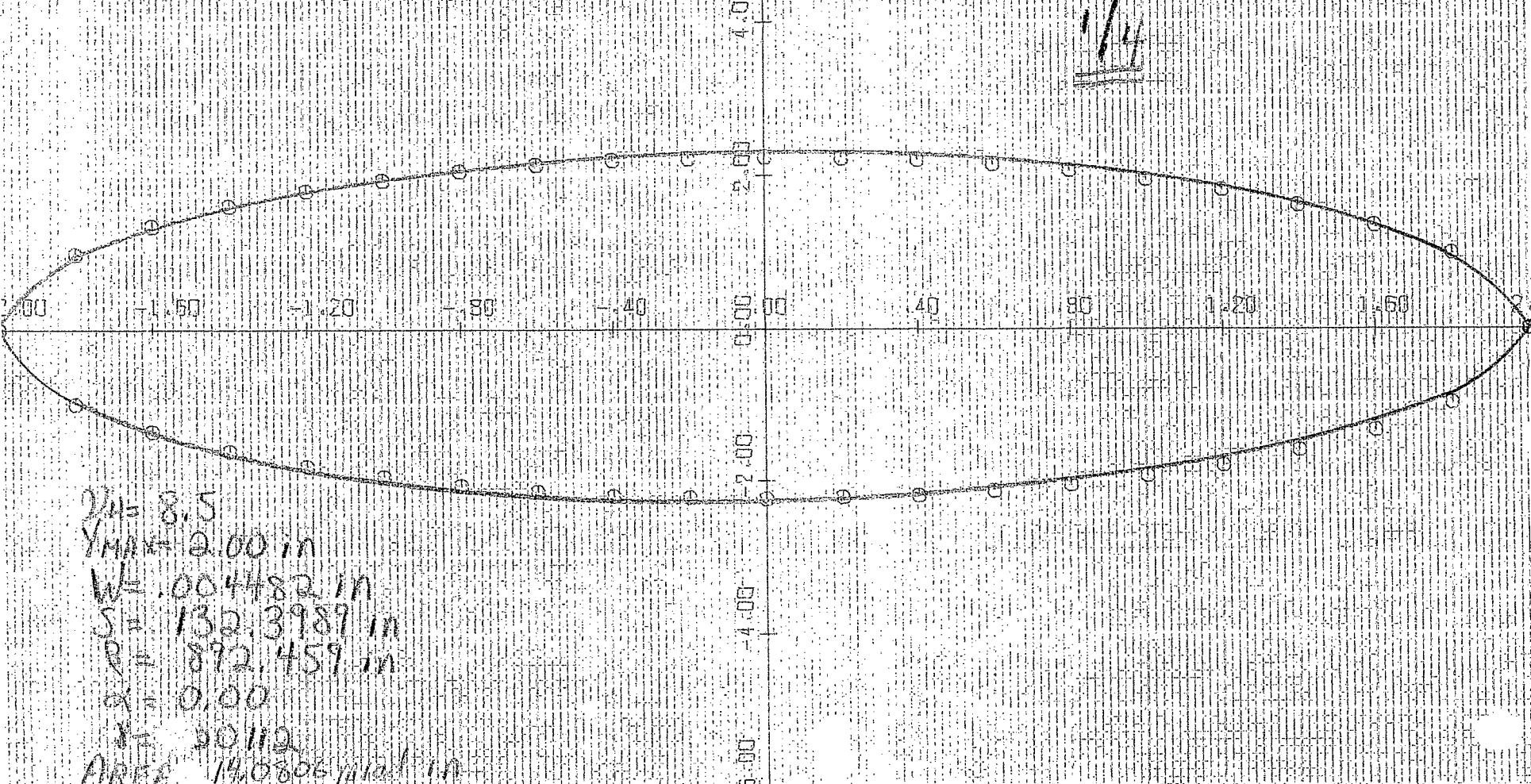


$$\begin{aligned}
 & x_{\max} = 0.85 \text{ m} \\
 & y_{\max} = 0.20 \text{ m} \\
 & S = 66.1975 \text{ m}^2 \\
 & R = 843.487 \text{ m} \\
 & d = -1.25398 \\
 & \theta = 0.0305 \\
 & \text{AREA} = 14.0806 \text{ m}^2 \text{ rad}^{-1}
 \end{aligned}$$

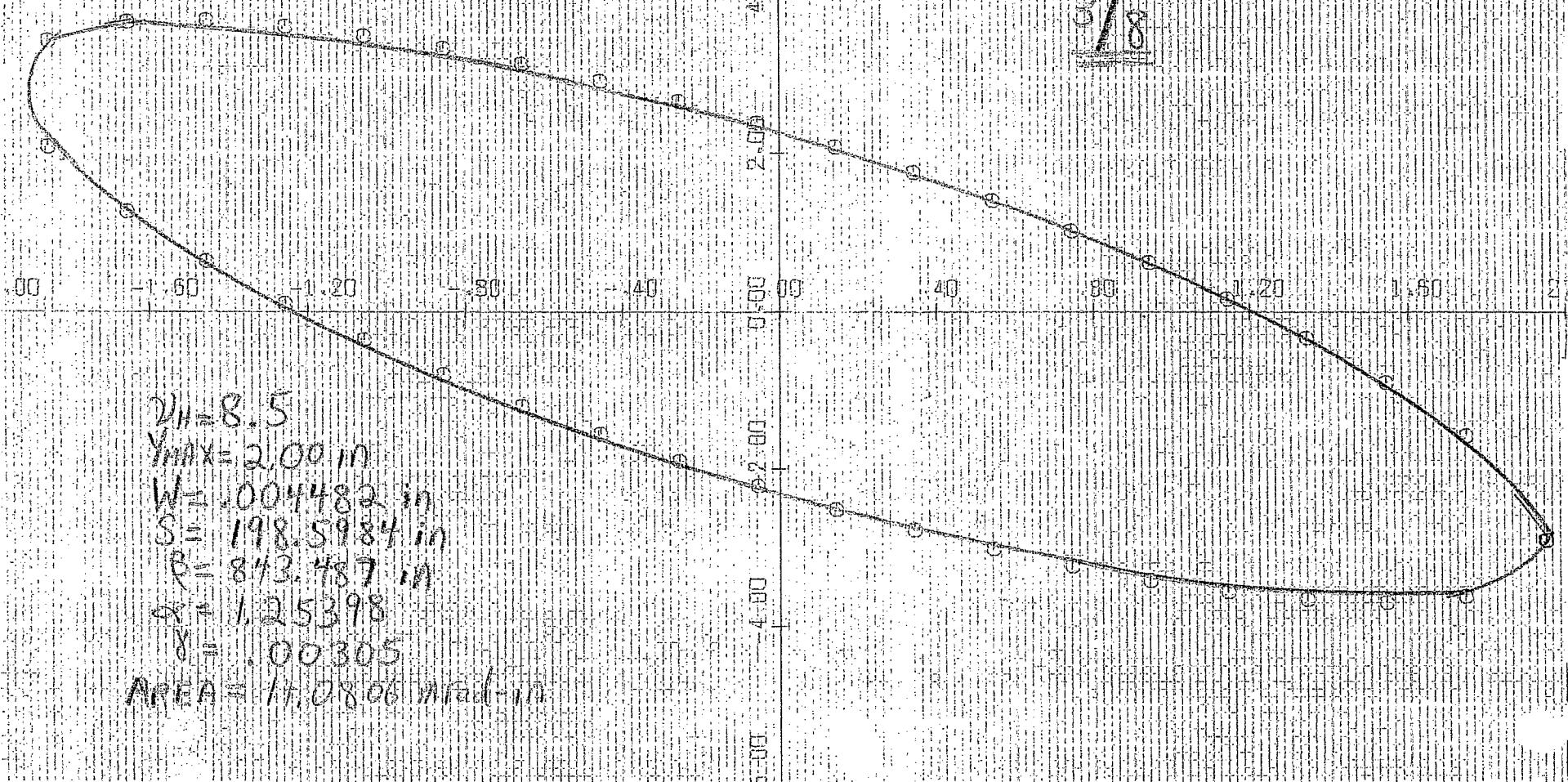
19

POSITION IN SUPER-SUPERIOR.

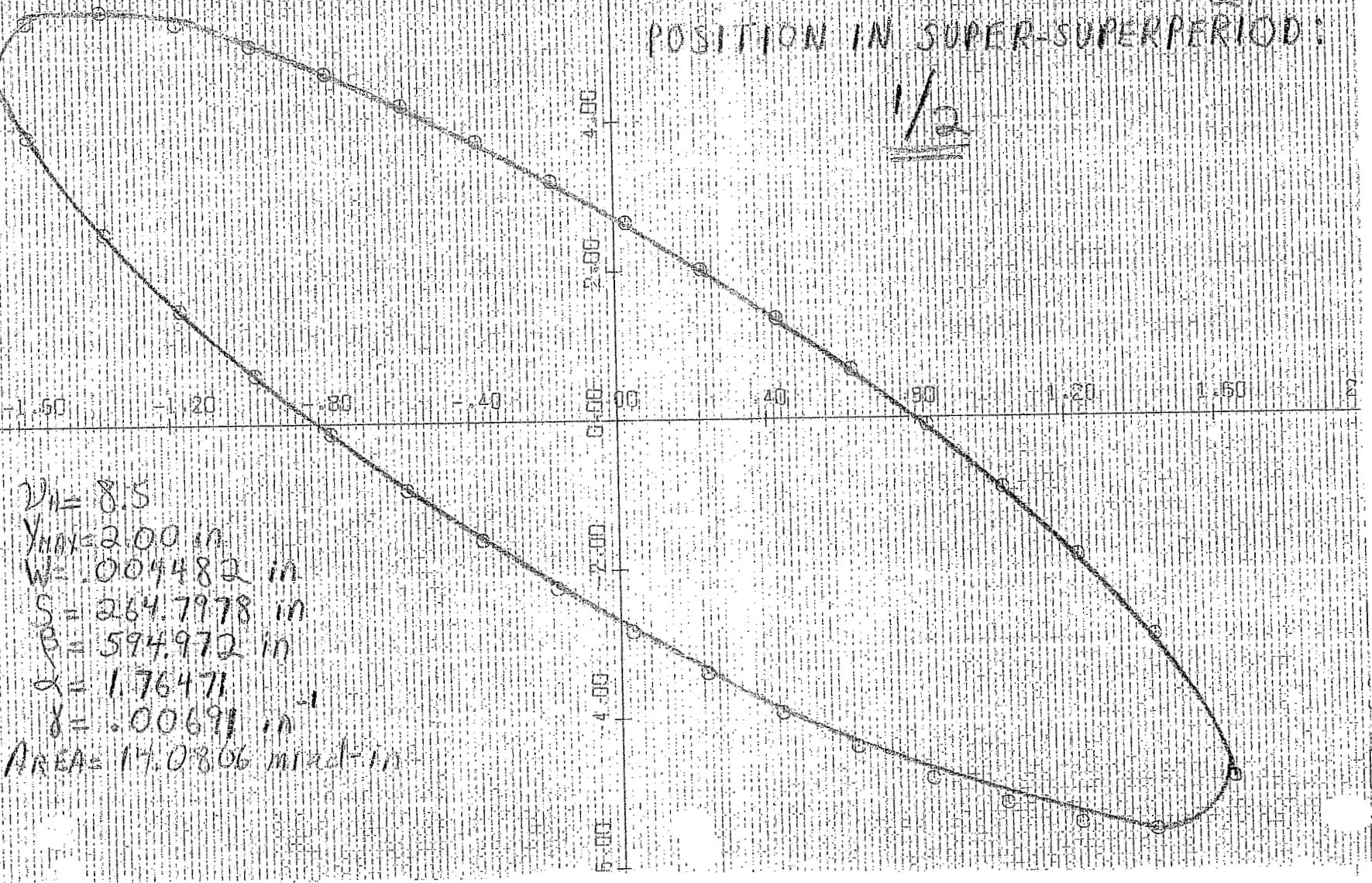
1/4



POSITION IN SUPER-SUPER PERIOD

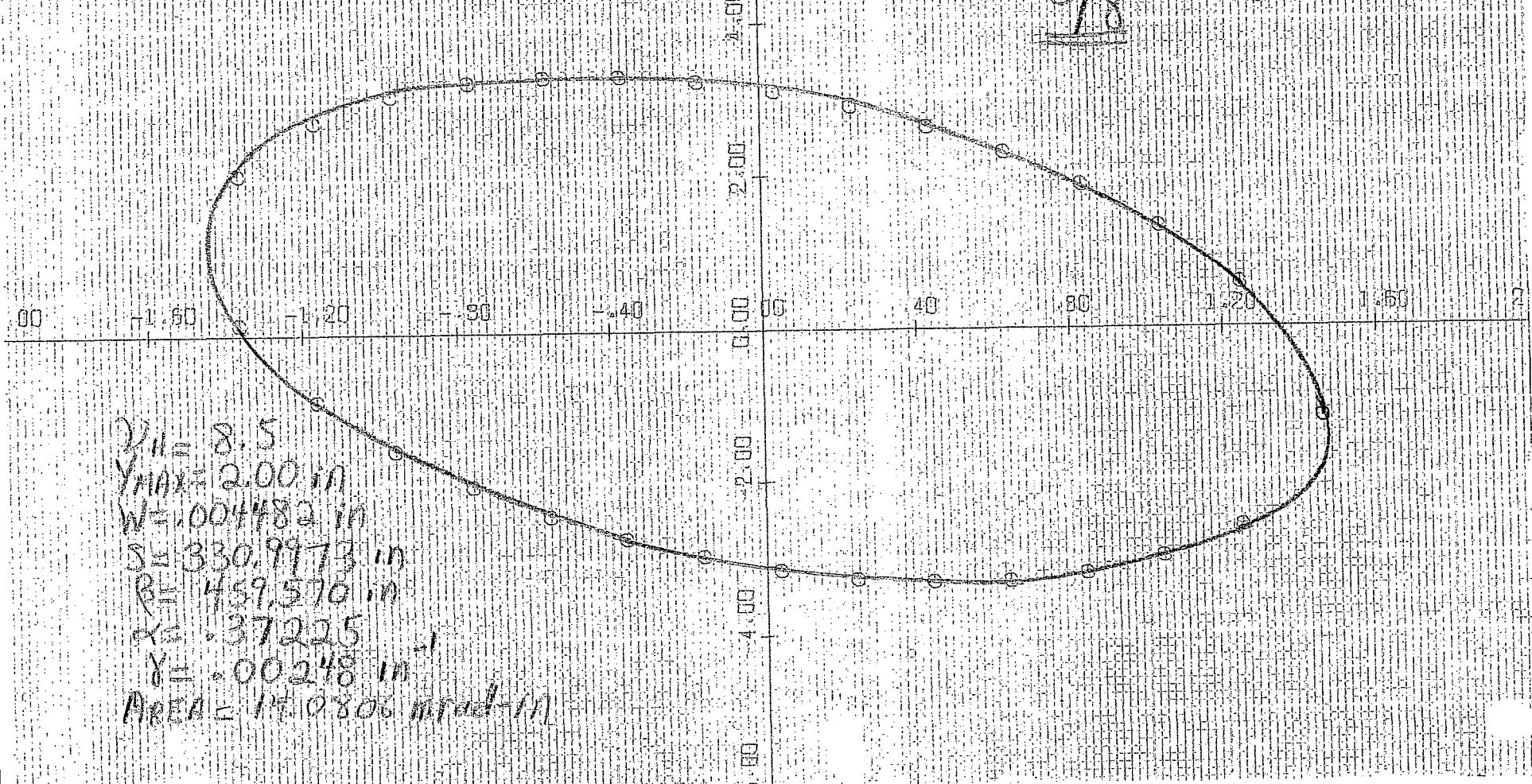


POSITION IN SUPER-SUPERPERIOD:



POSITION IN SUPER-SUPER PERIOD

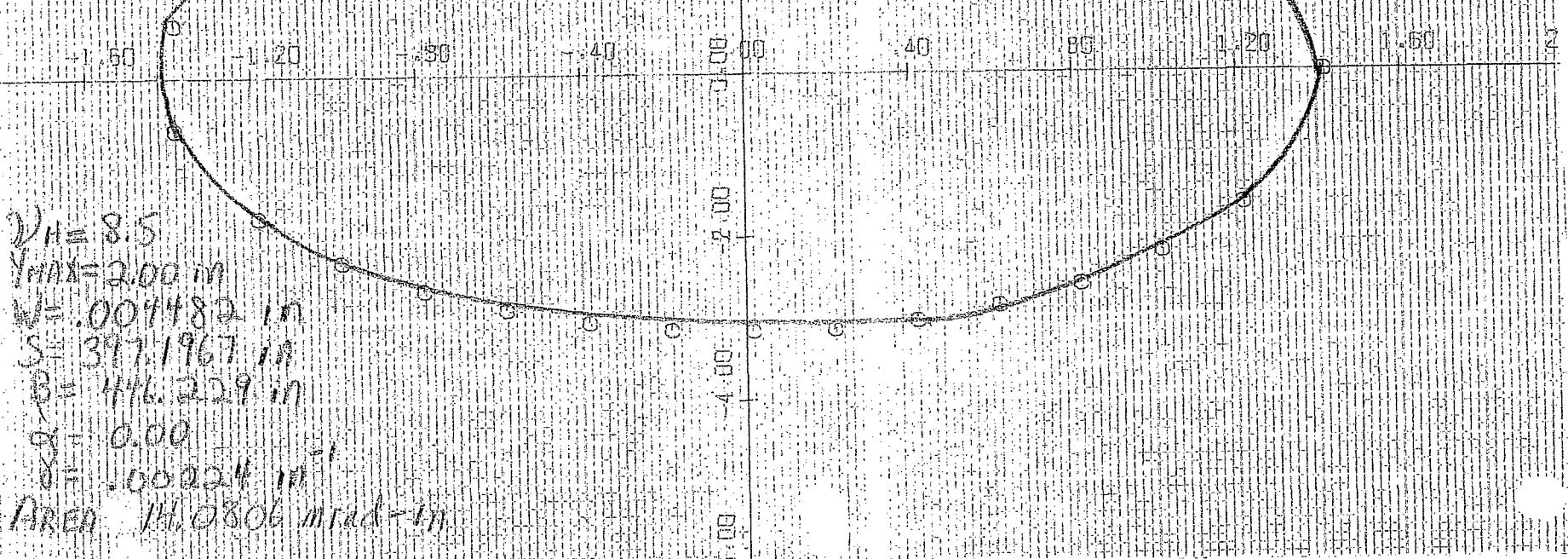
5/8



POSITION IN SUPER-SUPER PERIOD

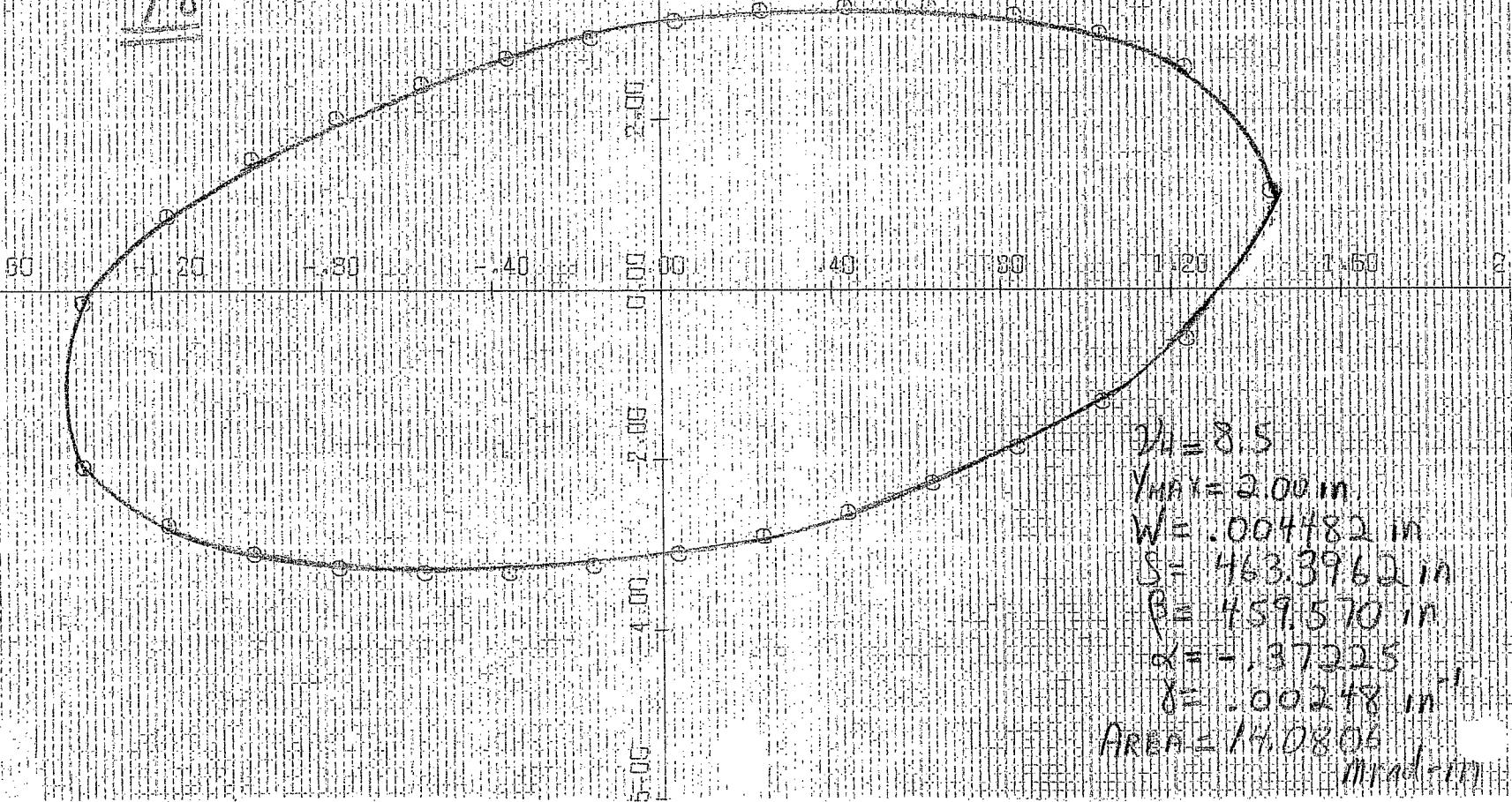
23

3
4



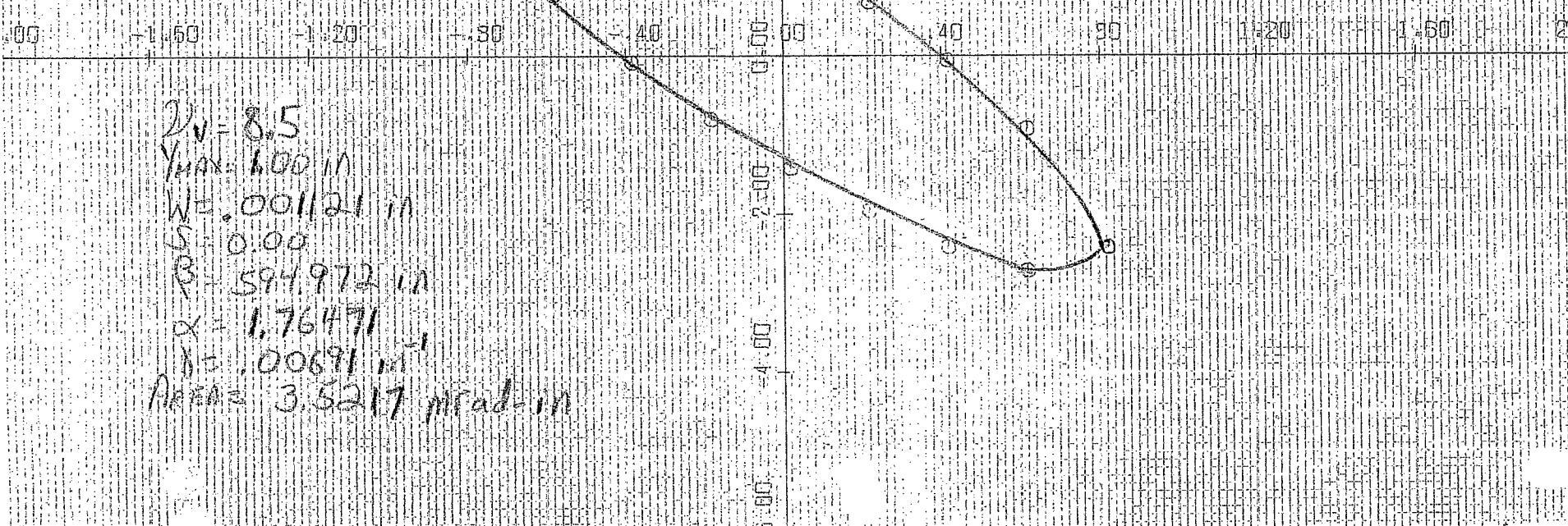
POSITION IN SUPER-SUPER PERIOD

1/8



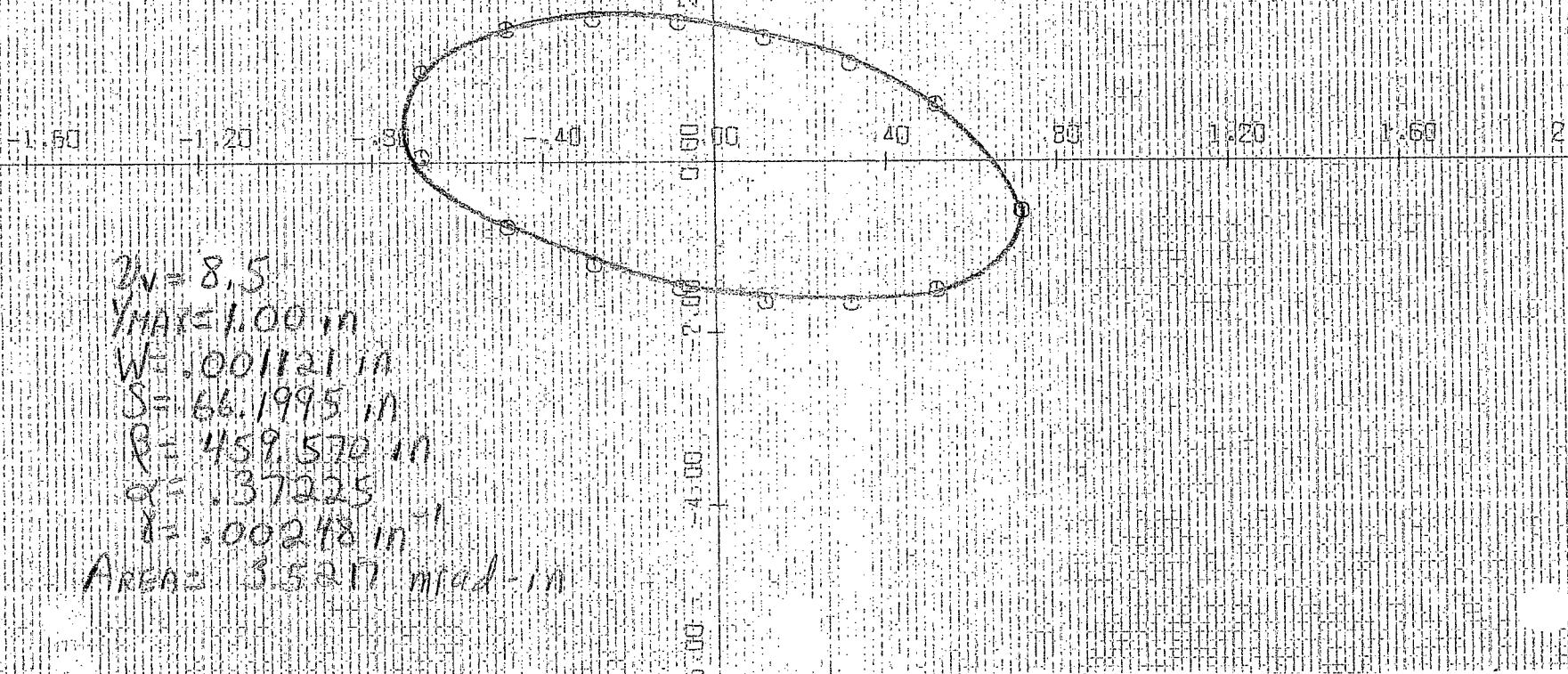
125

POSITION IN SUPER-SUPERPOSITION.



$\Sigma = 8.5$
 Year = 1.00 m
 N = 0.00
 S = 594.972 m
 X = 1.76471 m
 Y = 0.0691 m
 Position = 3.5217 m rad/m

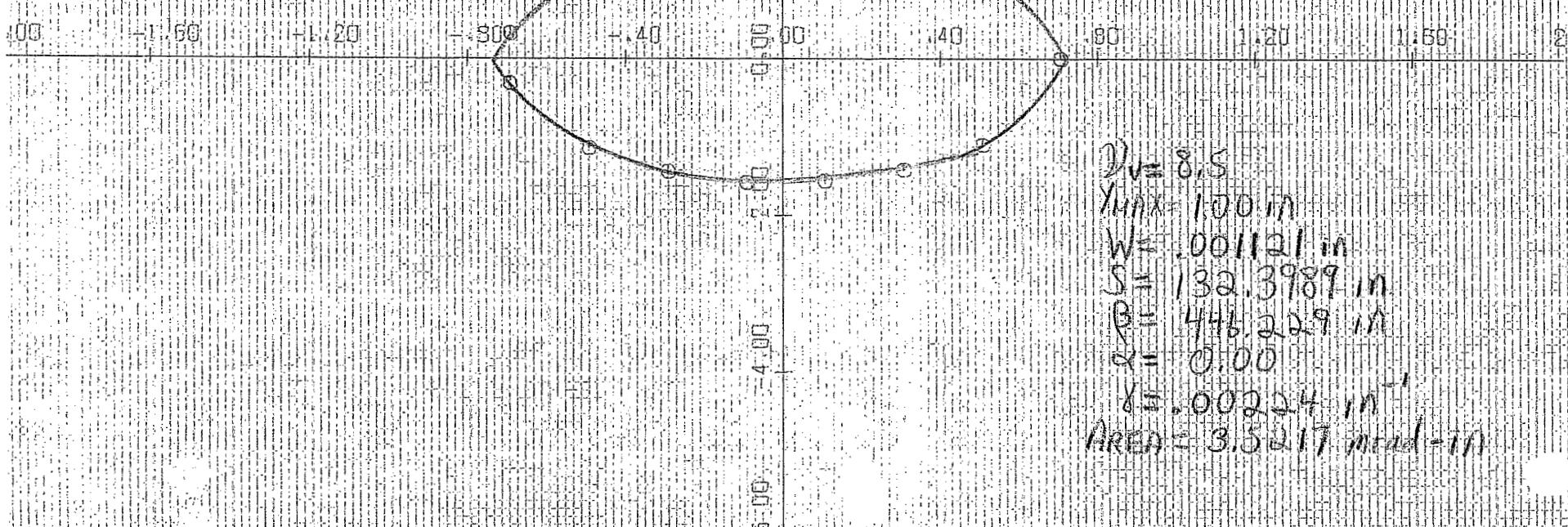
POSITION IN SUPER-SUPER PERIOD



27

POSITION IN SUPER-SUPER PERIOD

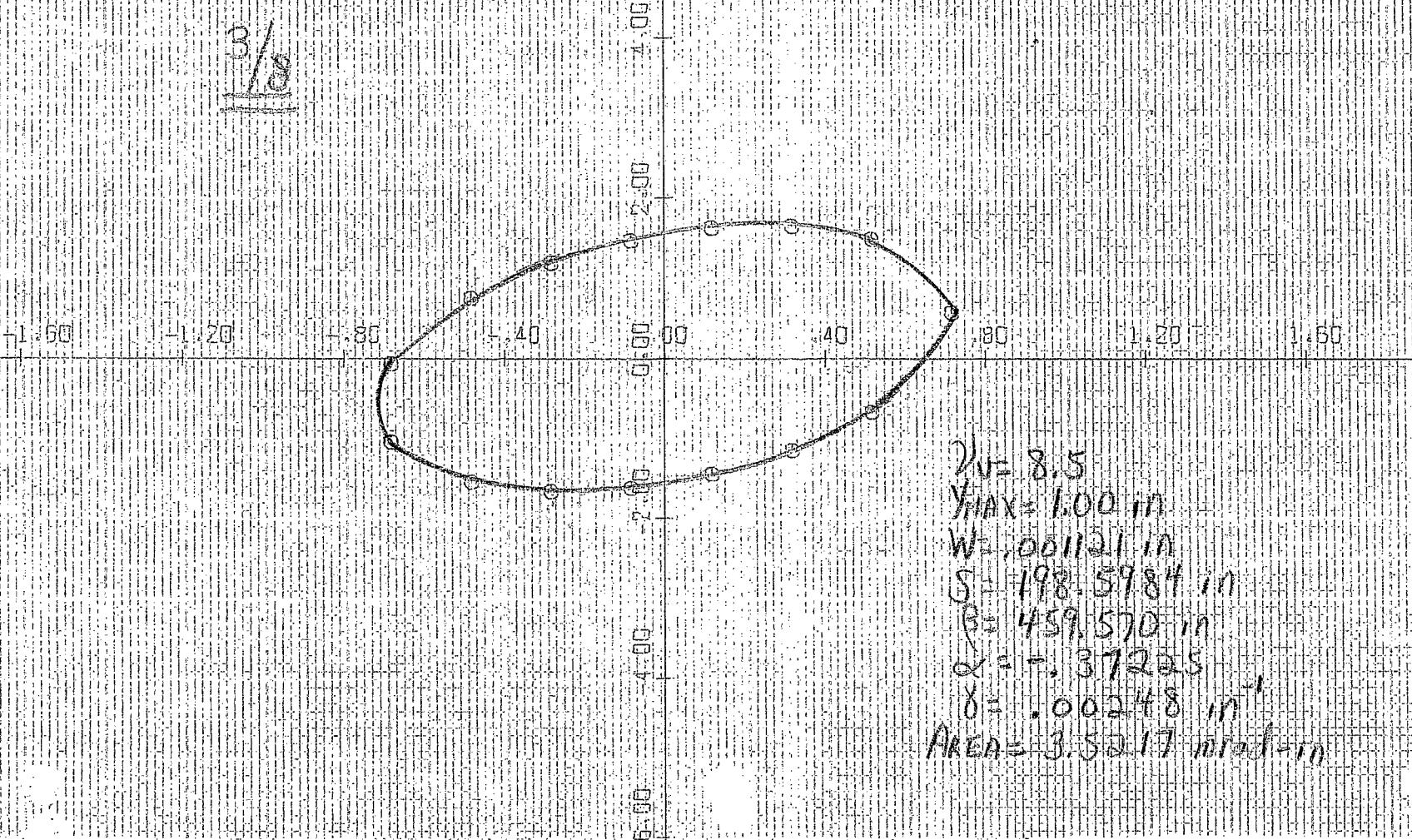
1/4



$$\begin{aligned}
 v &= 8.5 \\
 Y_{MAX} &= 100 \text{ m} \\
 Y_{MIN} &= -100 \text{ m} \\
 S &= 132.3989 \text{ m} \\
 Q &= 446.229 \text{ m} \\
 \alpha &= 0.00 \\
 \gamma &= 0.00244 \text{ m}^{-1} \\
 \text{AREA} &= 3.5417 \text{ m}^2 \text{ rad}^{-1}
 \end{aligned}$$

POSITION IN SUPER SUPER PERIOD

3/8



$$D = 8.5$$

$$Y_{MAX} = 1.00 \text{ m}$$

$$W = .001121 \text{ m}^3$$

$$S = 198.5984 \text{ m}$$

$$S_C = 459.5702 \text{ m}$$

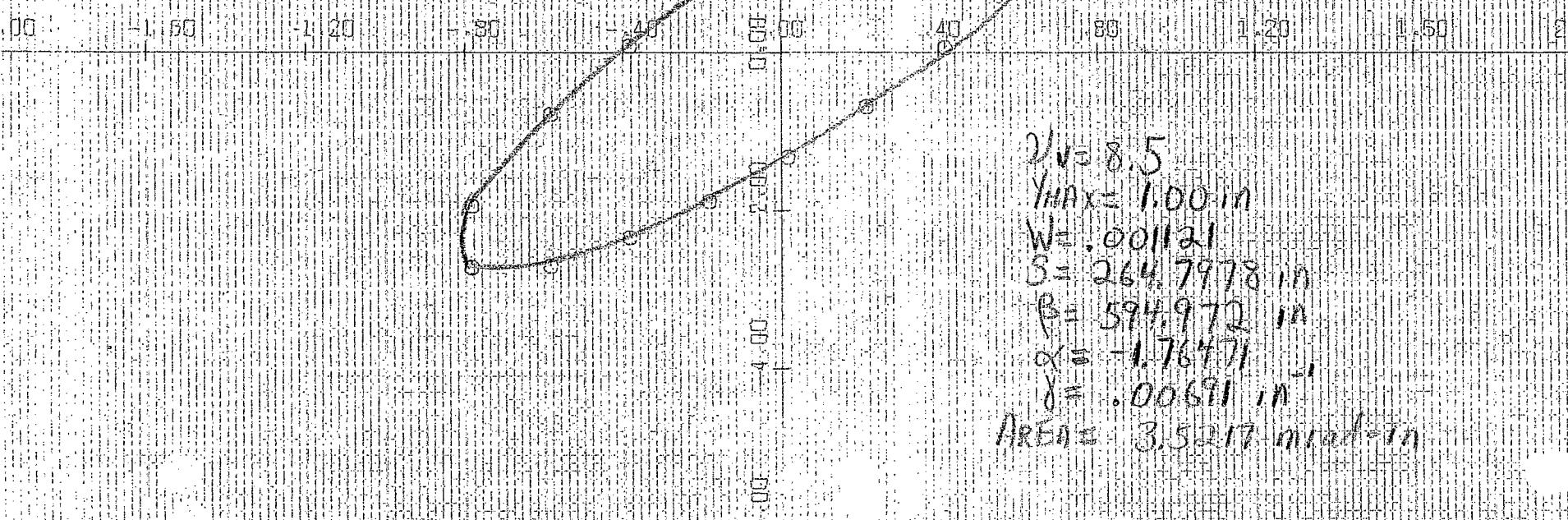
$$S_B = -.371205 \text{ m}$$

$$\alpha = .00248 \text{ m}^{-1}$$

$$AREA = 3.5017 \text{ m}^2 \cdot \text{m}^{-1}$$

POST TIDE IN SUPER-SUPER PERIOD

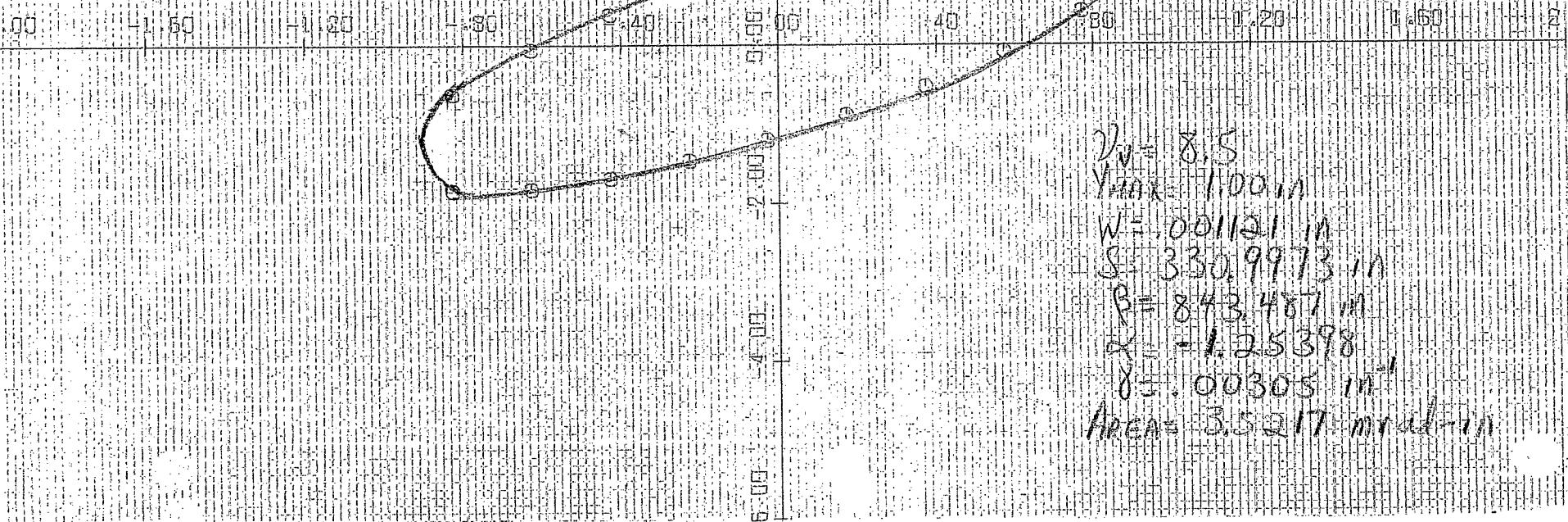
1/5



POSITION IN SURVEY SUPERPERIOD

5/8

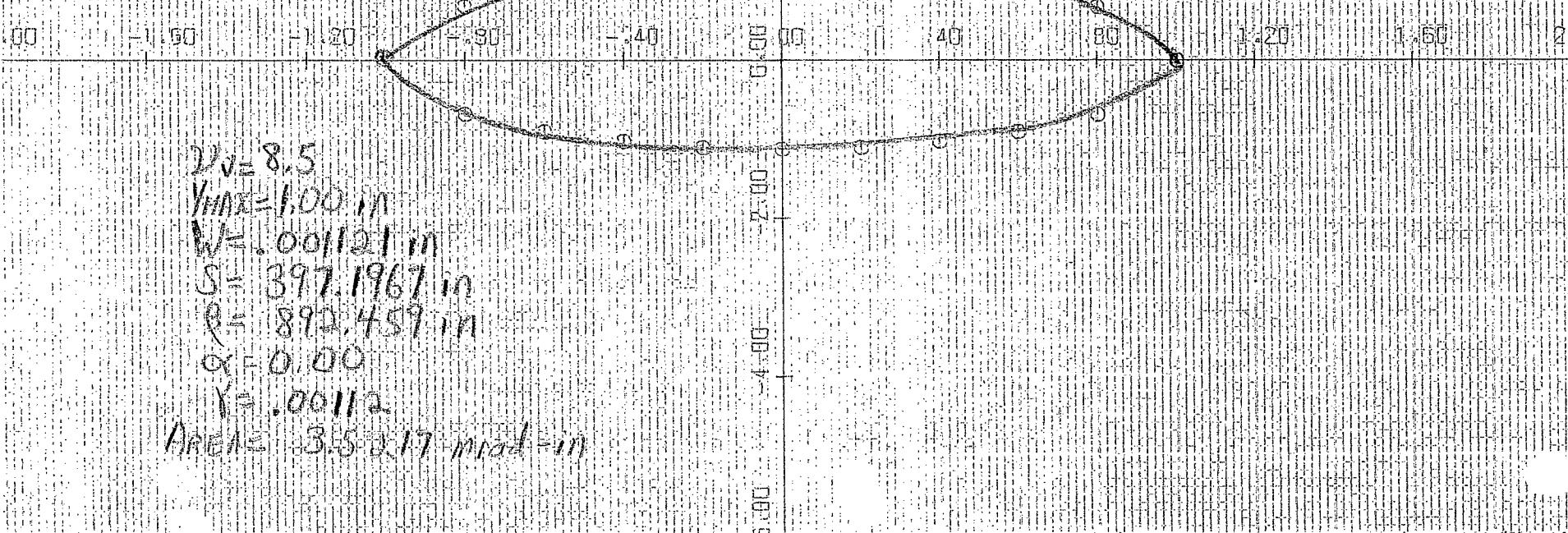
30



B1

NO. 111 IN SUPER-SUPER PERIOD

314



$$2\beta = 8.5$$

$$Y_{MAX} = 1.00 \text{ m}$$

$$W = 397.1967 \text{ m}$$

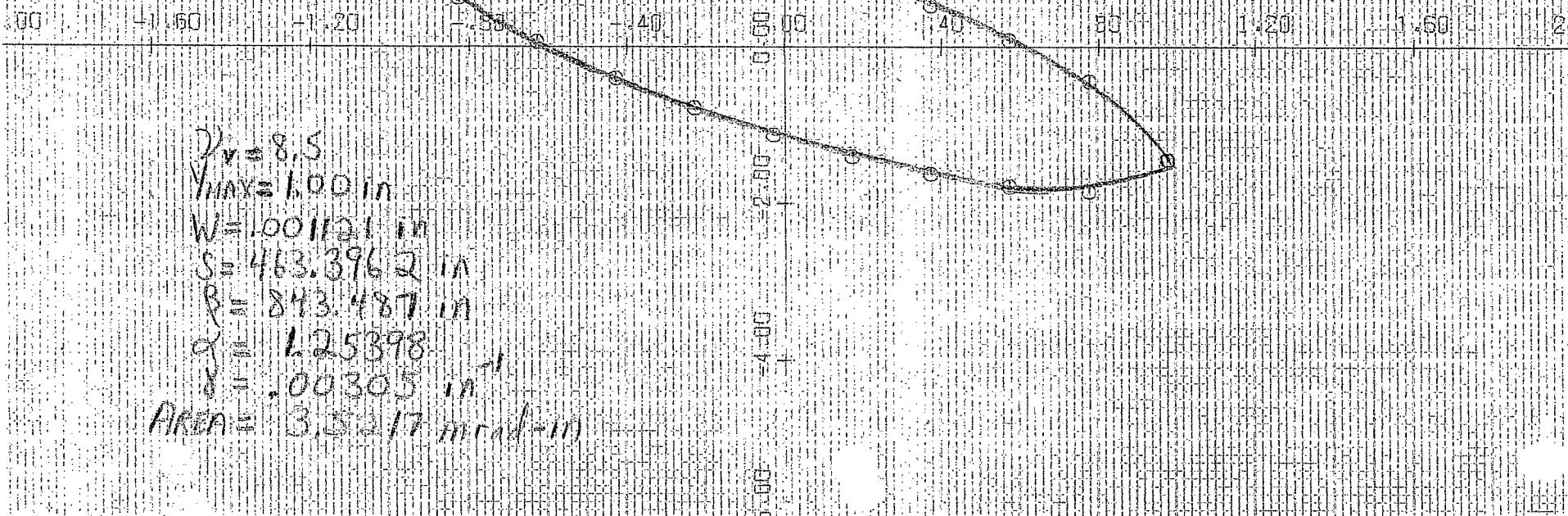
$$S = 0.00$$

$$\alpha = 0.00112$$

$$AREA = 3.5317 \text{ mrad/m}$$

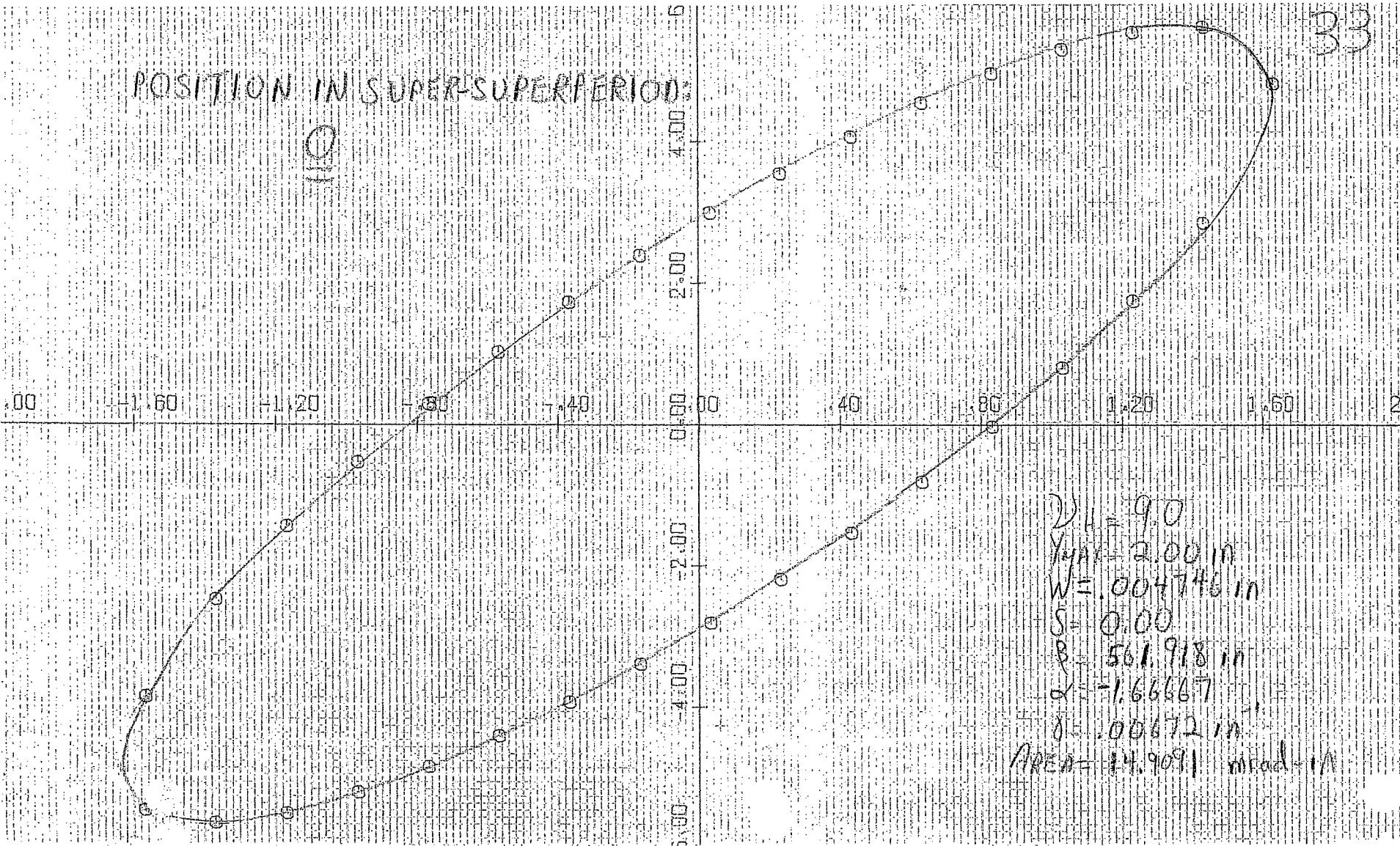
POSITION IN SUPER-SUPERPERIOD

7/8

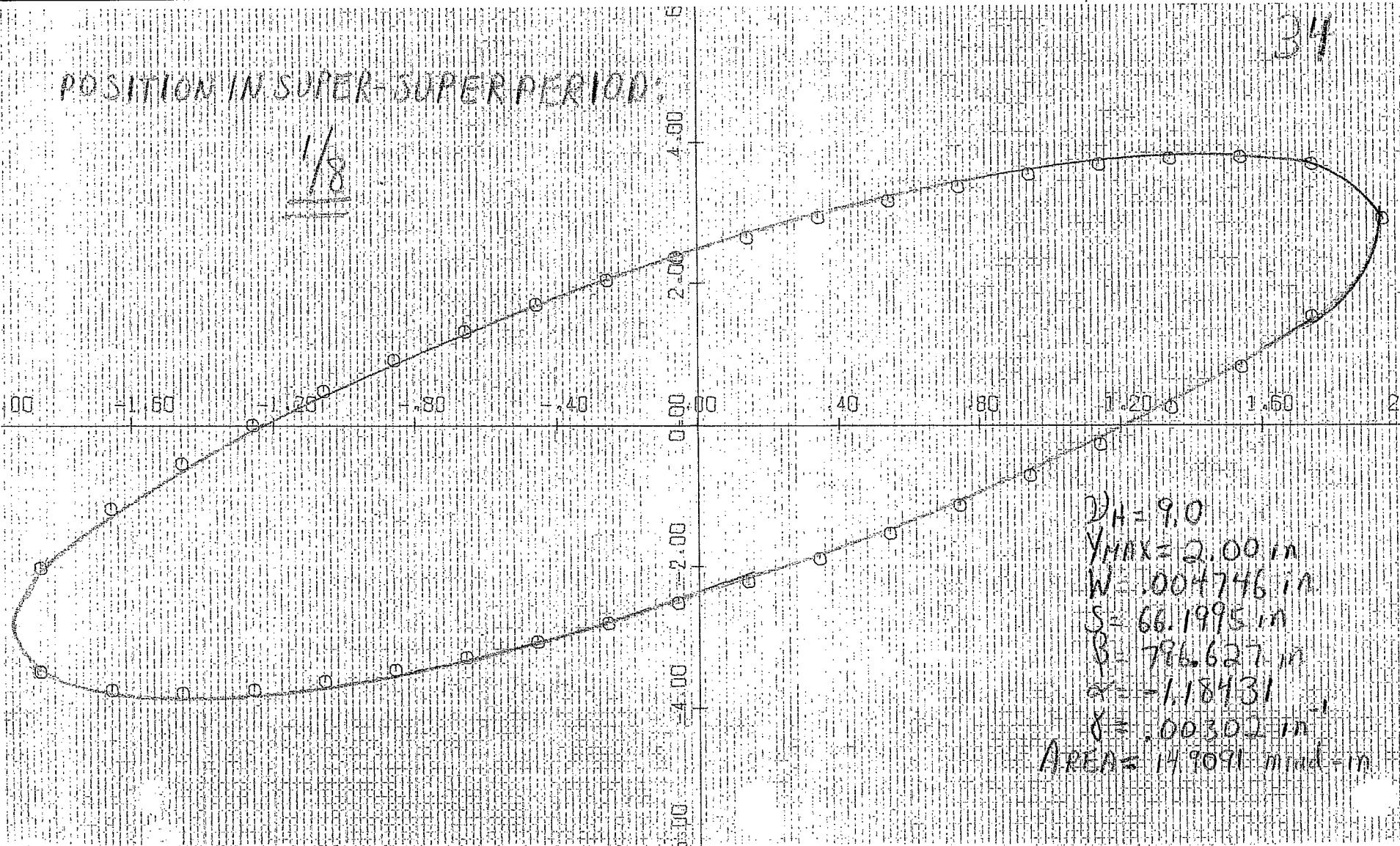


$$\begin{aligned}
 D &= 1.00 \\
 V_{MAX} &= 1.00 \\
 W &= 0.001121 \text{ m} \\
 R_u &= 463.396 \text{ m} \\
 S_u &= 843.487 \text{ m} \\
 L &= 1.25398 \text{ m} \\
 \theta &= 0.00305 \text{ rad} \\
 \text{AREA} &= 3.2517 \text{ m}^2/\text{m}
 \end{aligned}$$

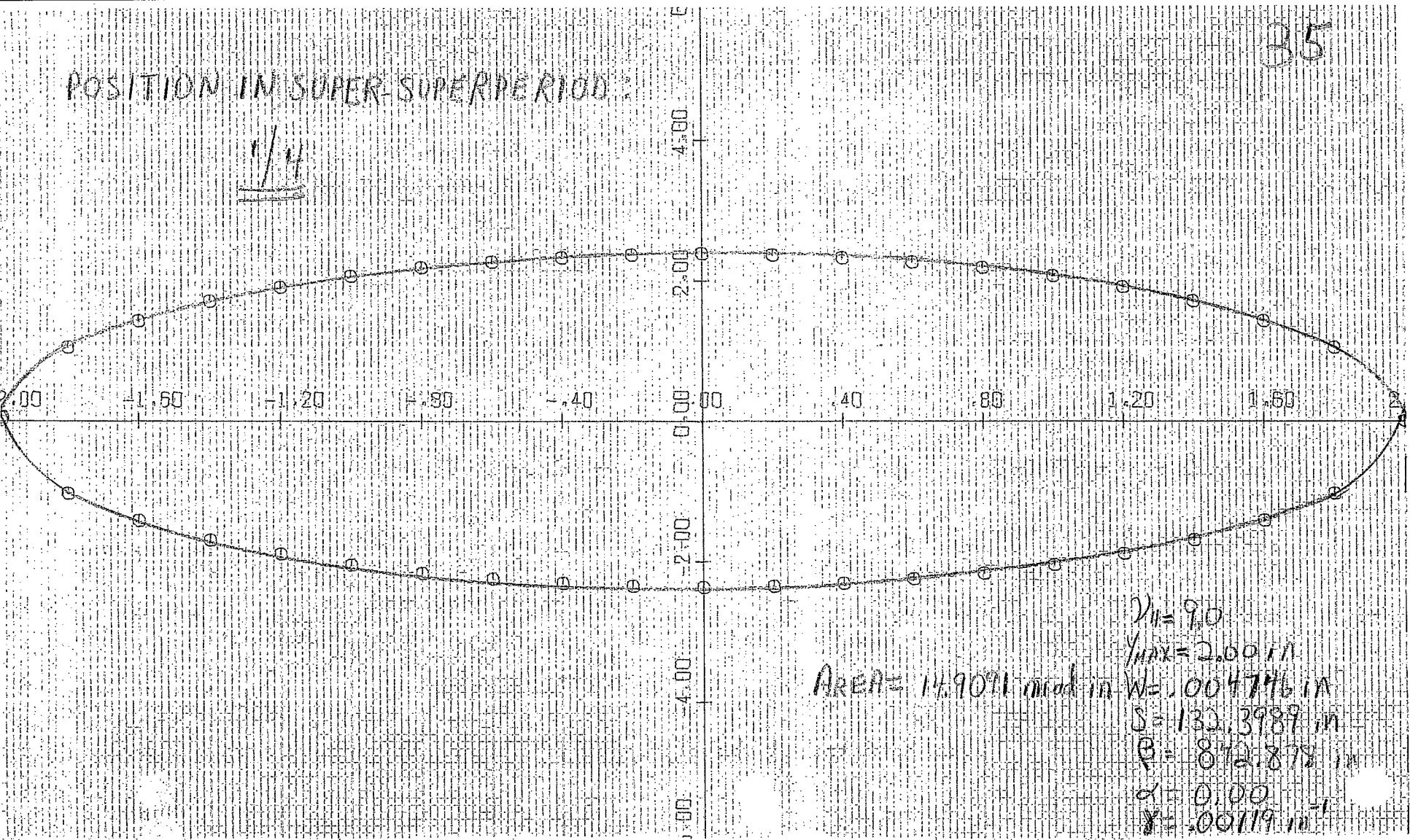
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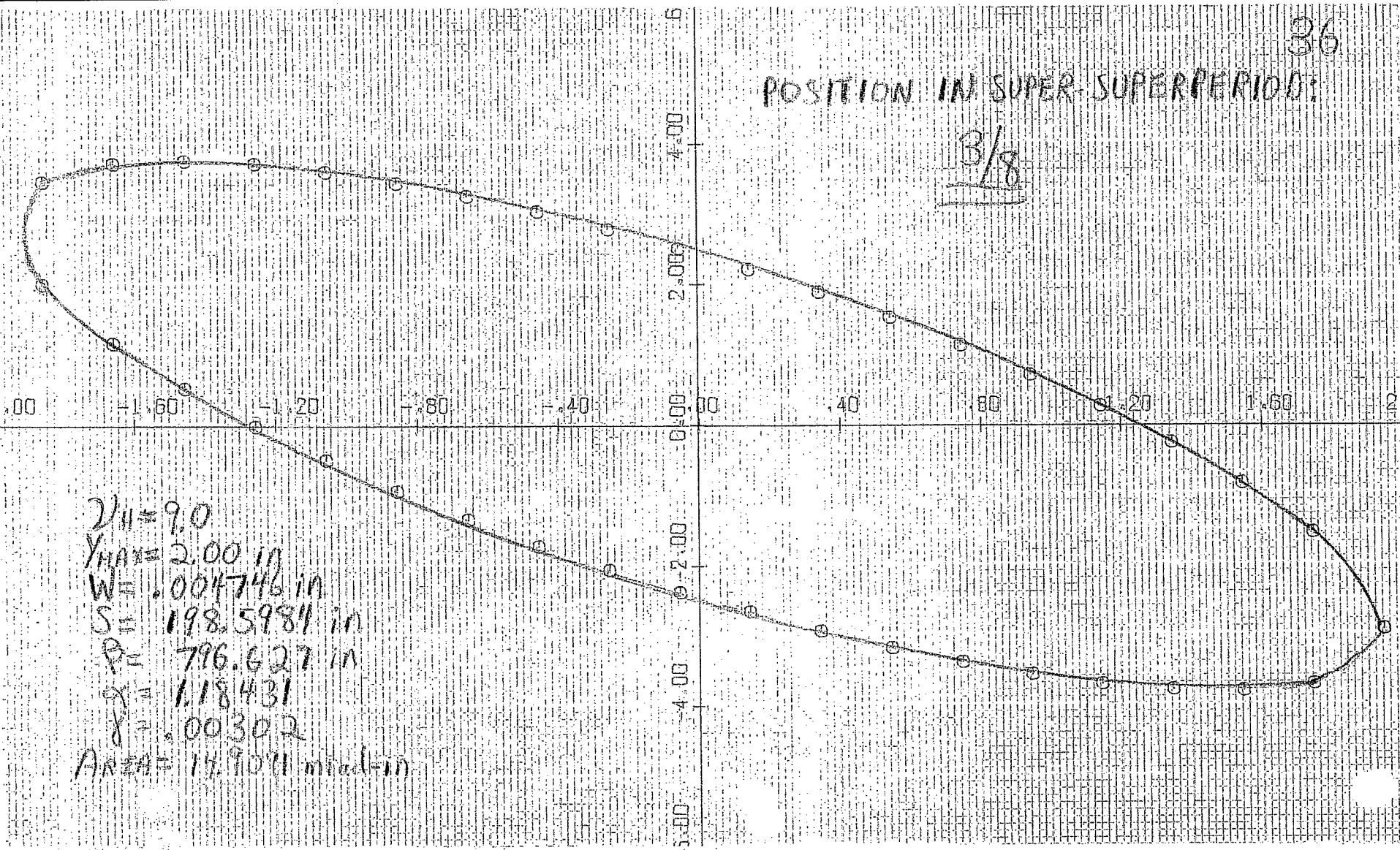


POSITION IN SUPER ELEVATION

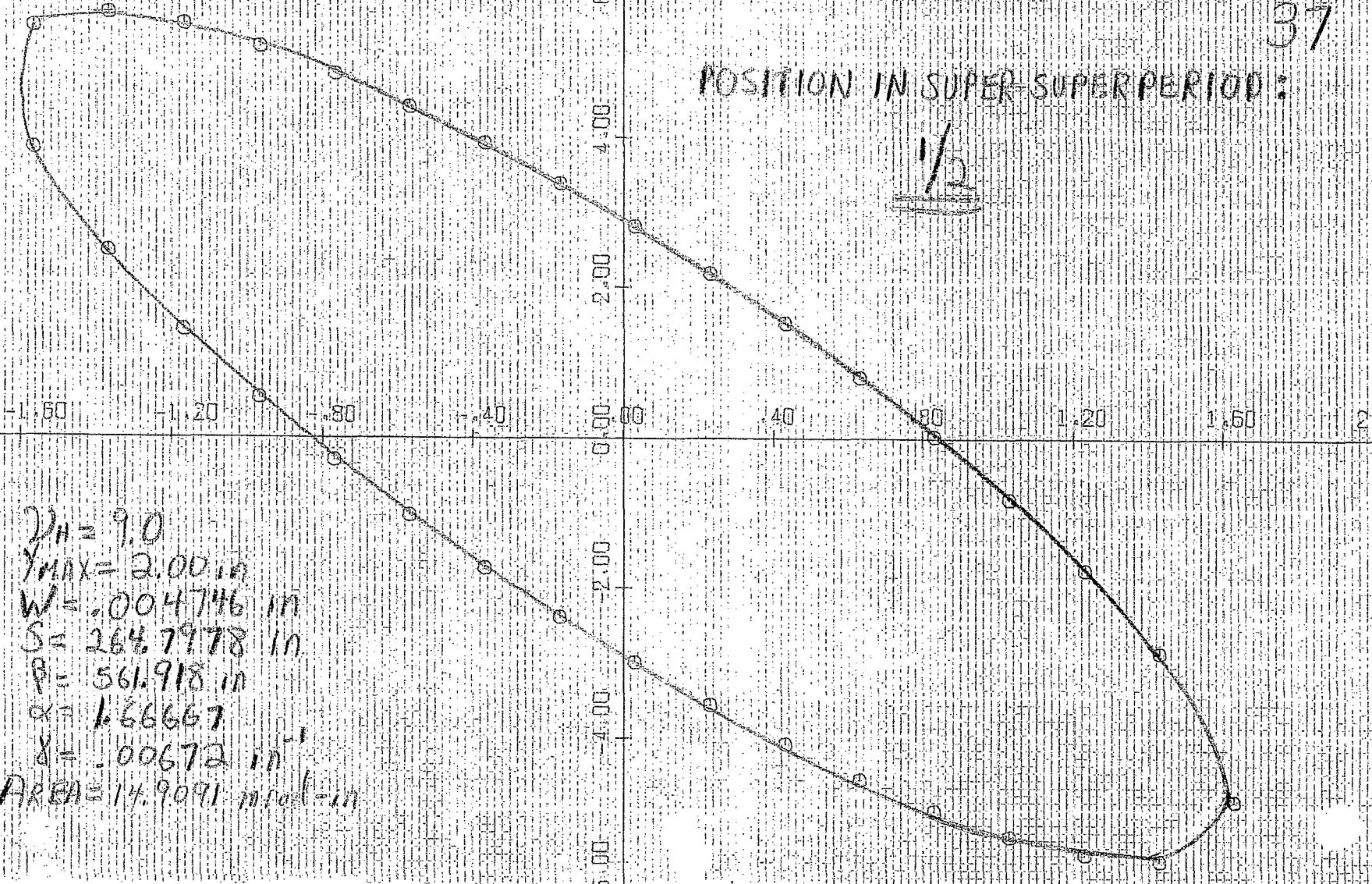


POSITION IN SUPER-SUPER PERIOD





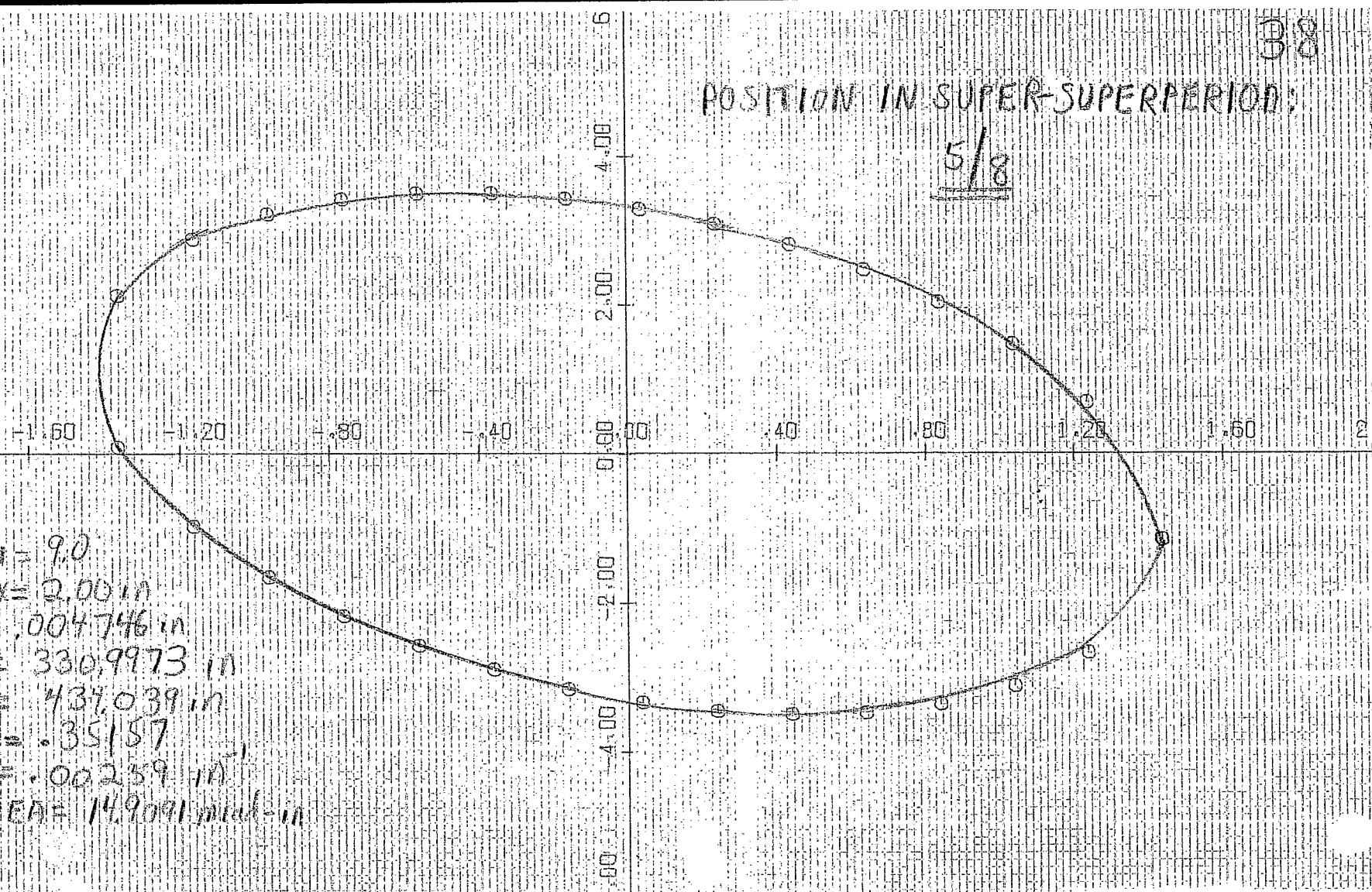
POSITION IN SUPER-SUPER PERIOD



POSITION IN SUPER-SUPERPERIOD

30

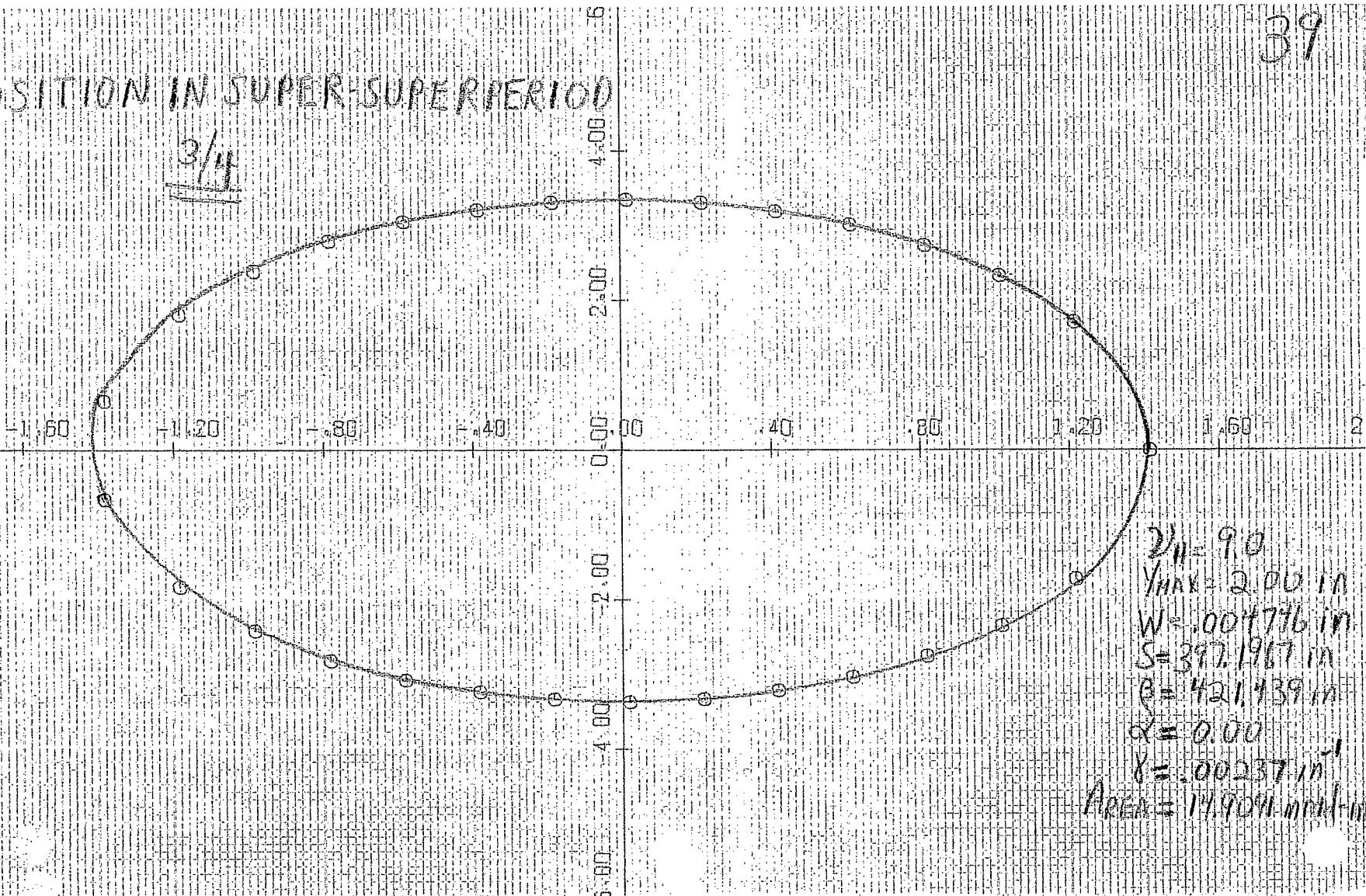
5
8



39

POSITION IN SUPER-SUPER PERIOD

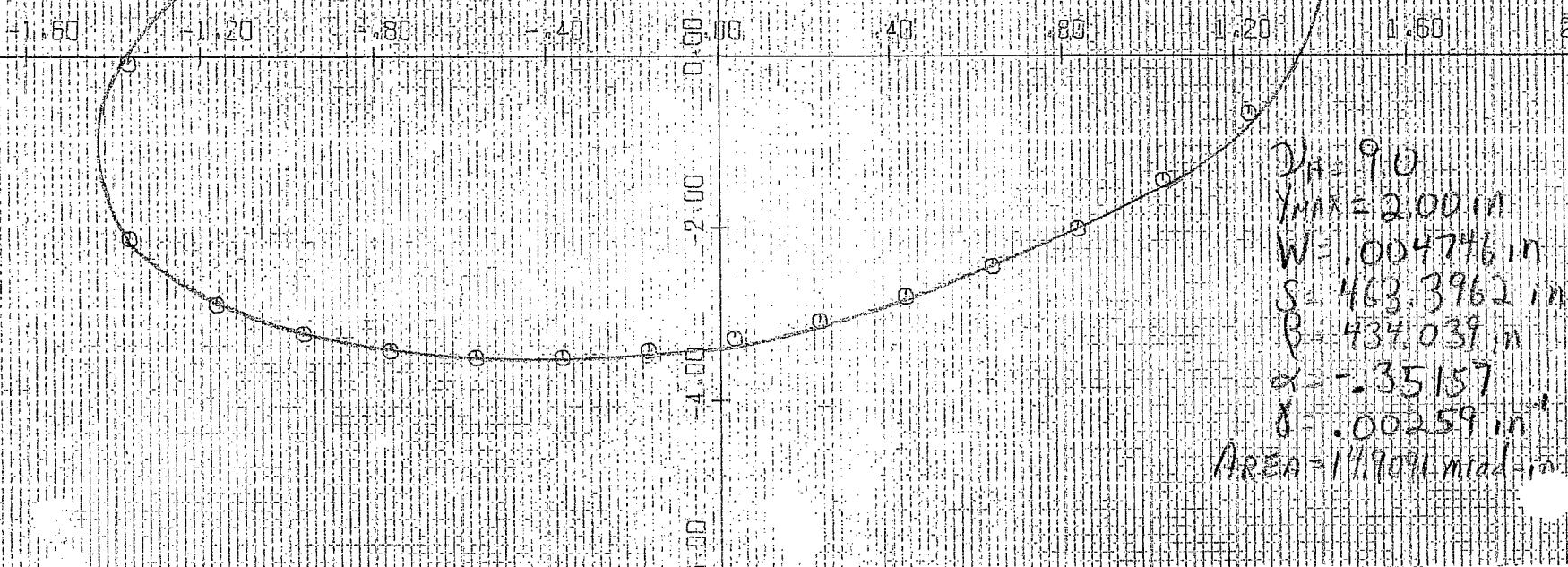
3/4



POSITION IN SUPER-SUPERPERIOD

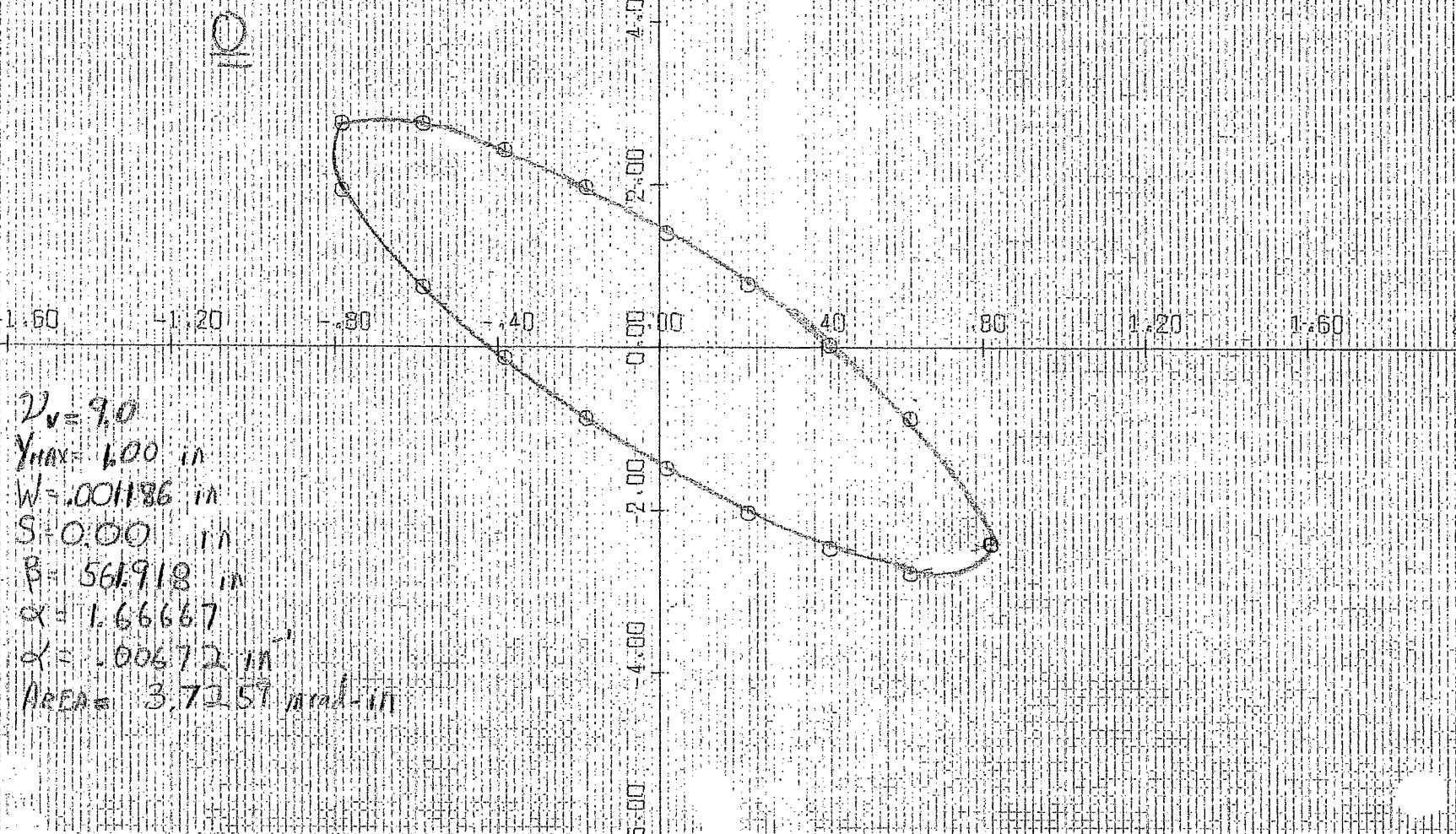
1/80

40



$$\begin{aligned}D_H &= 9.0 \\Y_{MAX} &= 2.00 \text{ in} \\W &= .004716 \text{ in} \\S_B &= 463.3962 \text{ in} \\X &= -35157 \\Y &= .00259 \text{ in} \\AREA &= 1419091 \text{ in}^2/\text{in}\end{aligned}$$

POSITION IN SUPER-SUPERPERIOD



POSITION IN SUPER-SUPER PERIOD

1/8

5

4
7

00

160

120

80

40

00

40

80

120

160

21

90
1,00 m

W =

001186 m

S =

66.1995 m

B =

431039 m

Y =

351571 m

A =

000571 m

AREA =

3.7259 m²



43

POSITION IN SUPER-SUPERPERIOD

1/4

1.20

1.60

1.20

1.20

5

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

40

800

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

00' 00"

1.20

1.60

21 9.0

Y MAX 1.00 m

W 0.01786 m

S 1.323989 m

B 4.21439

X 0.00

Y 0.00673 m

Z 3.7259 m

A 0.00

B 0.00

C 0.00

D 0.00

E 0.00

F 0.00

G 0.00

H 0.00

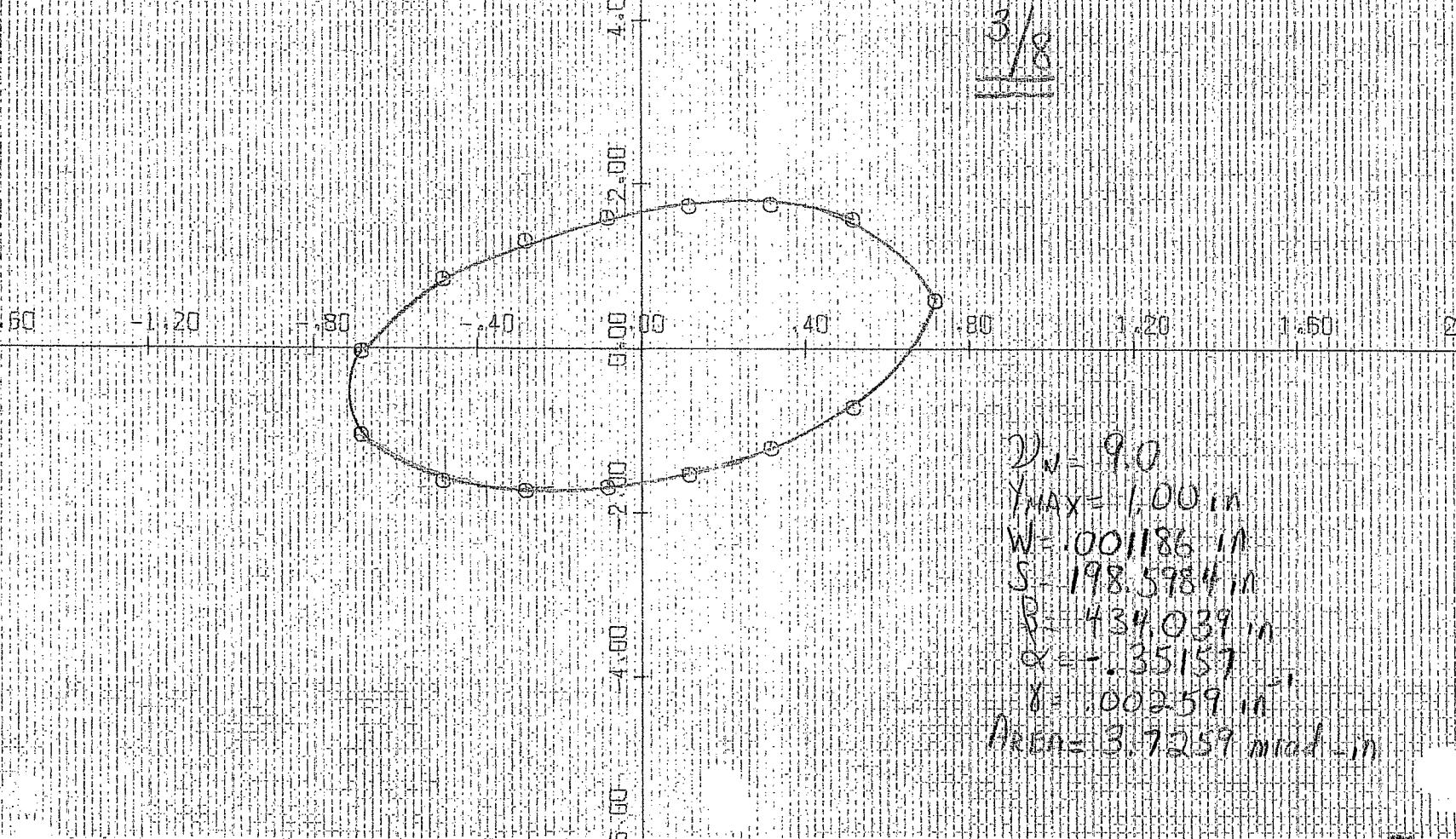
I 0.00

J 0.00

44

POSITION IN SUMO SUPER PERIOD

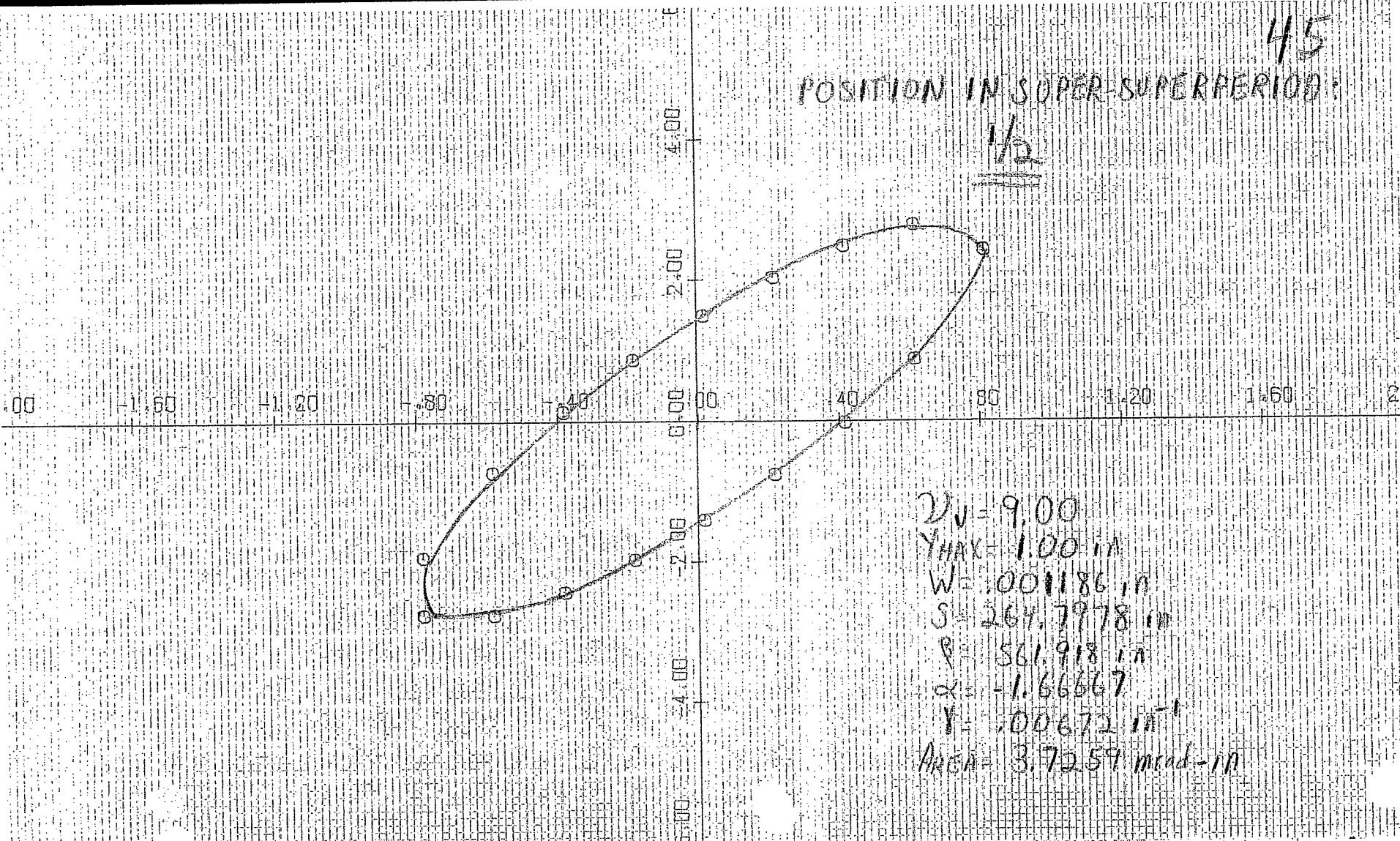
3/8



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POSITION IN SUPER-SUPERIOR

1/2



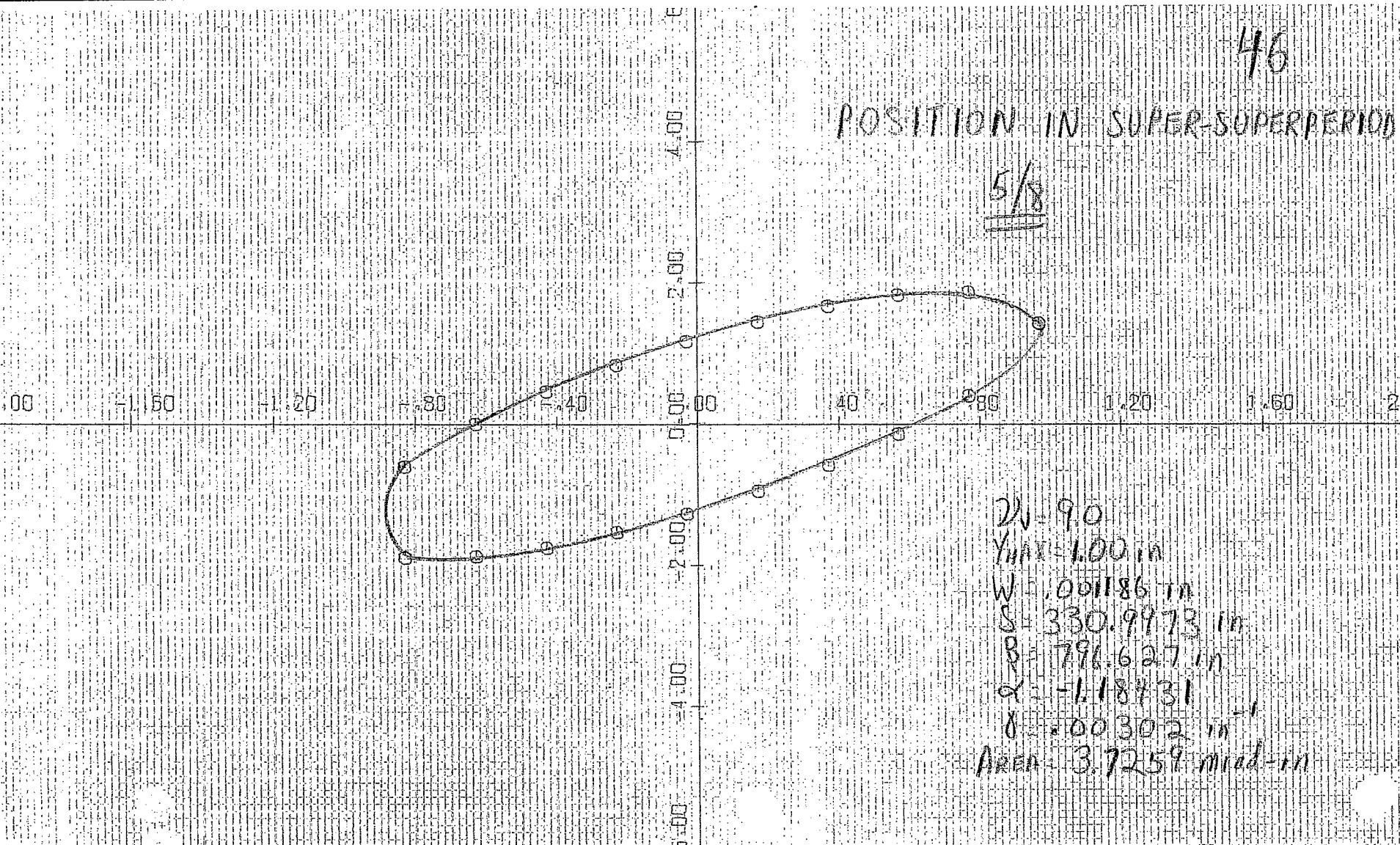
2.0
YMAX = 1.00
W
S
R
Y
-1.66667
-0.00672

AREA = 3.7359 mrad-mm

46

POSITION IN SUPER-SUPERPERIOD

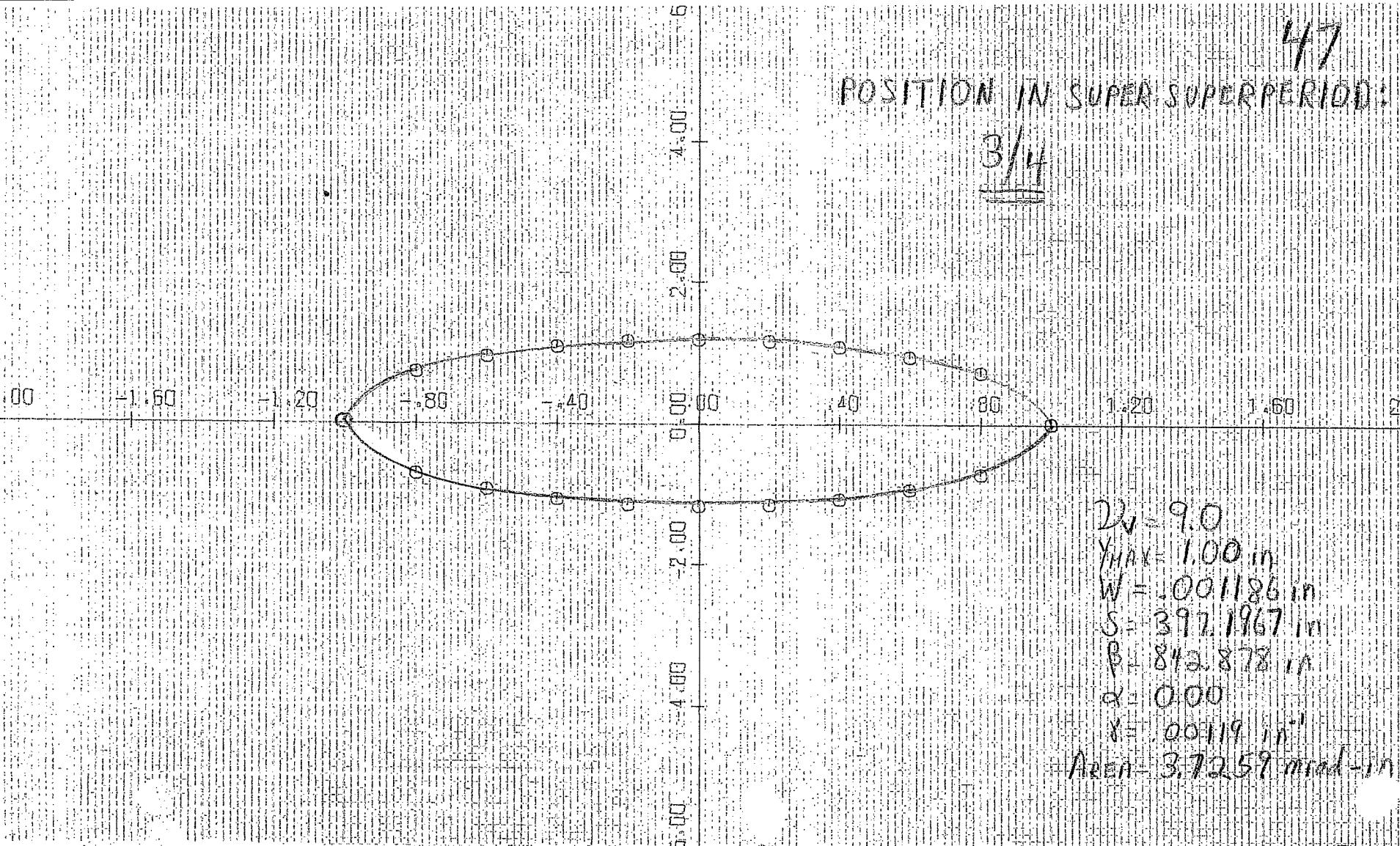
5/8



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POSITION IN SUPER-SUPERPERIOD:

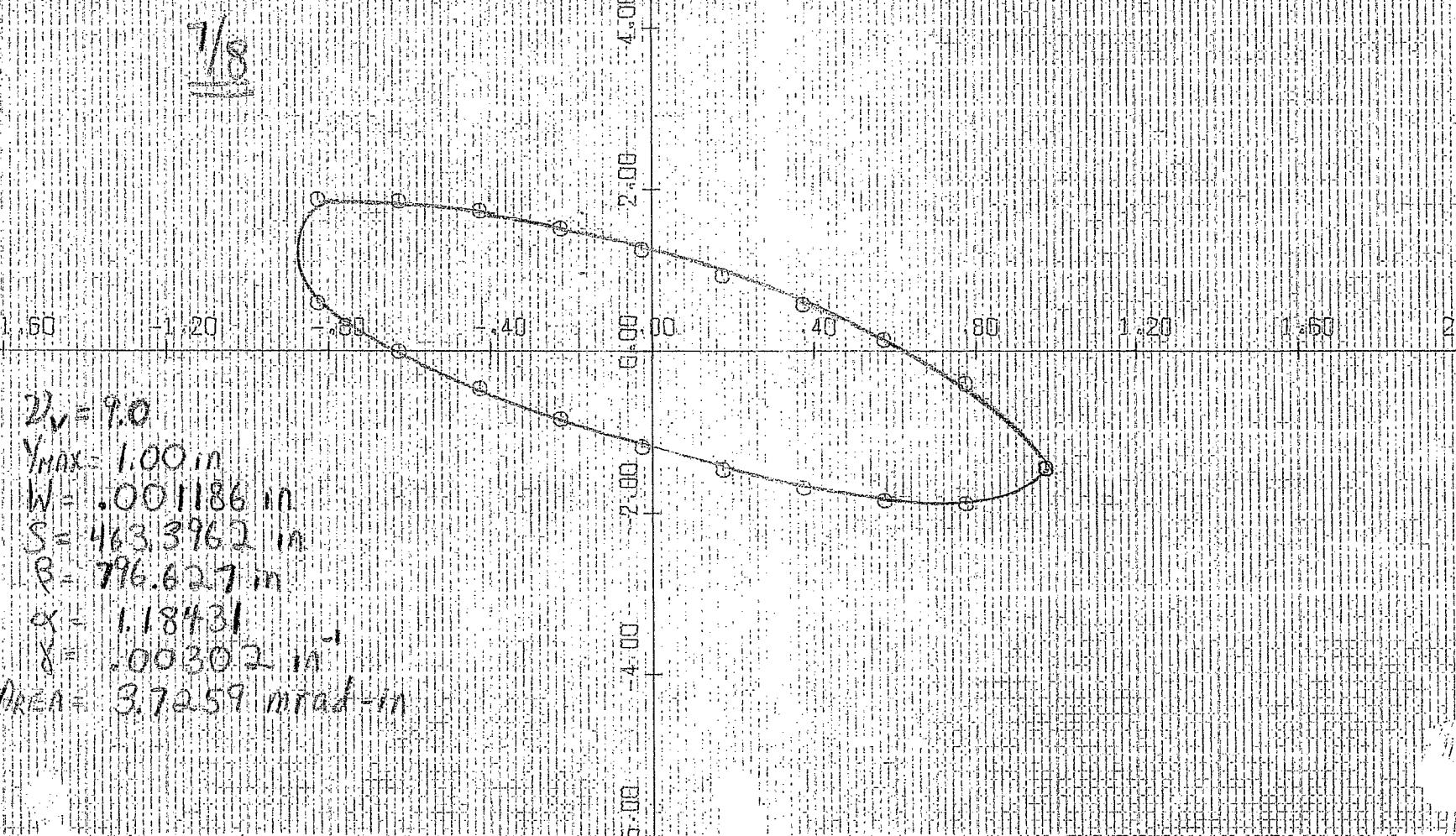
3/4



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POSITION IN SUPER-SUPERPOSITION

1/61



$$2r = 9.0$$

$$Y_{max} = 1.00 \text{ m}$$

$$W = .001186 \text{ m}$$

$$S = 463.3962 \text{ m}$$

$$796.627 \text{ m}$$

$$1.18431$$

$$.00302 \text{ m}$$

$$Area = 3.7259 \text{ mrad-m}$$