Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
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Upton, NY 11973

AGS Division Technical Note
No. 206

Some Beam Related rf Topics

Accelerator Department Lecture Series

September 8-14, 1984

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CERN, SPS RF Group

October 1, 1984

* ACCELERATOR DEPARTMENT LECTURES *

Some Beam Related RF Topics

by

Dr. Daniel Boussard

CERN, SPS RF Group

Time: 11:00 a.m., September 8-14, 1983

Place: Snyder Seminar Room, First Floor, Bldg. 911

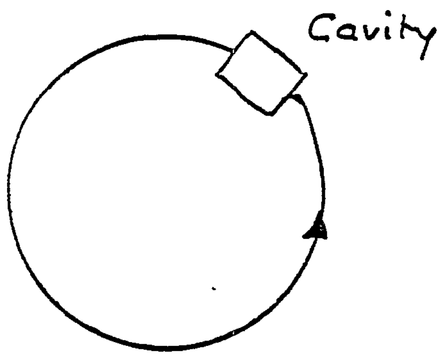
1. Longitudinal Phase Space for Beginners (Thursday)
2. Phase Space Manipulations (Friday)
3. Beam Control Systems (Monday)
4. Instabilities (Tuesday)
5. Two Examples of Beam Loading Compensation at CERN (Wednesday)

1. Longitudinal Phase Space for Beginners

- Stable and unstable points, trajectories in phase space.
- Stationary and accelerating buckets. Conservation of phase space area, adiabaticity, filamentation.

Longitudinal phase space

RF OFF



particle with energy $\gamma_0 m_0 c^2$
β_0
p_0

Magnetic field B_0 $\rightarrow$ $R_0 = \frac{L}{2\pi}$
f_0

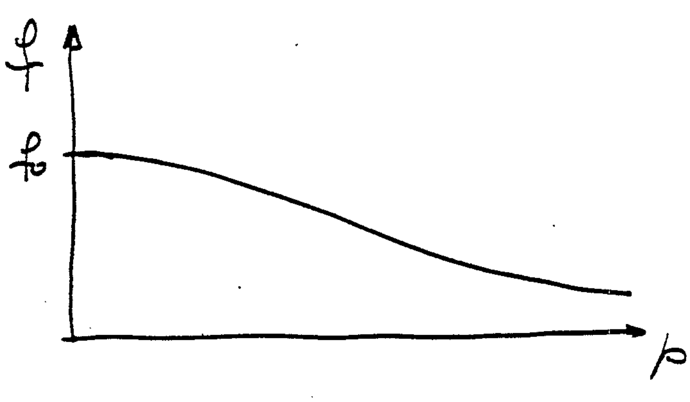
2nd particle with energy $\gamma_0 + \Delta\gamma$, $\beta_0 + \Delta\beta$, $p_0 + \Delta p$

ΔR ? Δf ?

Simple and unrealistic cases.

a) Uniform field $\rightarrow$ circular trajectories

$$\left.\begin{array}{l} f = \dfrac{\beta c}{2\pi\rho} \\ p = eB\rho \end{array}\right\} \qquad f = \frac{eB}{2\pi} \frac{1}{\sqrt{\frac{E_0^2}{c^2} + p^2}}$$



Wanted \ In terms of	β	cp	T	E	γ
$\beta =$	β	$[(E_0/cp)^2 + 1]^{-1/2}$ cp/E	$[1 - (1 + T/E_0)^{-2}]^{1/2}$	$[1 - (E_0/E)^2]^{1/2}$ cp/E	$(1 - \gamma^{-2})^{1/2}$
$cp =$	$E_0(\beta^{-2} - 1)^{-1/2}$ $E\beta$	cp	$[T(2E_0 + T)]^{1/2}$ $T[(\gamma + 1)/(\gamma - 1)]^{1/2}$	$(E^2 - E_0^2)^{1/2}$ $E\beta$	$E_0(\gamma^2 - 1)^{1/2}$
$E_0 =$	$cp/\beta\gamma$ $E(1 - \beta^2)^{1/2}$	$cp(\gamma^2 - 1)^{-1/2}$	$T/(\gamma - 1)$	$(E^2 - c^2p^2)^{1/2}$	E/γ
$T =$	$[(1 - \beta^2)^{-1/2} - 1]E_0$	$[E_0^2 + c^2p^2]^{1/2} - E_0$ $cp[(\gamma - 1)/(\gamma + 1)]^{1/2}$	T	$E - E_0$	$E_0(\gamma - 1)$
$\gamma =$	$(1 - \beta^2)^{-1/2}$	$cp/E_0\beta$ $[1 + (cp/E_0)^2]^{1/2}$	$1 + T/E_0$	E/E_0	γ

1.2 First Derivatives

Wanted \ In terms of	$d\beta$	$d(cp)$	$d\gamma = dE/E_0 = dT/E_0$
$d\beta =$	$d\beta$	$[1 + (cp/E_0)^2]^{-3/2}\ d(cp)/E_0$ $\gamma^{-3}\ d(cp)/E_0$	$\gamma^{-2}(\gamma^2 - 1)^{-1/2}\ d\gamma$ $\beta^{-1}\ \gamma^{-3}\ d\gamma$
$d(cp) =$	$E_0(1 - \beta^2)^{-3/2}\ d\beta$ $E_0\ \gamma^3\ d\beta$	$d(cp)$	$E_0\gamma(\gamma^2 - 1)^{-1/2}\ d\gamma$ $E_0\ \beta^{-1}\ d\gamma$
$d\gamma =$ $= dE/E_0 =$ $= dT/E_0 =$	$\beta(1 - \beta^2)^{-3/2}\ d\beta$ $\beta\gamma^3\ d\beta$	$[1 + (E_0/cp)^2]^{-1/2}\ d(cp)/E_0$ $\beta\ d(cp)/E_0$	$d\gamma$

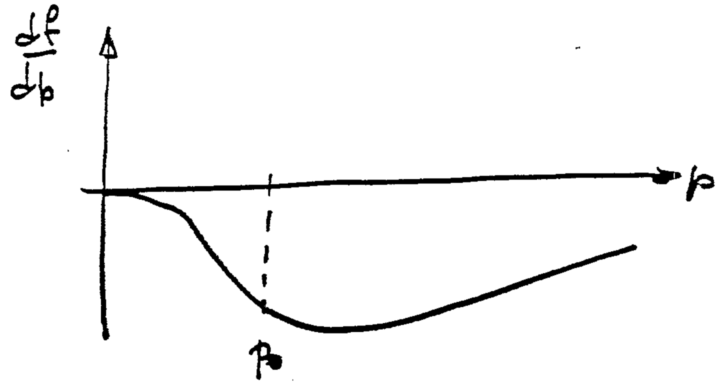
1.3 Logarithmic first derivatives

Wanted \ In terms of	$d\beta/\beta$	dp/p	dT/T	$dE/E = d\gamma/\gamma$
$d\beta/\beta =$	$d\beta/\beta$	$\gamma^{-2}\ dp/p$ $dp/p - d\gamma/\gamma$	$[\gamma(\gamma + 1)]^{-1}\ dT/T$	$(\gamma^2 - 1)^{-1}\ d\gamma/\gamma$ $(\beta\gamma)^{-2}\ d\gamma/\gamma$
$dp/p =$	$\gamma^2 d\beta/\beta$	dp/p	$[\gamma/(\gamma + 1)]\ dT/T$	$\beta^{-2}\ d\gamma/\gamma$
$dT/T =$	$\gamma(\gamma + 1)\ d\beta/\beta$	$(1 + \gamma^{-1})\ dp/p$	dT/T	$\gamma(\gamma - 1)^{-1}\ d\gamma/\gamma$
$dE/E =$ $d\gamma/\gamma =$	$(\beta\gamma)^2\ d\beta/\beta$ $(\gamma^2 - 1)\ d\beta/\beta$	$\beta^2\ dp/p$ $dp/p - d\beta/\beta$	$(1 - \gamma^{-1})\ dT/T$	$d\gamma/\gamma$

$$
\left\{
\begin{aligned}
\gamma_{\varpi}^{2} \frac{dR}{R} &= \frac{dp}{p} - \frac{dB}{B} \\
\frac{d\rho}{\rho} &= \underbrace{\frac{\gamma_{\varpi}^{2} - \gamma^{2}}{\gamma^{2}\gamma_{\varpi}^{2}}}_{\eta} \frac{dp}{p} + \frac{1}{\gamma_{\varpi}^{2}} \frac{dB}{B}
\end{aligned}
\right.
$$

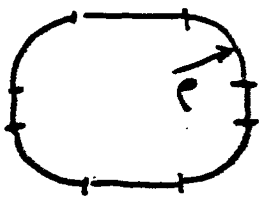
$$
\frac{dp}{p} = \gamma^{2} \frac{d\rho}{\rho} + \gamma^{2} \frac{dR}{R}
$$

$$
\frac{dB}{B} = \gamma^{2} \frac{d\rho}{\rho} + (\gamma^{2} - \gamma_{\varpi}^{2}) \frac{dR}{R}
$$



always negative

b) Uniform magnetic field + straight sections



straight sections : L

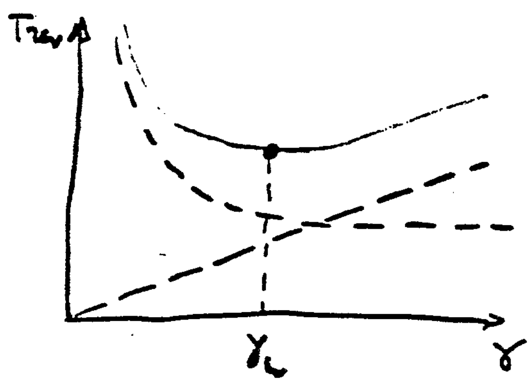
arcs : $2\pi\rho$

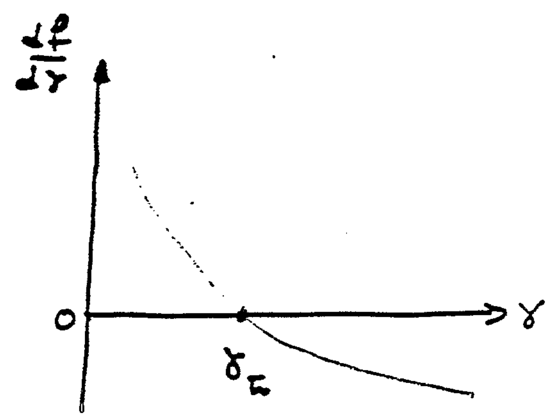
$$T_{rev} = \frac{L + 2\pi\rho}{\beta c}$$

with $\rho = \frac{p}{eB}$ and $cp = E\beta = E_0\beta\gamma$

$$T_{rev} = \frac{2\pi E_0}{eBc^2}\gamma + \frac{L}{c}\cdot\frac{1}{\beta}$$

increases — decreases with energy





$\frac{dT}{d\gamma} = 0$ at γ_{tr}

changes sign

$$\frac{dT_{rev}}{d\gamma} = \frac{2\pi E_0}{eBc^2} - \frac{L}{c\beta^2}\frac{d\beta}{d\gamma} \qquad \frac{d\beta}{d\gamma} = \frac{1}{\beta\gamma^3}$$

γ_{tr} determined by $\frac{dT_{rev}}{d\gamma} = 0$

$$(\beta\gamma)^3 = \frac{LeBc}{2\pi E_0}$$

With $\begin{cases} p = m_0 c\beta\gamma \\ p = eB\rho_0 \end{cases}$

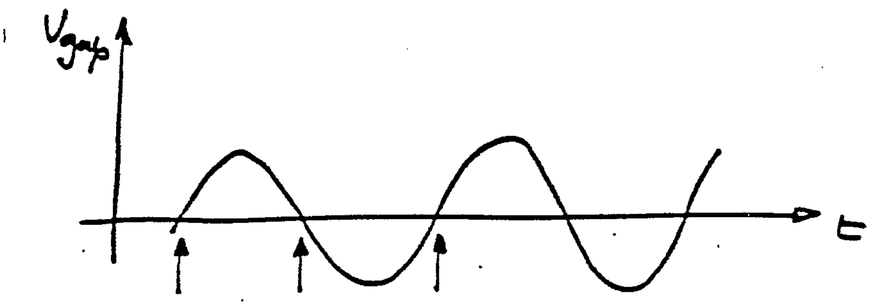
$$(\beta\gamma)^2_{t} = \frac{L}{2\pi\rho_0}$$

<u>γ_{tr} depends on lattice geometry</u>

For a smooth machine $\gamma_{tr} \sim \nu_x$

Let's turn $\boxed{\text{RF ON}}$ at $f_{RF} = h f_0$ → integer; harmonic number

1) <u>Synchronous particle</u> : we know its motion for ever



$\phi = 0°$
$\phi = 180°$
$f_0 = f_{RF}/h$

2) <u>Non synchronous particle</u>

2 effects : gap + drift space

machine

$$\begin{matrix} \Delta\phi \\ \Delta E \end{matrix} \xrightarrow{\text{gap}} \begin{matrix} \Delta\phi \\ \Delta E + eV\sin\Delta\phi \end{matrix} \xrightarrow[\text{space}]{\text{drift}} \begin{matrix} \Delta\phi + k(\Delta E + eV\sin\Delta\phi) \\ \Delta E + eV\sin\Delta\phi \end{matrix}$$

i th turn — i+1 th turn

simul.

$$\begin{matrix} X \\ Y \end{matrix} \xrightarrow{\text{gap}} \begin{matrix} X \\ Y = Y + V\sin X \end{matrix} \xrightarrow{\text{drift}} \begin{matrix} X = X + K * Y \\ Y \end{matrix}$$

<u>Phase space plot</u> $\Delta\phi$, ΔE (ΔR, Δf).
X , Y

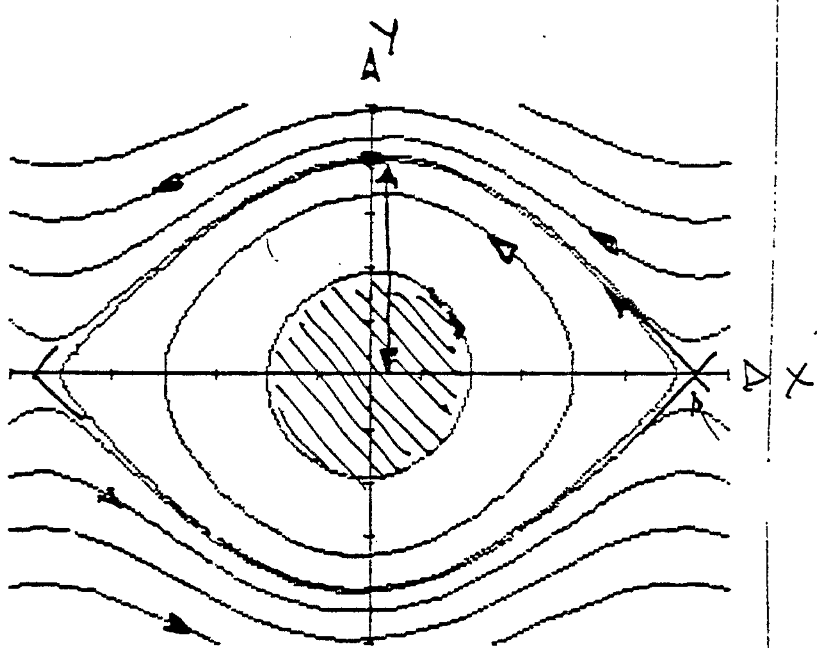
Projections on axes : phase oscillation

$\left.\begin{matrix} \text{energy} \\ \text{radius} \\ \text{frequency} \end{matrix}\right\}$ oscillation

2 regions of phase space

Closed trajectories — SEPARATRIX — Open trajectories

stable point : elliptical trajectories
unstable point : hyperbolic trajectories.




```
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15 XAXIS 0,.5
20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
45 V=V0*SIN(X)
50 Y=Y+V
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 45
70 END
```

Bucket parameters

10) Synchrotron frequency.

$f_s \ll f_r$

energy gain / turn $= eV \sin\phi$

" " / unit time $= eV \sin\phi \cdot f_r = \frac{d\,\Delta E}{dt}$

$$\Delta E \longrightarrow \Delta f = \frac{1}{2\pi}\frac{d\phi}{dt}$$

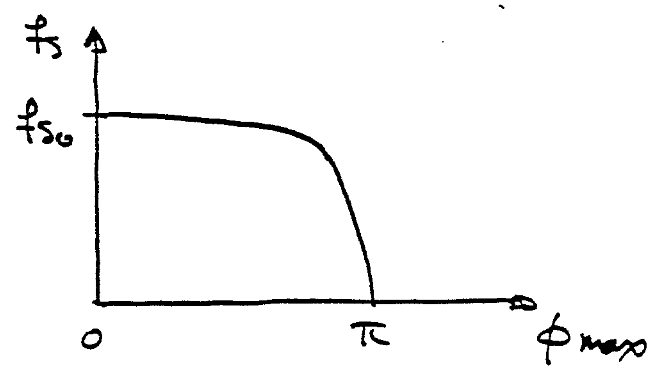
$$\begin{cases} \dfrac{d\Delta E}{dt} = a \sin\phi \\[2ex] \dfrac{d\phi}{dt} = b\,\Delta E \end{cases}$$

– Small oscillations $\qquad \sin\phi \sim \phi$

$$\frac{d^2\phi}{dt^2} - \underbrace{ab}_{\omega_s^2}\,\phi = 0$$

$$\boxed{f_{s_0} = \frac{\omega_s}{2\pi} = f_{RF\infty}\sqrt{\frac{\eta e V}{2\pi E h}}}$$

– Large oscillations

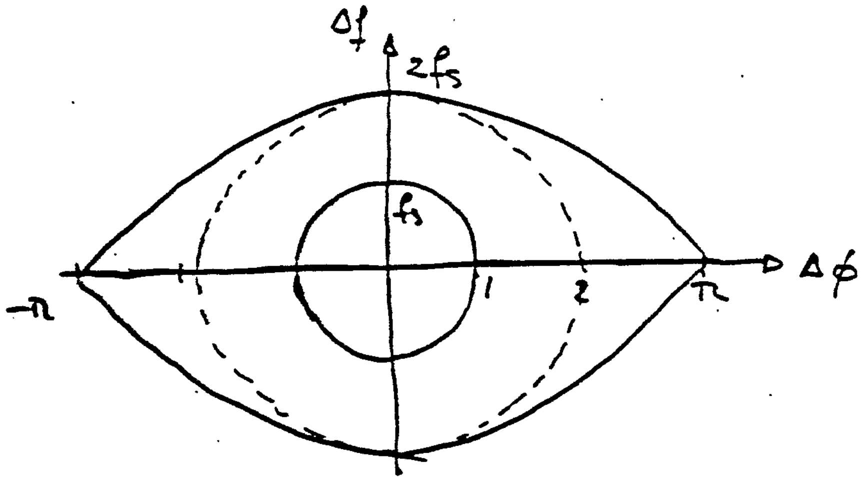


7.

2°) Bucket height, area

Verify that $\Delta E = \Delta E_m \cos\frac{\phi}{2}$ satify the equations for a particular value of ΔE_{max} : bucket height.

$$\Delta E_{max} \longrightarrow \Delta f_{max} = 2 f_{s0}$$

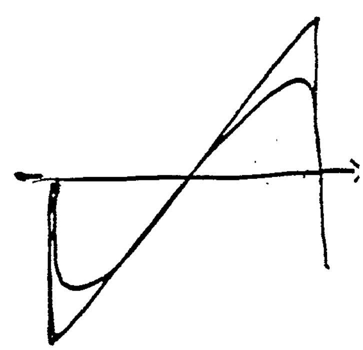


$$\text{bucket area} = \frac{2}{\pi} \times 2\pi \times 2 \text{ bucket height}$$

Area inside a given trajectory $\longrightarrow$ use tables

3°) Non sinusoidal buckets.

- 2nd harmonic   flat-topped bunches
- 3rd harmonic   linearized
- missing bucket



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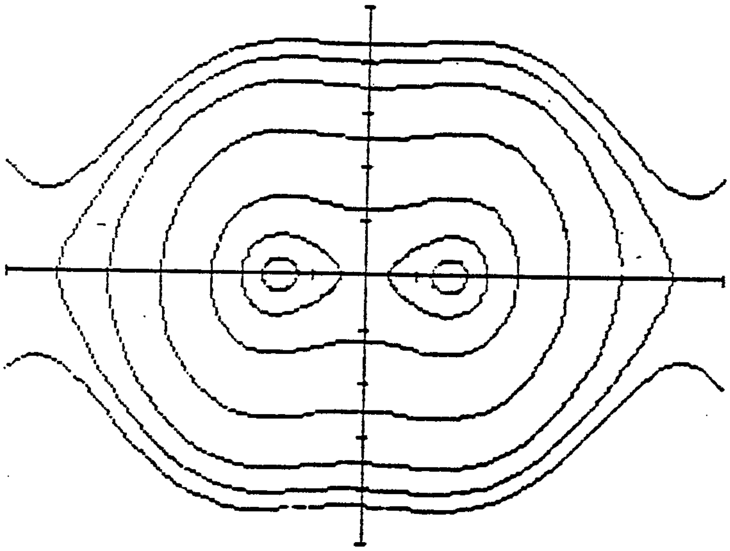

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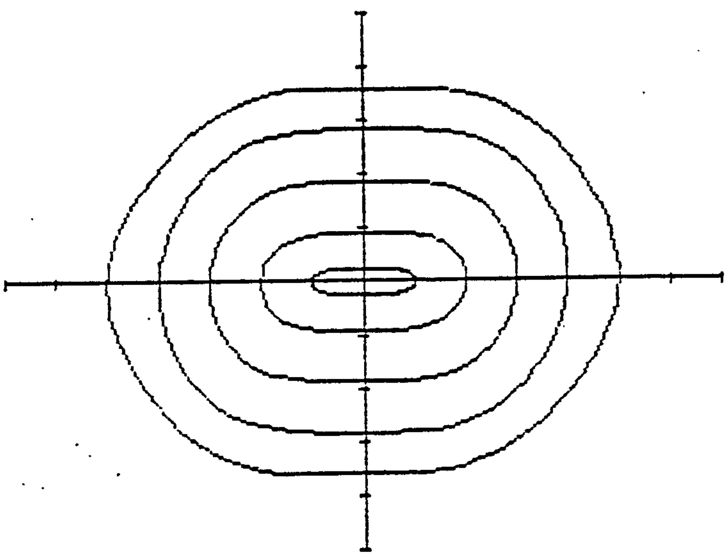
15 XAXIS 0,.5
20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
45 V=V0*SIN(X)-.75*V0*SIN(2*X
50 Y=Y+V
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 45
70 END
```



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25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
45 V=V0*SIN(X)-V0/2*SIN(2*X)
50 Y=Y+V
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 45
70 END
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```

5

Bucket area	half bucket height	coordinates
$\sqrt{heV}\cdot\alpha(\Gamma_s)\cdot\frac{16\gamma}{h}\sqrt{\frac{1}{2\pi E\lvert\eta\rvert}}$	$\sqrt{heV}\cdot y\cdot\frac{\beta\gamma}{h\Omega\lvert\eta\rvert}B$	$\left(\frac{\Delta p}{m_0 c}\right)-\varphi$
$\sqrt{heV}\cdot\alpha(\Gamma_s)\cdot\frac{16\beta}{h}\sqrt{\frac{E}{2\pi\lvert\eta\rvert}}$	$\sqrt{heV}\cdot y\cdot\frac{\beta^2 E}{h\Omega\lvert\eta\rvert}B$	$(\Delta E)-\varphi$
$\sqrt{heV}\cdot\alpha(\Gamma_s)\cdot\frac{16\alpha R}{h\beta}\sqrt{\frac{1}{2\pi\lvert\eta\rvert E}}$	$\sqrt{heV}\cdot y\cdot\frac{\alpha R}{h\Omega\lvert\eta\rvert}B$	$(\Delta R)-\varphi$
$\sqrt{heV}\cdot\alpha(\Gamma_s)\cdot\frac{16\beta}{h^2\Omega}\sqrt{\frac{E}{2\pi\lvert\eta\rvert}}$	$\sqrt{heV}\cdot y\cdot\frac{\beta^2 E}{h^2\Omega^2\lvert\eta\rvert}B$	$\left(\frac{\Delta E}{h\Omega}\right)-\varphi$

(13)

$$\Omega = \frac{\beta c}{R} \quad ; \quad B = \frac{\Omega}{\beta}\sqrt{\frac{|\eta|}{\pi E}}$$

$$T_s = \frac{1}{\sqrt{heV}} \qquad T_n \quad \frac{1}{B}$$

$$f_s = \sqrt{heV} \qquad f_n \quad B$$

	φ_1	φ_2	γ	ε	T_n	f_n
$\varphi_3 = 0°$	180.	-180.0	1.414214	1.000001	********	.0000000
	175.	-175.0	1.412868	.995225	25.57024	.0391080
	170.	-170.0	1.408332	.983557	21.67561	.0461348
	165.	-165.0	1.402115	.966537	19.41534	.0514951
	160.	-160.0	1.392728	.945028	17.83824	.0560593
	155.	-155.0	1.380691	.919668	16.62970	.0601334
	150.	-150.0	1.366025	.890980	15.65253	.0638630
	145.	-145.0	1.348759	.859421	14.85237	.0673293
	140.	-140.0	1.328926	.825401	14.16787	.0705822
	135.	-135.0	1.306563	.789298	13.57698	.0736541
	130.	-130.0	1.281713	.751464	13.06047	.0765669
	125.	-125.0	1.254423	.712235	12.60457	.0793363
	120.	-120.0	1.224745	.671927	12.19509	.0819733
	115.	-115.0	1.192736	.631845	11.83628	.0844860
	110.	-110.0	1.158456	.589279	11.51009	.0868803
	105.	-105.0	1.121971	.547509	11.21572	.0891606
	100.	-100.0	1.083350	.505804	10.94530	.0913300
	95.	-95.0	1.042668	.464420	10.73768	.0933909
	90.	-90.0	1.000000	.423607	10.48823	.0953450
	85.	-85.0	.955429	.383598	10.28878	.0971932
	80.	-80.0	.909039	.344621	10.10749	.0989365
	75.	-75.0	.860919	.306890	9.94281	.1005752
	70.	-70.0	.811160	.270608	9.79340	.1021096
	65.	-65.0	.759856	.235968	9.65814	.1035396
	60.	-60.0	.707107	.203149	9.53604	.1048653
	55.	-55.0	.653011	.172322	9.42628	.1060864
	50.	-50.0	.597672	.143640	9.32813	.1072027
	45.	-45.0	.541196	.117250	9.24098	.1082139
	40.	-40.0	.483690	.093280	9.16425	.1091197
	35.	-35.0	.425262	.071850	9.09754	.1099198
	30.	-30.0	.366025	.053064	9.04046	.1106139
	25.	-25.0	.306092	.037012	8.99267	.1112017
	20.	-20.0	.245576	.023773	8.95391	.1116830
	15.	-15.0	.184592	.013410	8.92398	.1120576
	10.	-10.0	.123257	.005972	8.90271	.1123253
	5.	-5.0	.061687	.001495	8.89000	.1124860
	4.	-4.0	.049355	.000957	8.88647	.1125053
	3.	-3.0	.037020	.000538	8.88729	.1125203
	2.	-2.0	.024681	.000239	8.88644	.1125310
	1.	-1.0	.012341	.000060	8.88594	.1125374

Accelerating bucket

$\dot{B} \neq 0 \qquad \dot{f} \neq 0 \qquad R = \text{const.}$

$\dot{B} = 0 \qquad \dot{f} \neq 0 \qquad \dot{R} \neq 0$

$\dot{B} = 0 \qquad \dot{f} = 0 \qquad \dot{R} = 0$ (electrons)

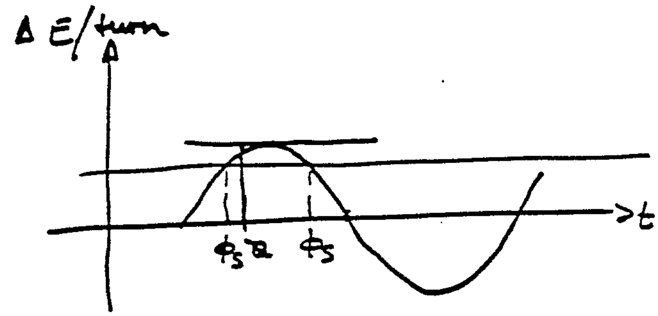
in all cases energy gain/turn $\neq 0$ for synchronous particle (constant phase)

- $\dot{B} \neq 0 \quad \dot{R} = 0$

$$\left.\begin{array}{l} \dfrac{dp}{dt} = e\rho\dfrac{dB}{dt} \\ \downarrow \\ 1\ \text{turn} = \dfrac{2\pi R}{\beta c} \end{array}\right\} \Delta E = 2\pi R \rho \dot{B}$$

- electrons

$$\Delta E/\text{turn} = \frac{1}{3\varepsilon_0}\,\frac{e^2}{R}\left(\frac{E}{m_0c^2}\right)^4$$



difference in energy / ~~…~~ synchronous particle

$$\Delta E = \underset{\substack{\uparrow \\ \text{current} \\ \text{particle}}}{eV\sin\phi} - \underset{\substack{\uparrow \\ \text{synchronous} \\ \text{particle}}}{eV\sin\phi_s} \simeq \underset{\substack{\downarrow \\ f_s \simeq f_{s,\,\phi_s=0^\circ} \times\sqrt{\cos\phi_s}}}{eV\cos\phi_s \cdot \Delta\phi}$$

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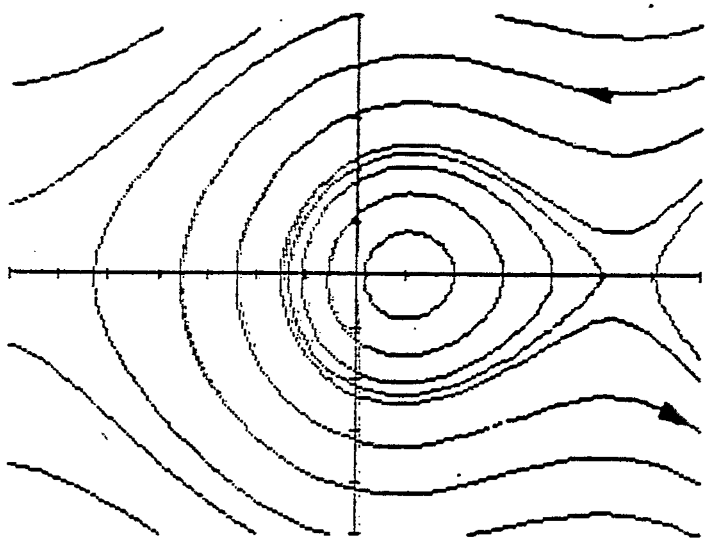

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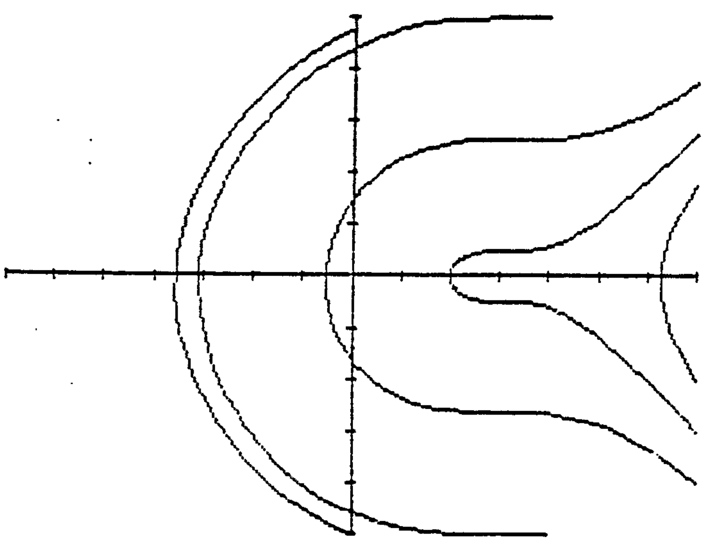
15 XAXIS 0,.5
20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
45 V=V0*(SIN(X)-SIN(.523))
50 Y=Y+V
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 45
70 END
```

(30°)



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20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
45 V=V0*(SIN(X)-SIN(PI/2))
50 Y=Y+V
52 PLOT X,Y.
55 X=X+K*Y
65 GOTO 45
70 END
```

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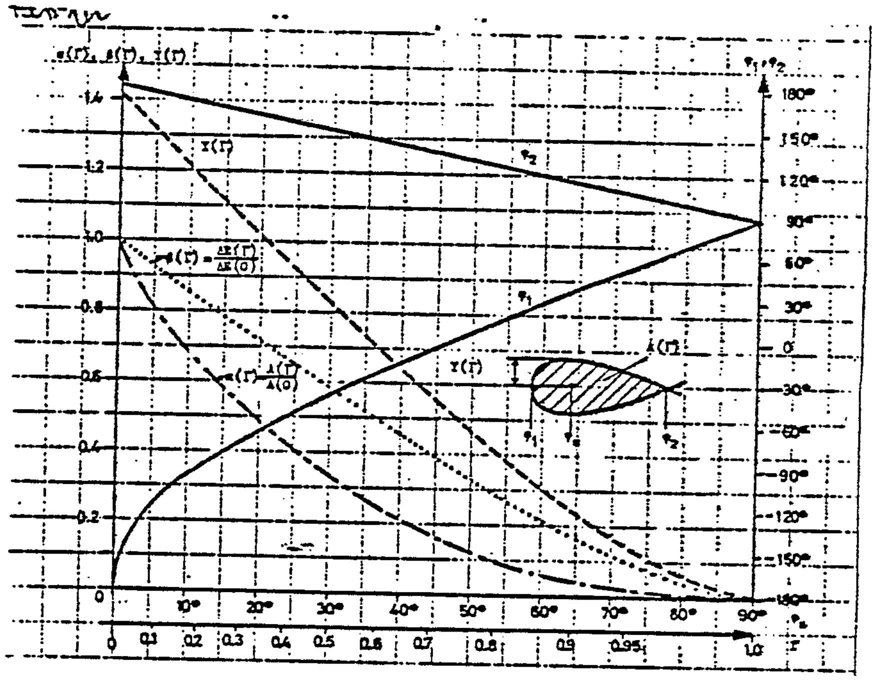
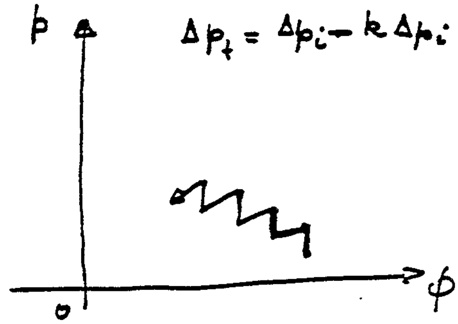


$T = \sin \phi_s$

Approximate formula (G Dôme)

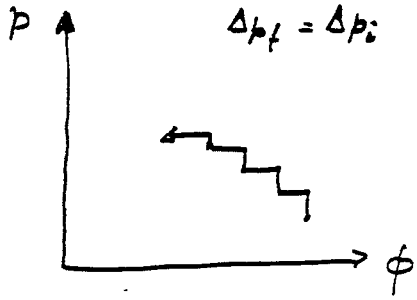
$$\alpha(\phi_s) \simeq 0.3 \left(\frac{\pi}{2} - \phi_s\right)^{5/2}$$

The case of electrons. $\phi_s \neq 0$



electrons

relative energy loss depends on energy difference



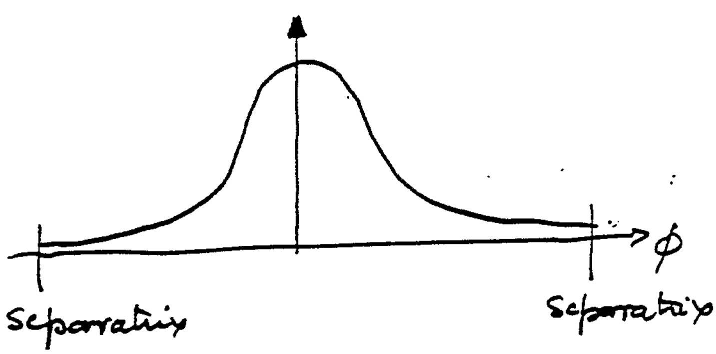
protons

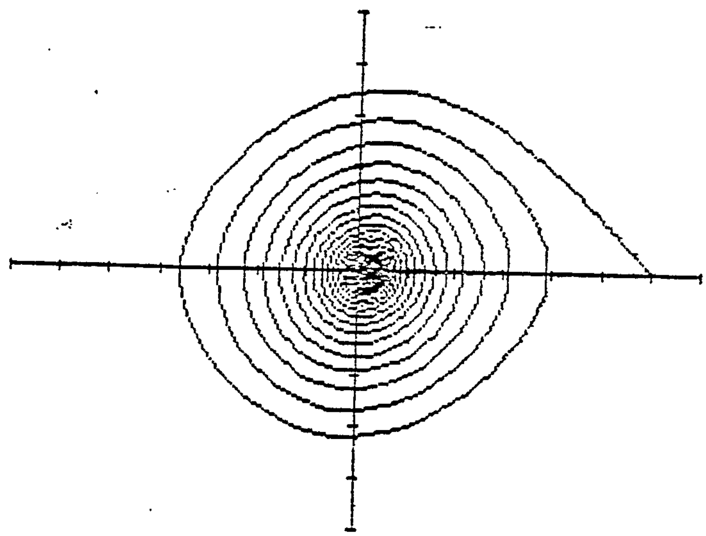
no energy loss

$\downarrow$

natural damping
\+ quantum noise } gaussian probability

Many electrons → gaussian distribution





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15 XAXIS 0,.5
20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
40 K=-.01
45 V=V0*(SIN(X)-SIN(.1))
50 Y=Y+V
51 Y=Y-.005*Y
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 45
70 END
```

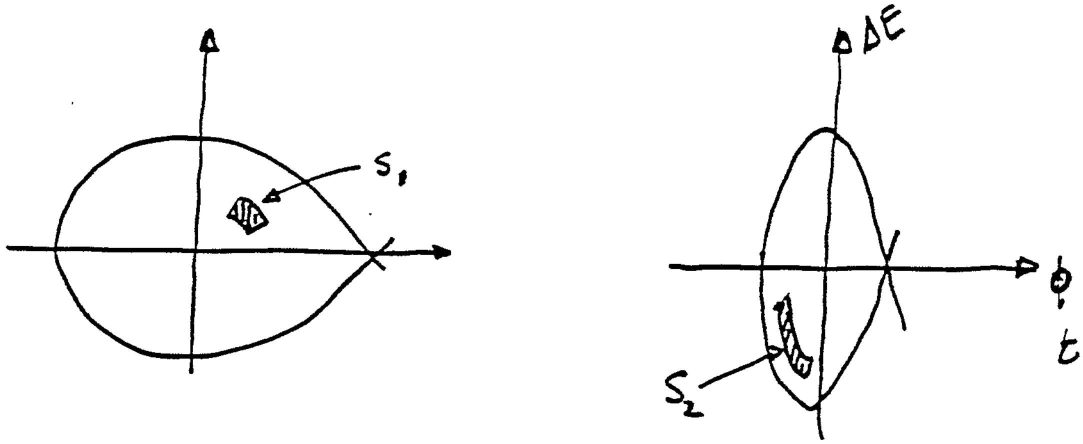
average energy loss

energy dependent radiation loss

<u>Many particles</u> (protons)

No damping at all $\longrightarrow$ complete memory

$\longrightarrow$ conservation of phase space area (Liouville theorem)



$S_1 = S_2$

$\longrightarrow$ Conservation of phase space density

But:

- proper choice of variables : $\begin{cases} \phi \ , \ \Delta\beta\gamma\left(\frac{\Delta p}{m_0 c}\right) & : \text{mrad} \\ t \ , \ \Delta E & : \text{eV.s} \end{cases}$ (or)

- possible exchange between transverse & longitudinal phase planes

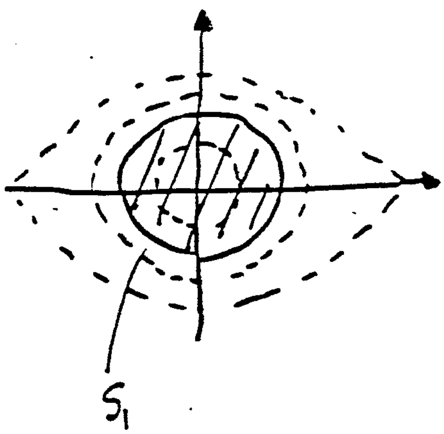
- Ways to cheat Liouville theorem
  - H^- injection
  - stochastic cooling

Matched beam. (uniform density case)

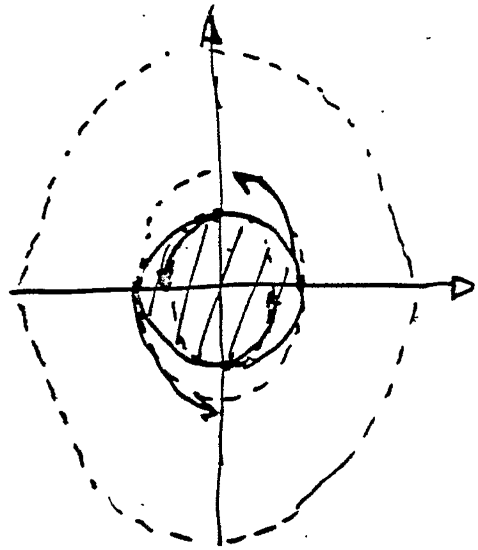
frontier of beam = 1 particular trajectory

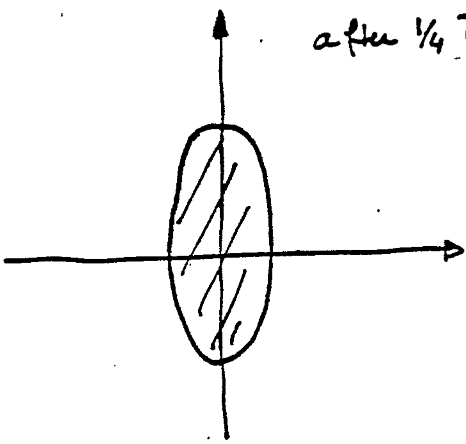
↓

macroscopic steady situation.



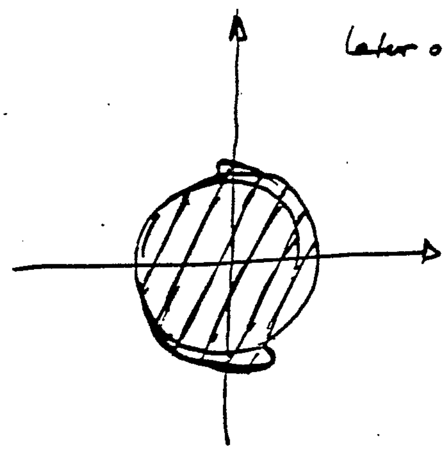
increase V_{RF} or transfer from machine A to B

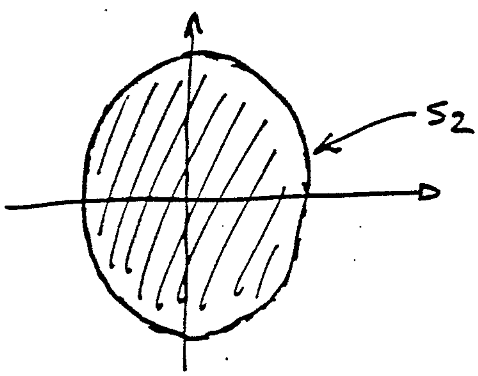




after $1/4\ T_s$

later on





Filamentation or Dilution because $\boxed{f_s \neq f_{s0}}$

$S_2 > S_1$

2. Phase Space Manipulations

- o Debunching, capture, transition, controlled blow-up.
- o Normal modes of oscillation. Beam signals, beam transfer functions.

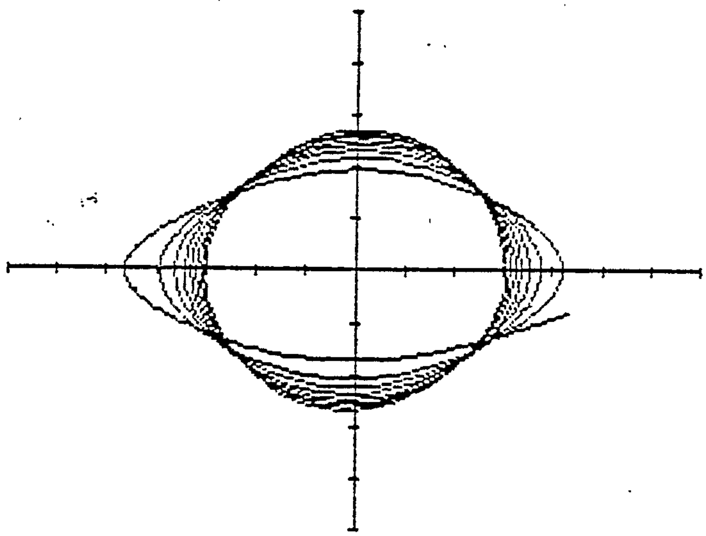
Conclusion : beam area (beam emittance) can only grow
(like entropy)

- How to measure beam emittance ?
 a) matched beam (no oscillation)
 b) bunch length , V_{RF} , $\dot{B}$
 c) Reduce V_{RF} until $I_b \searrow$: beam emittance = bucket area (acceptance)

Adiabaticity

Change parameters slowly enough :
(typical time scale = T_s)
- during acceleration
- when making RF manipulations.

Then : the beam remains matched
conservation of emittance

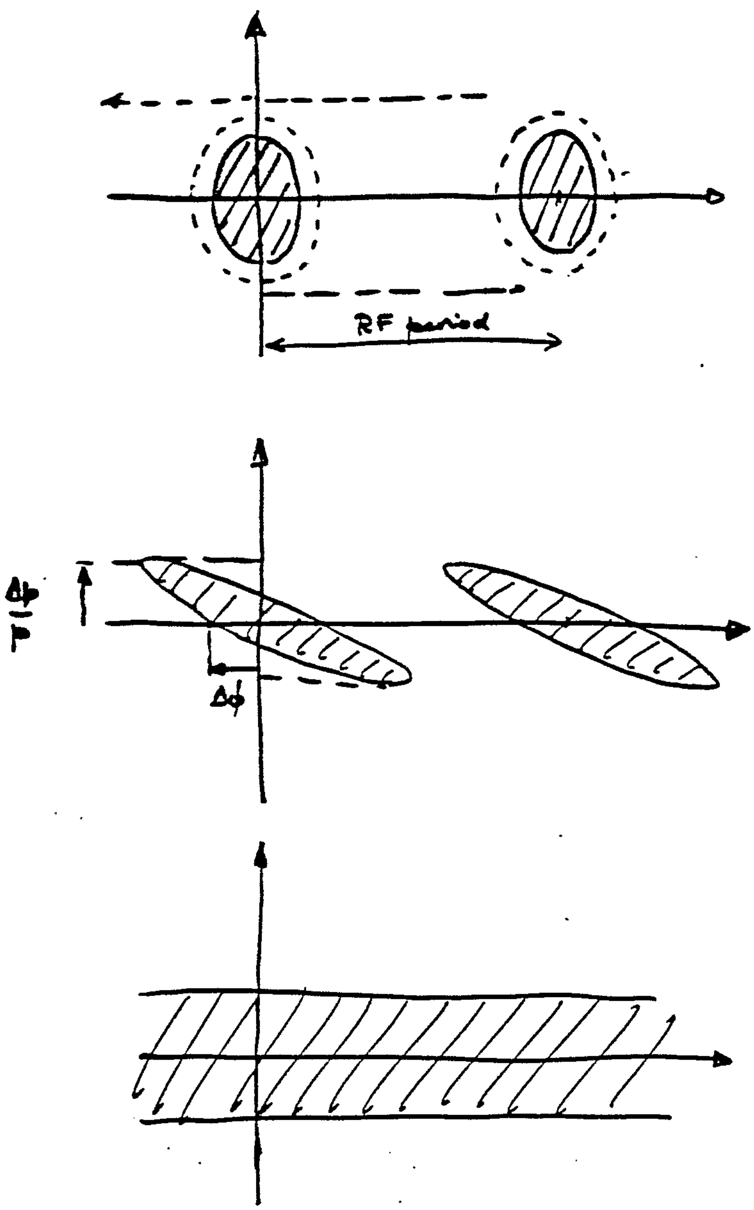


```
 5 GCLEAR
10 SCALE -3.5,3.5,-25,25.
15 XAXIS 0,.5
20 YAXIS 0,5
25 V0=1
30 INPUT X
35 INPUT Y
38 MOVE X,Y
40 K=-.01
42 V0=V0-.001
45 V=V0*SIN(X)
50 Y=Y+V
52 PLOT X,Y
55 X=X+K*Y
65 GOTO 42
70 END
```

← slow decrease of V

Phase space manipulations.

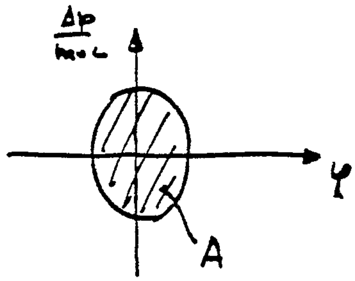
1°/ Debunching (RF cut off)

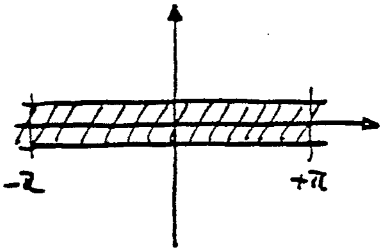


Debunching time

$$t_{db} \simeq \frac{\pi - \Delta\phi}{2\pi f_{RF}\, \eta\, \frac{\Delta p}{p}}$$

2°) Debunching (adiabatic)



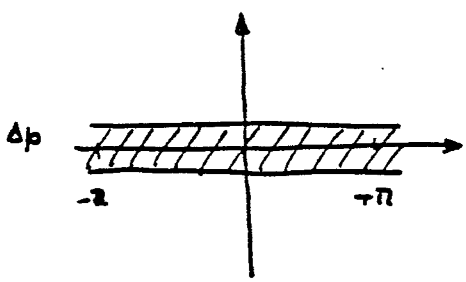


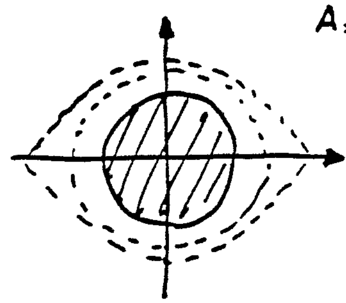
$$\frac{\Delta p}{m_0 c} = A/2\pi$$

but of academic interest $f_s \to 0$ $\quad T \to \infty$

3°) Adiabatic capture

- ideal





$A = 2\pi \cdot \frac{\Delta p}{m_0 c}$

$V_{RF} = 0$ $\qquad V_{RF} \neq 0 \quad \phi_s = 0$

- practical $\quad V_i \neq 0$

iso adiabatic law: $\left.\frac{dA}{A}\right)_{bucket} = \alpha_c \frac{dt}{T_s}$

$\alpha_c < 0.5$

As $f_s = kA \qquad \frac{dA}{dt} = \alpha_c k A^2$

$$A(t) = \frac{A_1}{1 - \frac{t-t_1}{t_2-t_1}\left(\frac{A_2-A_1}{A_2}\right)}$$

A_1, A_2 initial, final bucket areas

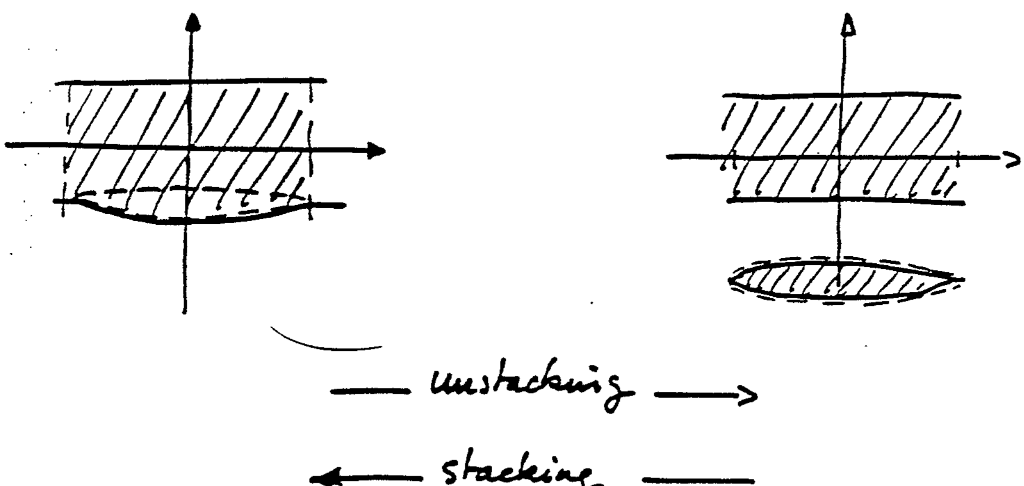
5

See : 1983 US conf p2280.

Capture efficiencies > 90% , time ~ few T_{S_2} , $A_2/A_1 \simeq .4$
for synchrotrons.

For storage rings A_2/A_1 much larger : perfectly reversible operation

Example : unstacking , stacking



May need very low voltages (few Volts in AA) → use missing bucket scheme to carry smaller emittances.

- AGS at injection $\phi_s \neq 0$ efficiency ~ 75 to 80%
more complicated simulation.

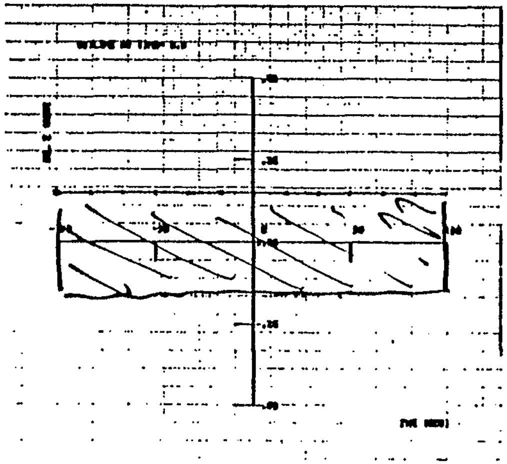


Fig. 1a

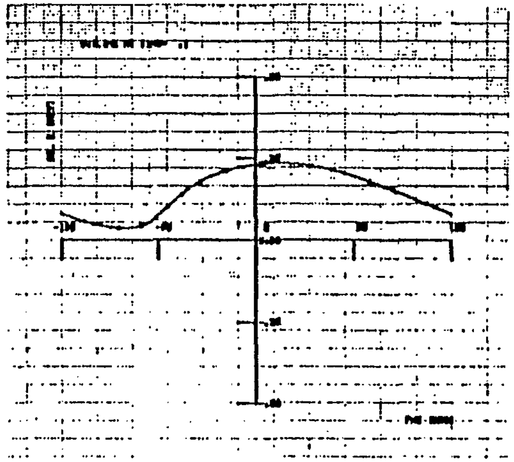


Fig. 1b

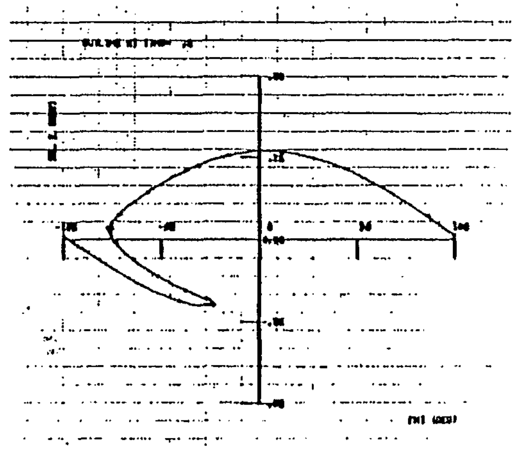


Fig. 1c

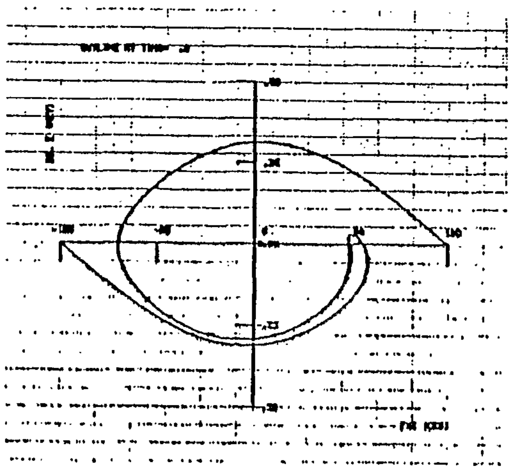


Fig. 1d

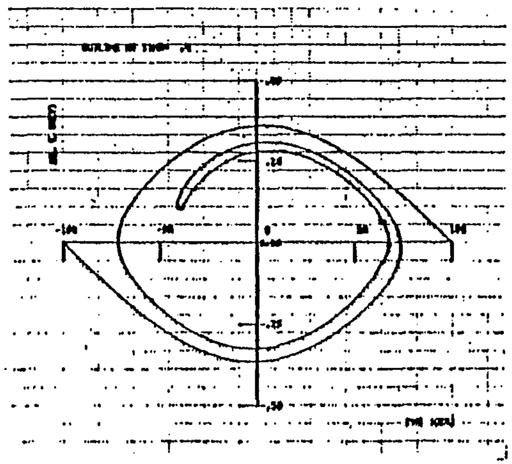


Fig. 1e

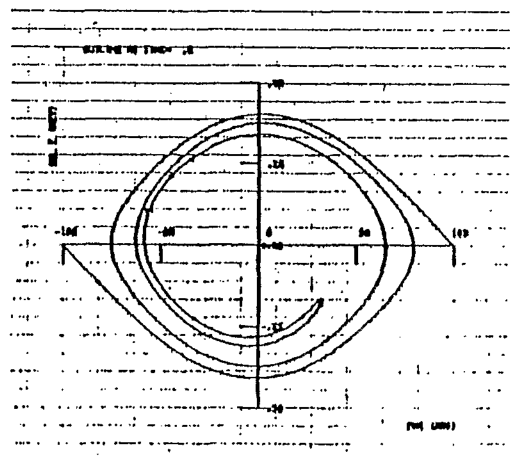


Fig. 1f

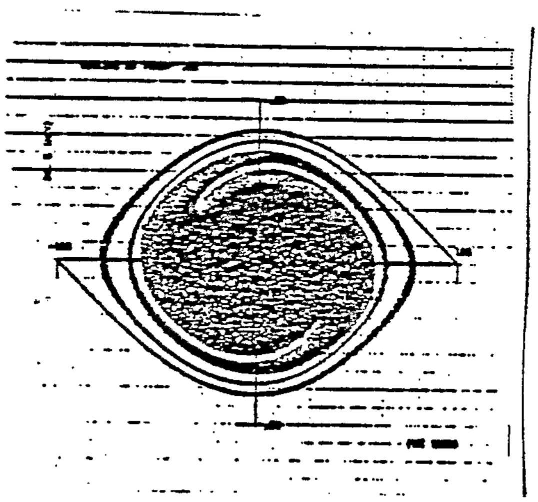
Linear voltage rise $V_2/V_1 = 12$ in $2.7\ T_s$ (2)

$\phi_s = 0.5°$

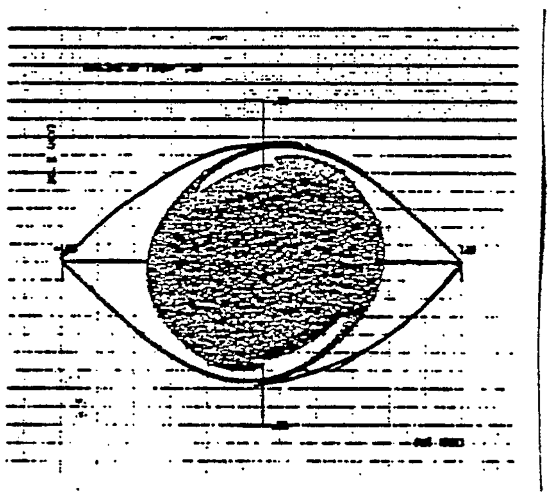
from CERN.MPS/BR 73-17

5"

Final bunch shapes

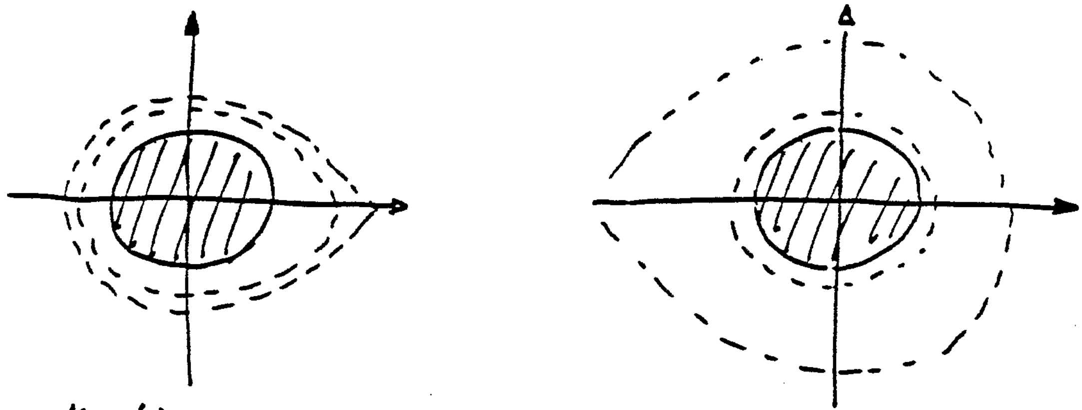


Linear rise



iso ~~Adiabatic~~ rise

Injection matching



Machine A — Machine B

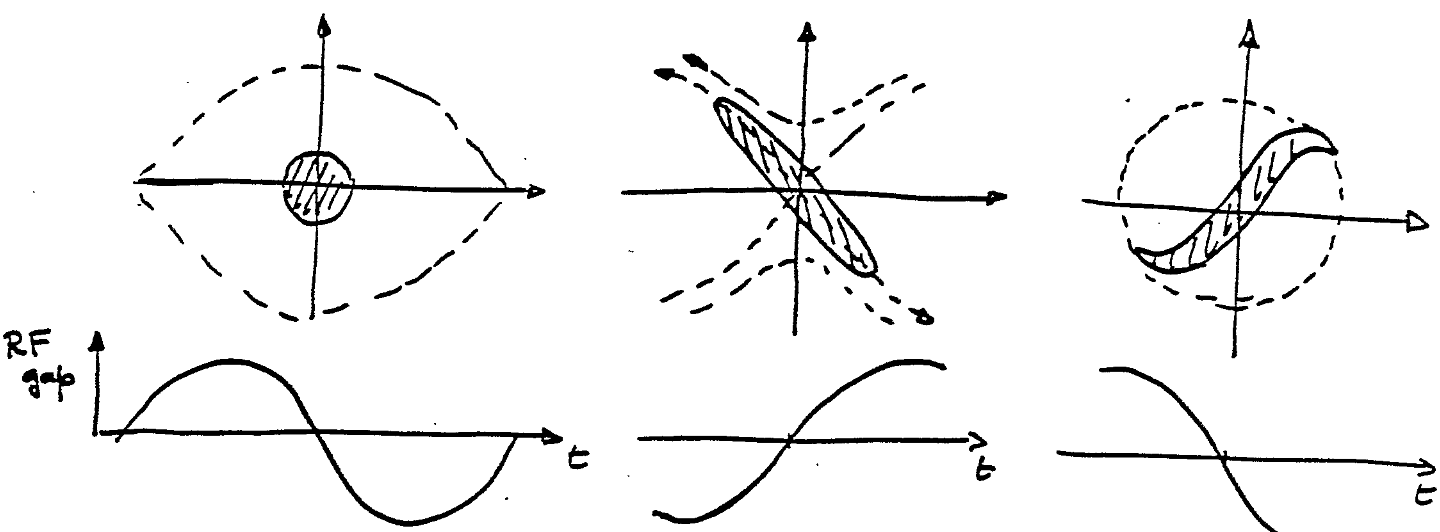
h or f_{RF} may be different (ex : $h = 20$ to $h = 6$ in CPS)

Variant with $\frac{1}{4}$ Ts rotation (compare with $\lambda/4$ matching in RF engineering).

$$f_{s_1} \; f_{s_3} = f_{s_2}^{\,2}$$

matching: f_{s_1} before, f_{s_3} after; f_{s_2} during $T_s/4$

Controlled blow-up



180° phase jump — reverse phase jump

Table I - Main Ring Parameters for Antiproton Production

Proton beam kinetic energy at extraction	120.	GeV
Number of Booster batches accelerated	1	
Number of proton bunches	82	
Total number of protons per batch	$2.\times10^{12}$	
Main Ring Cycle Time	2.0	s
Longitudinal emittance, 95% of beam at 120 GeV	.3	eV-s
Momentum aperture, $\Delta p/p$ at 120 GeV (full width)	.6	%
RF harmonic number (h) t 1113 br RF frequency at 120 GeV	53.1035	MHz
Revolution period at 120 GeV	20.96	ms
Booster batch length	1.56	µs
Transition gamma (γ_T)	18.75	
Mixing factor, $\eta=\gamma_T^{-2}-\gamma^{-2}$ at 120 GeV	.0028	
Maximum RF voltage	4.0	MV
RF voltage, start of debunching	1.0	MV
RF voltage, end of debunching	27.0	kV
Time required for debunching	100.	ms
Time required for rotation	1.446	ms

Table II-Debuncher Parameters

Kinetic energy	8.0	GeV
Number of antiproton bunches	82	
Total number of antiprotons ($\Delta p/p$=3%)	$7.\times10^{8}$	
Momentum aperture, $\Delta p/p$ (full width)	4.	%
Bunch width (full)	<1.	ns
Transition, γ (γ_T)	7.661	
Mixing factor, $\eta=\gamma_T^{-2}-\gamma^{-2}$	.006	
RF frequency	53.1035	MHz
RF harmonic number (h)	90	
Revolution period	1.695	us
Maximum RF voltage	5.0	MV
RF voltage, end of rotation	122.5	kV
RF voltage, end of debunching	5.0	kV
Time required for rotation	.103	ms
Time required for debunching	12.712	ms

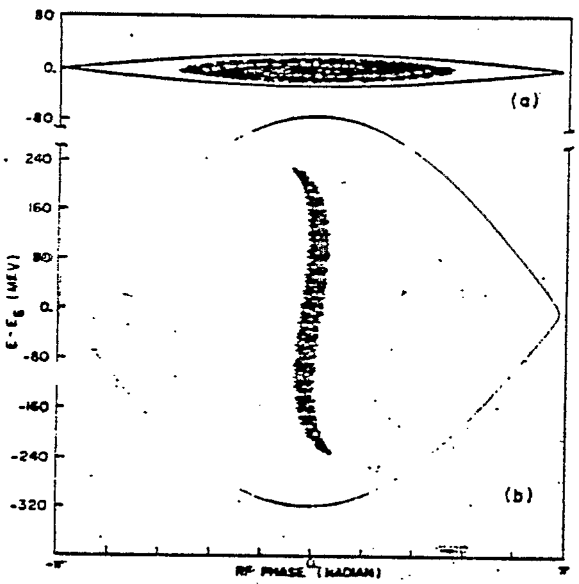


Fig. 1: Main Ring bunch rotation simulation

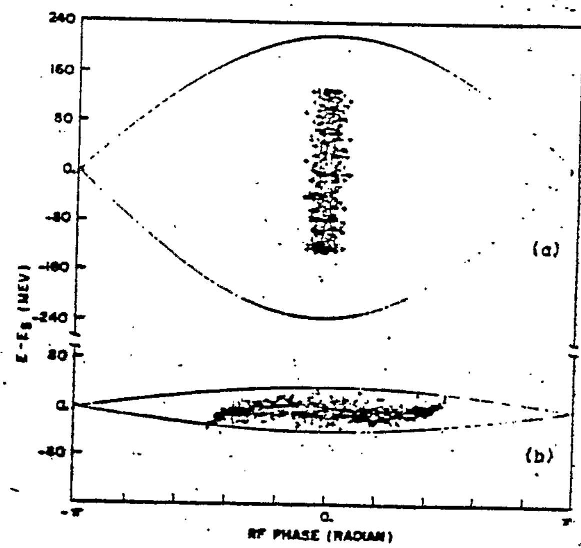


Fig. 3: Debuncher bunch rotation simulation

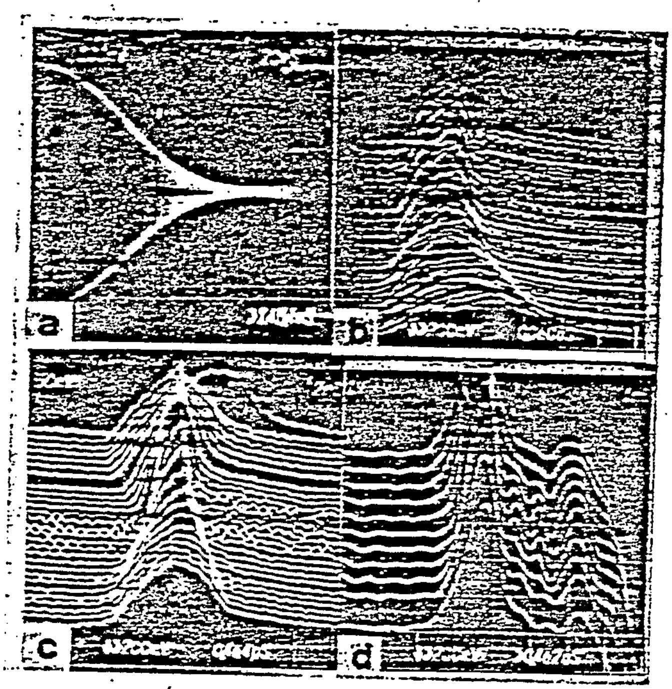


Fig. 2: Bunch rotation in Main Ring

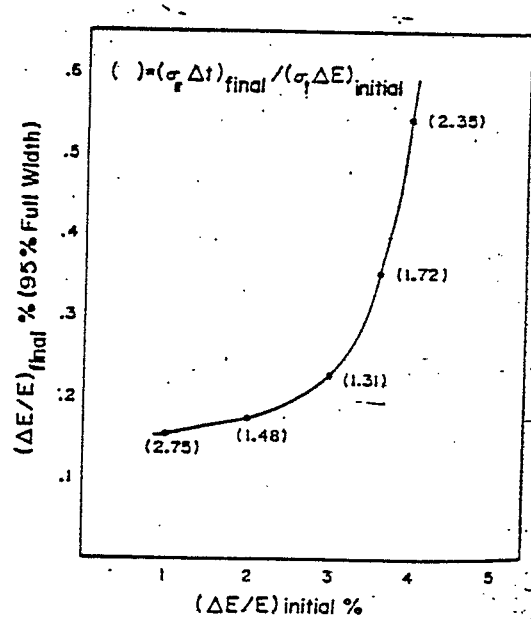


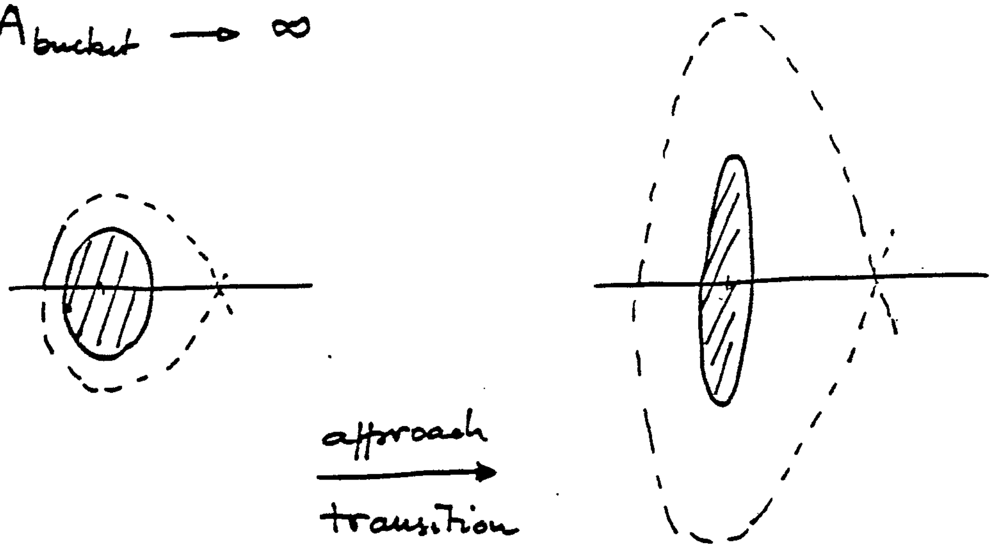
Fig. 4: Debunching efficiency

Griffin et al. US 1983 Conf p2632

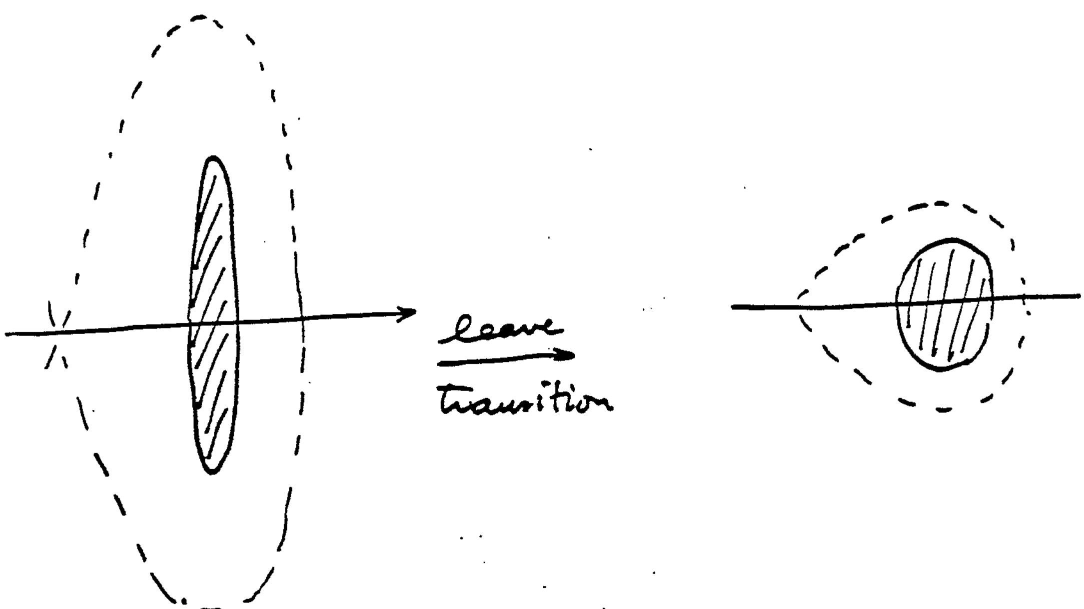
<u>Transition</u> $\gamma = \gamma_{tr}$ $\eta \to 0$

$f_s \to 0$: non adiabatic

$A_{bucket} \to \infty$

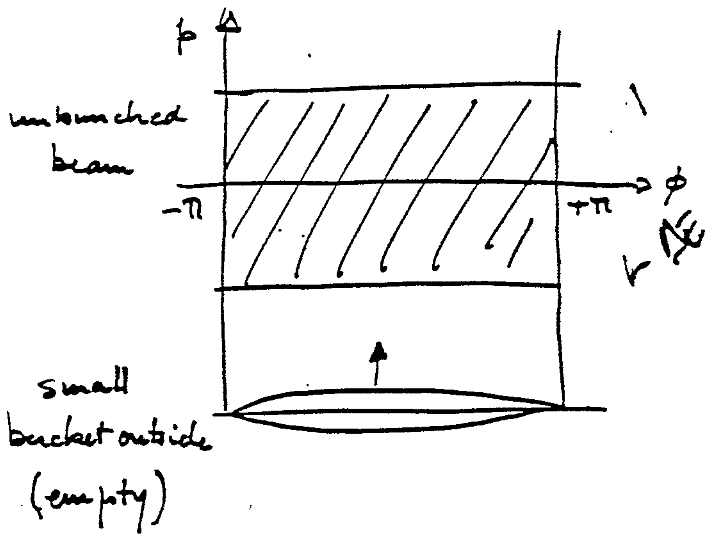


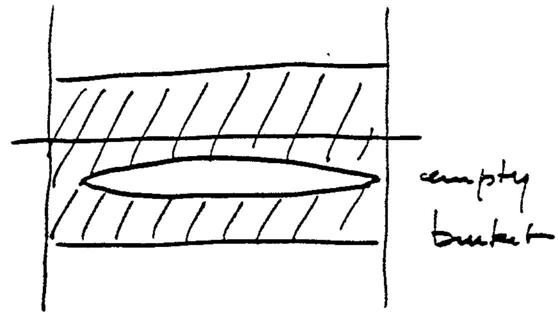
— exchange: stable and unstable points abruptly
(shift RF phase by $\pi - 2\phi_s$)



For a zero current beam geometric blow up very small.

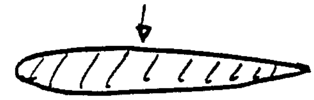
Example of an exotic RF manipulation





I_b (AC component)

$= -I_b$ (full bucket)



with same density

Energy given to full bucket = $S \times D \times \Delta E$

Surface   density   energy displacement

= Energy lost by full beam

= Average energy loss × number of particles

$= \delta E \times 2\pi \times \Delta E \times D$

$$\boxed{\delta E = \frac{S}{2\pi}}$$

→ phase displacement acceleration

Another example of sophisticated manipulation : merging of 2×5 bunches in the CPS

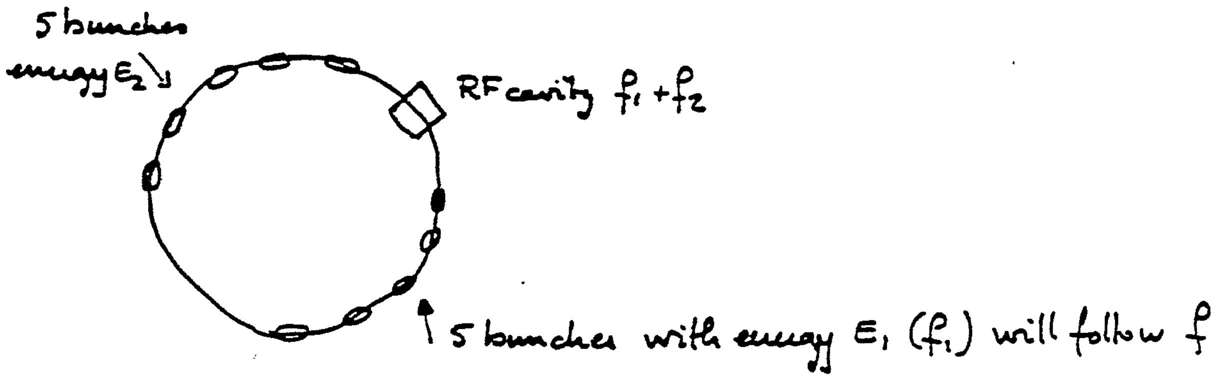
Feed the cavity with 2 different frequencies f_1 and f_2

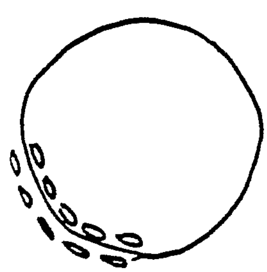
$$f_2 - f_1 = \Delta f$$

Very complicated and non stationary phase space but, if

$$\Delta f \gg 4\ f_s \qquad \text{(F. Mills, computer simulations)}$$

particles in bucket f_1 do not see f_2 and vice-versa.





After a time $\sim 1/\Delta f$: merging

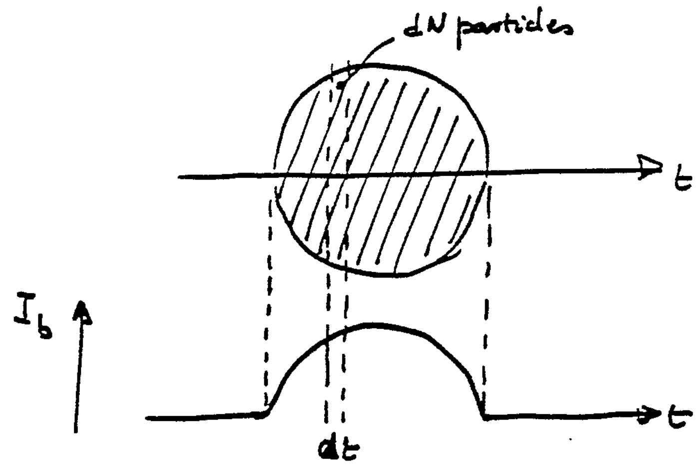
beam occupies only $1/4$ of circumference
= AA circumference.

How to produce 5 bunches with energy E_1 and 5 bunches with energy E_2?

Synthesize a missing bucket waveform (amplitude modulated RF wave) by combining $h = 20$ (carrier)

$$\left.\begin{array}{l} h = 19 \\ h = 21 \end{array}\right\} \text{sidebands}$$

Steady state beam currents.



projection on time axis

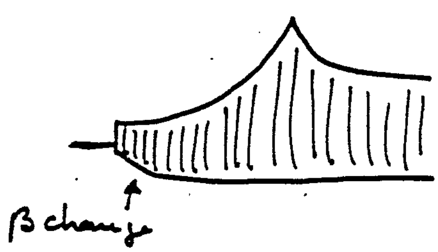
$$I_b = \frac{dN}{dt}$$

Monitored directly by a wall monitor

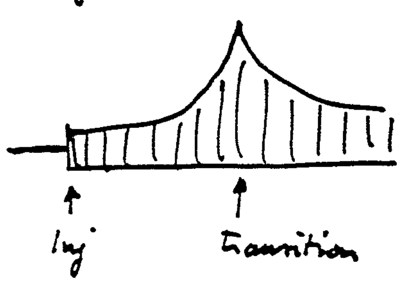
$$I_{wall} = -I_b$$

With an electrostatic monitor : charge instead of current.

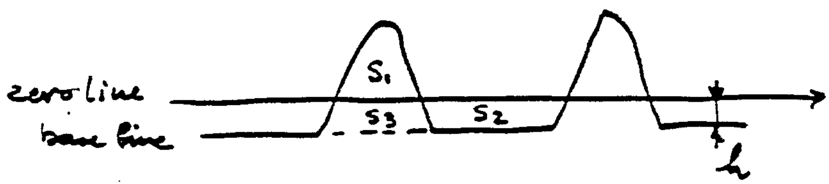
Note the difference with β change (acceleration)



No losses during acceleration



With AC coupled detectors, the base line height (h) is a measure of the beam current (or charge) in the bunch

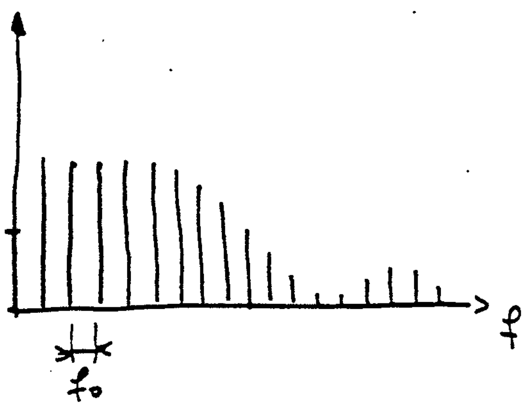


$S_1 = S_2$ (AC coupling)
$S_1 + S_3$ = beam charge
$= h \times$ period

→ Measure of capture efficiency.

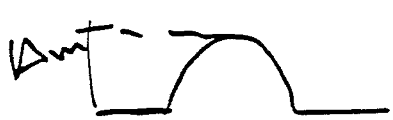
<u>Beam current spectrum</u>.

a) only one bunch : lines spaced by f_0 (f_r)



Amplitudes depends on distribution inside the bunch → bunch shape

examples : cosine shaped bunches length t_0,

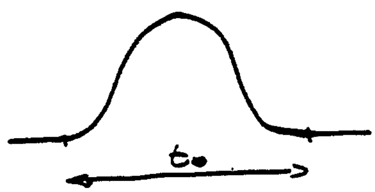


$$A_{DC} = \frac{2}{\pi} A_m \frac{t_0}{T}$$

$$A_n = 2 A_{DC} \left| \frac{\cos(n\pi \frac{t_0}{T})}{1 - (2n \frac{t_0}{T})^2} \right|$$

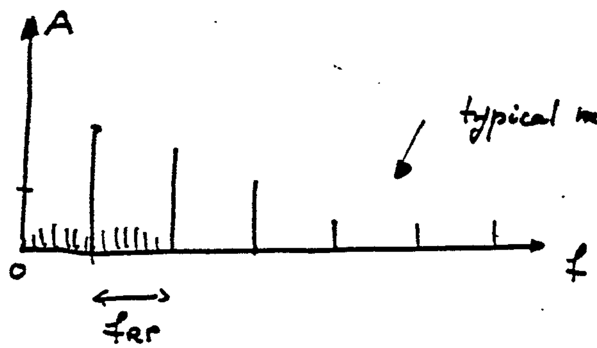
$\cos^2$ shaped bunches

$$A_{DC} = \frac{1}{2} A_m \frac{t_o}{T}$$



$$A_n = 2 A_{DC} \frac{\sin\left(n\pi \frac{t_o}{T}\right)}{\left(n\pi\frac{t_o}{T}\right)\left[1 - \left(n\frac{t_o}{T}\right)^2\right]}$$

b) h identical bunches : lines spaced by $h \times f_o = f_{RF}$



Steady state components of beam current will develop steady state voltages in the ring impedances.

- in the cavities (beam loading effect)
- in the vacuum chamber (space charge, inductive wall)

Consequences: - distorsion of bucket trajectories
- energy exchange between RF system and beam.

Energy exchange with RF cavities.

Two extreme cases:

- Narrow band cavity $\Delta f_{3dB} < f_0$

Beam induced voltage = sinusoid

Power received by beam $V \sin\phi_s \, I_b$

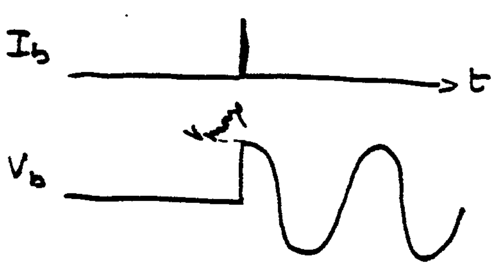
Power delivered by RF $\frac{1}{2} V I_{b_{RF}} \cos\phi$

ϕ = phase of beam current RF harmonic:

$$\cos\phi = \sin\phi_s \frac{2 I_b}{I_b}$$

- Wide band cavity $\Delta f_{3dB} \gg f_0$ or f_{rep}, short bunch

transient has decayed to zero at the next bunch passage



$$V_{max} = \frac{q}{C}$$

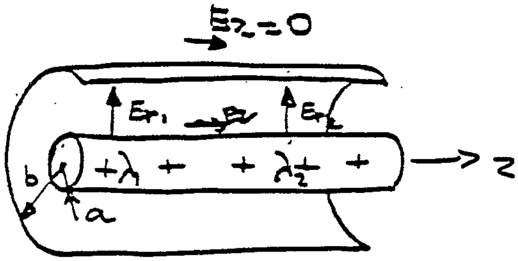
$$W_{cav} = \frac{1}{2} C V_m^2 = \frac{1}{2} q V_{max}$$

W lost by beam = q V seen by beam

$$V = \frac{1}{2} V_{max}$$

Fundamental theorem of beam loading (P. Wilson)

Outside the RF cavities.



$$E_r = \frac{e\lambda}{2\pi\varepsilon_0}\ \frac{1}{r}$$

If :- $\lambda_1 \neq \lambda_2$ $\qquad E_{r_1} \neq E_{r_2}$

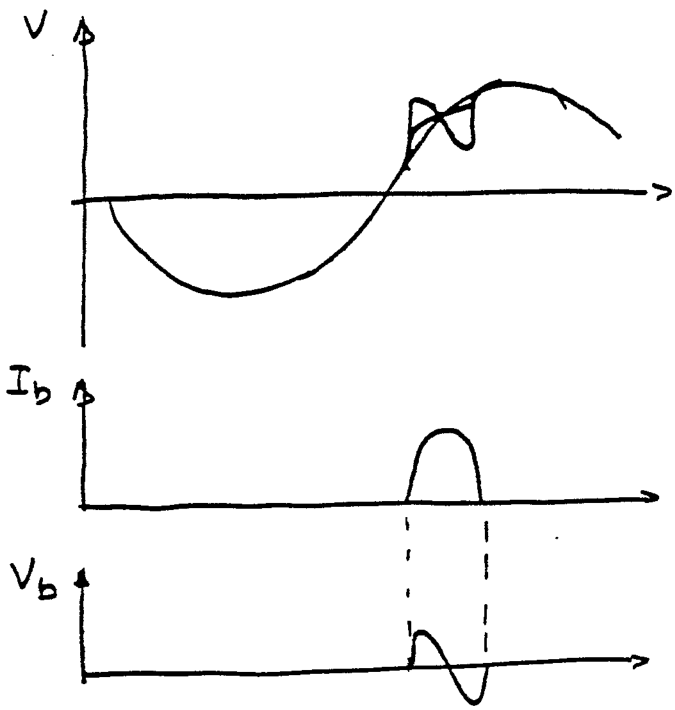
- Perfectly smooth, conducting wall :

$E_z = 0$ on wall

Then $\qquad E_{z\,beam} \neq 0 \qquad E_{z\,beam} \propto \dfrac{d\lambda}{dz}$

$$\boxed{E_z = -\frac{e}{4\pi\varepsilon_0}\ \frac{d\lambda}{dz}\ \left(1-\beta^2\right) g_0} \qquad g_0 = 1 + 2\ln\frac{b}{a}$$

electric $\quad$ magnetic $\quad$ components



beam current

space charge voltage

Consequences: Reduction of bucket area (below transition)

important at low energy

3.2.2 <u>Reduction of bucket area due to space charge effects</u> (below transition)

This reduction can be obtained from Fig. III.3.2.2, where

$$A\,A_{sp.c.} = 4\pi\, h\, g_o\, E_o\, r_p\, N/(\beta e V\, \gamma^2)$$

with

[15,3]

N = number of accelerated particles

$g_o = 1 + 2 \ln$ (vacuum chamber diameter/beam diameter)

r_p = classical proton radius

and E_o and eV are in the same units (as are r_p and R).

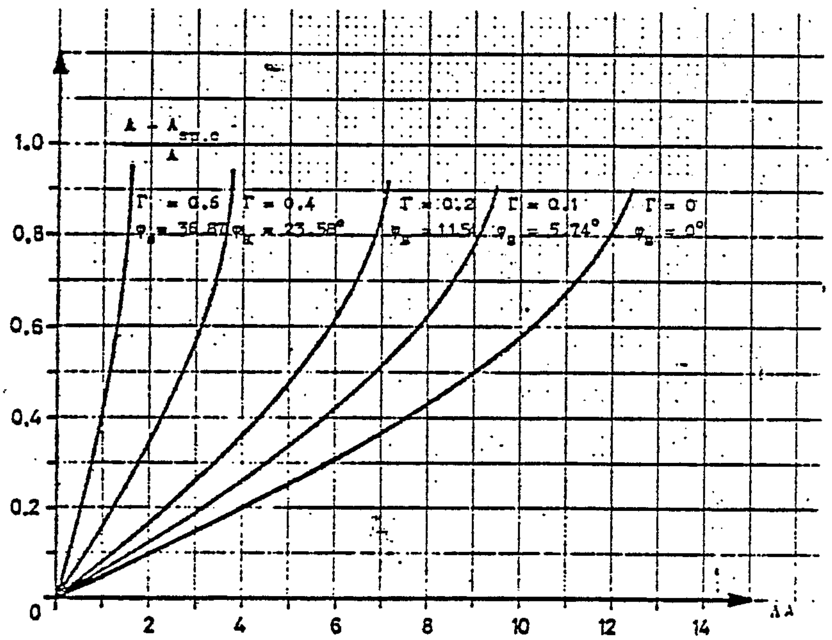


Fig. III.3.2.2 $(A - A_{sp.c})/A = f(A\,A_{sp.c.})$ (for constant density in phase space)

For $\varphi_s = 0°$ (and a $\cos^2$ distribution in real space) one has

[18, Appendix IV]

$$A_{sp.c.}/A = [1 - g_s\, e\, h\, N/(4\pi\, \varepsilon_o\, \gamma^2\, R\, V)]^{1/2}.$$

where V is in volts.

If the vacuum chamber wall is not perfectly smooth:

- cross section discontinuities
- boxes
- high order modes of RF cavities

represent it by a reactance, usually and inductance at low frequencies

Additional E_z field on the wall: $E_z = L \frac{dI_b}{dt} \sim L \frac{d\lambda}{dz}$

inductance / meter

Finally. for a parabolic bunch:

$$V_{z\,max} = \frac{3 I \ell}{2\pi^2 M R} \left(\frac{2\pi R}{\ell}\right)^3 \left(\frac{g_0 Z_0}{2\beta\gamma^2} - \omega_0 L\right)$$

strong bunch length dependance

ordinary space charge

inductive wall

Consequences: change bucket area

" synchrotron frequency of individual particles

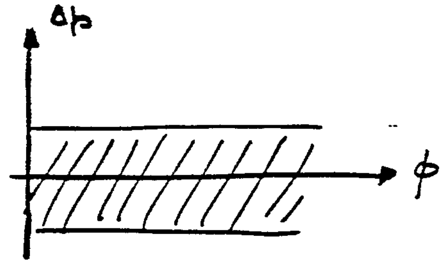
power losses (important for e^+e^- machines)

3. Beam Control Systems

- Damping of dipole oscillations: description of various schemes (multibunch, single bunch).
- Low frequency corrections: radial loop, frequency loop, synchronization loop.
- Quadrupole mode damping.

Few words on Schottky signals.

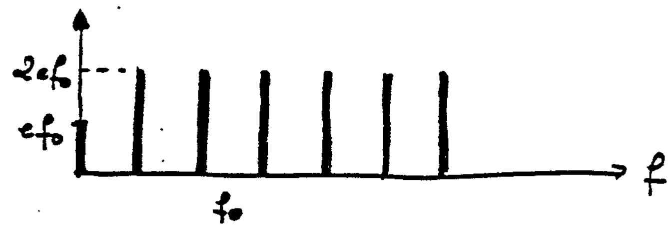
DC beam (RF OFF $\dot{B} = 0$)



no AC component

But : 1 particle

I_b spectrum



In a monitor

$$P_{DC} = R\, e^2 f_0^2$$
$$P_{Line} = \frac{1}{2} 4 R\, e^2 f_0^2$$

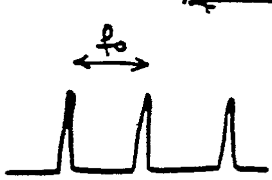
Many particles

$$P_{DC} = R\, (N e f_0)^2$$
$$P_{Line} = \frac{1}{2} N \times (4 R e^2 f_0^2)$$

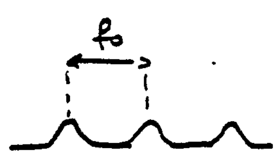
$$P_{schottky} = \frac{2}{N} P_{DC} \qquad = \text{constant / Line.}$$

Closer look at a line (number n)

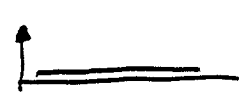
measurement of $\frac{\Delta p}{p}$, distribution ($\sqrt{D}$)
(even possible in a pulsed machine
→ highest n to decrease analysis time)



low n

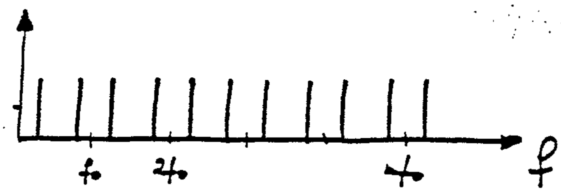


intermediate n

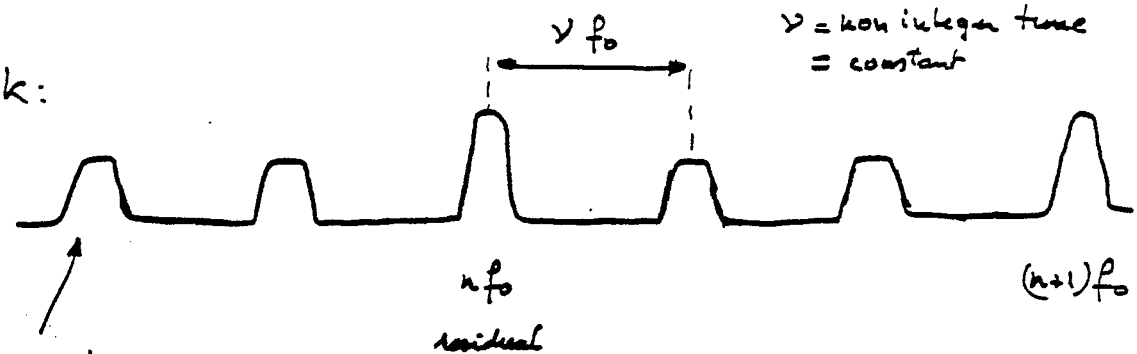


high n (overlaps)

With a transverse monitor:



Closer look:



Power prop. to average transverse oscillation amplitude

↓

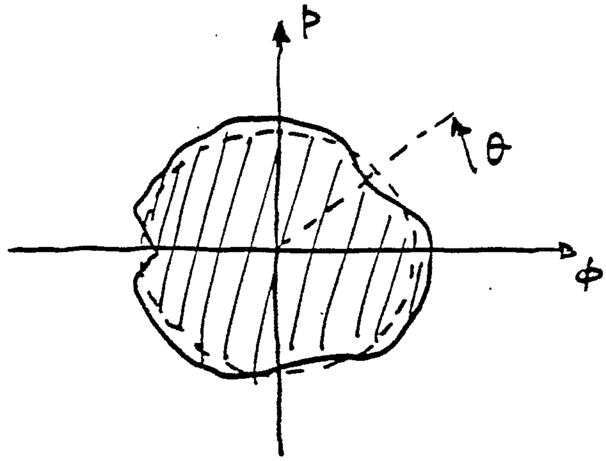
measurement of transverse emittance

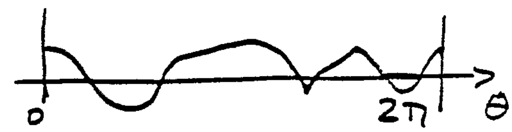
At high frequencies (n large) complete overlap of betatron bands: new information on individual particles is available each turn

→ good <u>mixing</u> situation

Perturbed beam currents

Simplified approach : constant density distribution





1) Represent the beam boundary by its Fourier components:

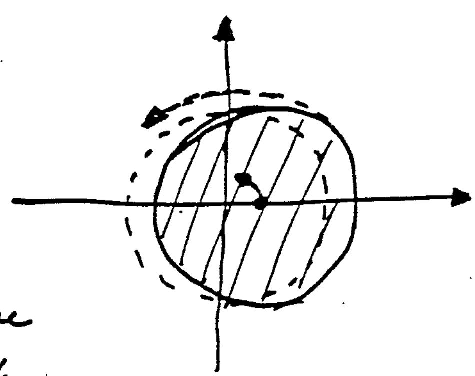
Numerology:

$m=0$	steady state
$m=1$	dipole
$m=2$	quadrupole
$m=3$	sextupole
"	"

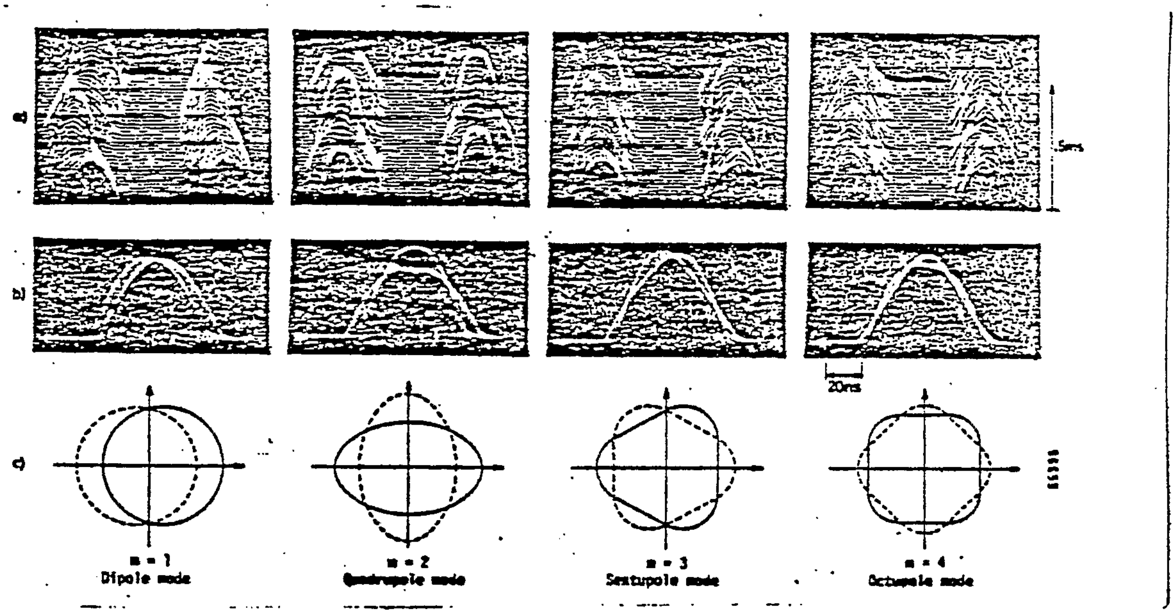
2) Study the evolution of each mode separately

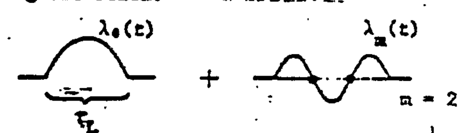
For example:

Dipole mode

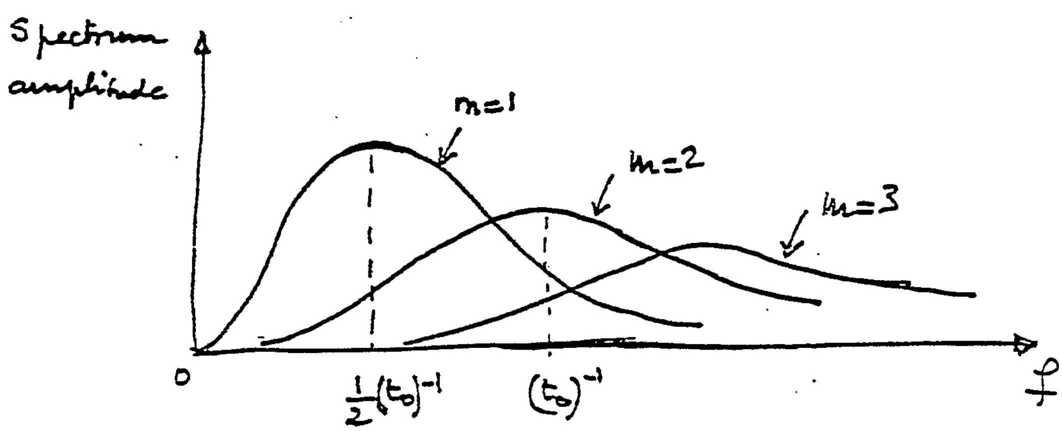


Described by the motion of the center of gravity of bunch





two zeros



t_0 = bunch length

Perturbation current spectra for various modes

5.

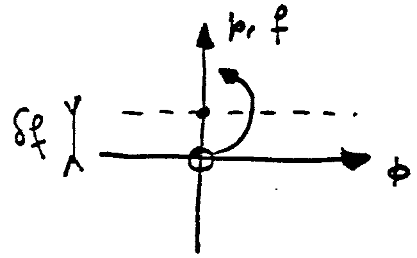
Dipole mode beam transfer function.

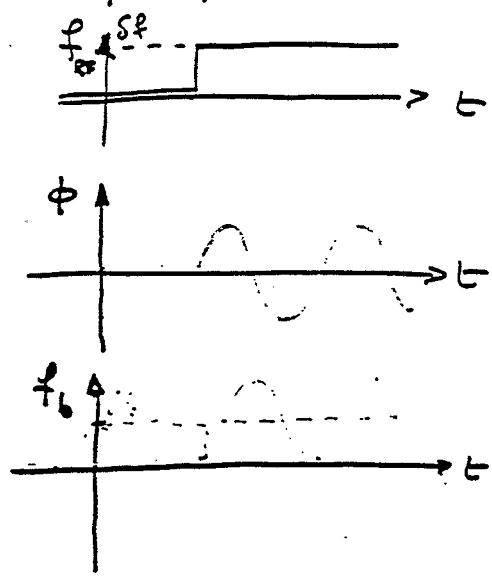
Phase (frequency) perturbation on RF

$\downarrow$

(frequency)
Phase perturbation on beam

Test with step function on RF frequency:



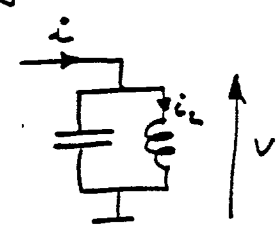


$\phi = \phi_b - \phi_{RF}$

From synchrotron oscillation equations

$$\boxed{\begin{aligned} \phi &= \frac{j\omega}{\omega_s^2 - \omega^2}\, \delta\omega_{RF} \\ \delta\omega_b &= \frac{\omega_s^2}{\omega_s^2 - \omega^2}\, \delta\omega_{RF} \end{aligned}}$$

Analogy with LC circuit:



$i \longrightarrow \delta\omega_{RF}$

$V \longrightarrow \phi$

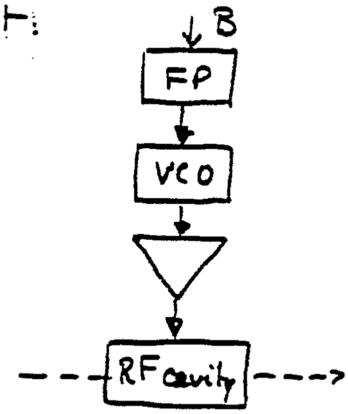
$i_L \longrightarrow \delta\omega_b$

$\omega_s^2 = \frac{1}{LC}$

No damping unless you wait for filamentation (blow-up).

LOW LEVEL RF SYSTEMS

The simplest:



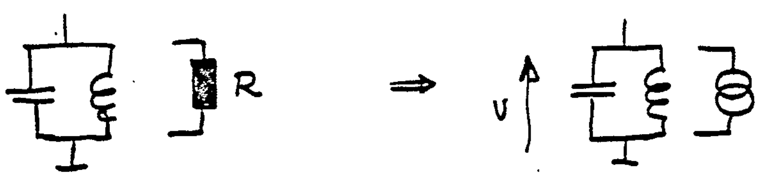
VCO = voltage controlled oscillator

FP = frequency program

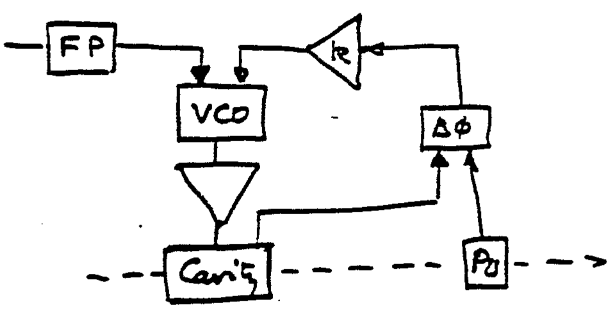
But : errors in FP
noise, ripple in VCO, magnet
tolerances at transition

} very large blow ups → losses

We need damping!



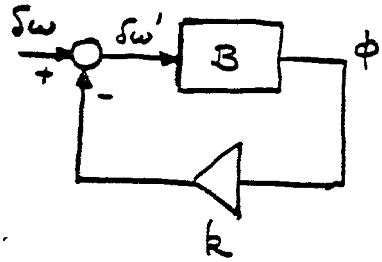
$i = kv \Rightarrow \delta\omega_{RF} = k\phi$



$\Delta\phi$ = phase detector

Damping of <u>dipole mode</u> (not individual particles)

Loop equations:



$$B = \frac{j\omega}{\omega_s^2 - \omega^2}$$

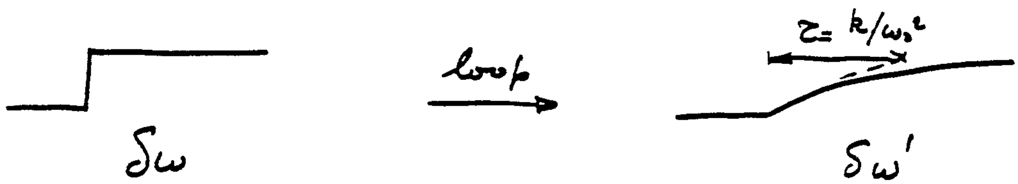
$$\phi = B\,\delta\omega \times \frac{1}{1 + kB}$$

$$\boxed{\phi = \frac{j\omega\,\delta\omega}{\omega_s^2 - \omega^2 + j\omega k}}$$

damping term

Usually the system is strongly overdamped (k large)

$$\delta\omega' = \frac{\delta\omega}{1 + k\,\frac{j\omega}{\omega_s^2 - \omega^2}} \simeq \frac{\delta\omega}{1 + j\,\omega \cdot \frac{k}{\omega_s^2}} \qquad \text{for } \omega < \omega_s$$



The effect of the loop is to make beam perturbation <u>adiabatic</u>

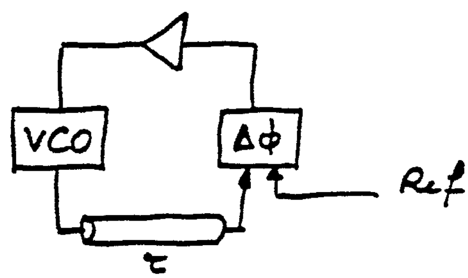
Bandwidth considerations

f_{rev} > unity gain frequency > f_s

present analysis incorrect   large k at f_s

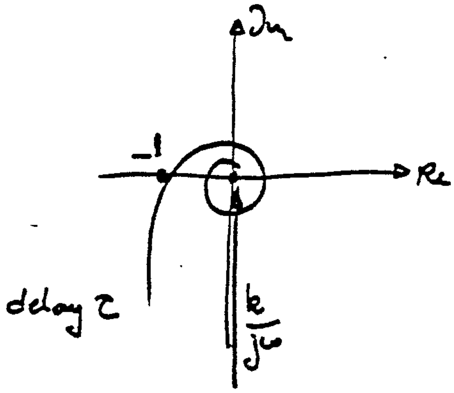
For $\omega > \omega_s$ $B = \dfrac{j\omega}{\omega_s^2 - \omega^2} \simeq \dfrac{1}{j\omega}$

beam equivalent to an external oscillator → classical phase lock circuit



– Delay ~ 1 turn   limits bandwidth to $f_m < \dfrac{1}{4\tau}$

$f_m < f_r/4$



Nyquist plot

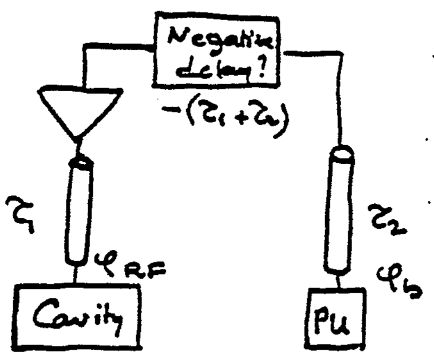
– Cavity bandwidth: $\Delta f_{3dB} > f_s$ (limitation for electron machines)

- Filtering of PU signals:
  - tunable filters (self-tracking amplifier)
  - heterodyne filtering (SPS)
  - sample and hold techniques (single bunches)
    - ↳ equivalent low pass filter
- Phase advance networks to optimise loop response

<u>The AGS low level system</u> $k \to \infty$

$$\phi = \phi_b - \phi_{RF} = \frac{j\omega\ \delta\omega}{\omega_s^2 - \omega^2 + j\omega k} \to 0 \quad \text{if } k \to \infty$$

Take beam RF component and feed RF cavities directly

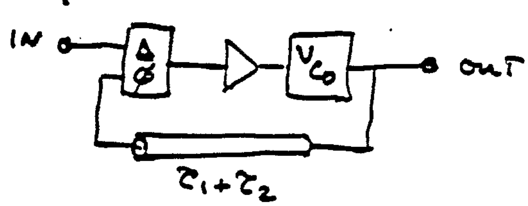


Required circuit characteristic:

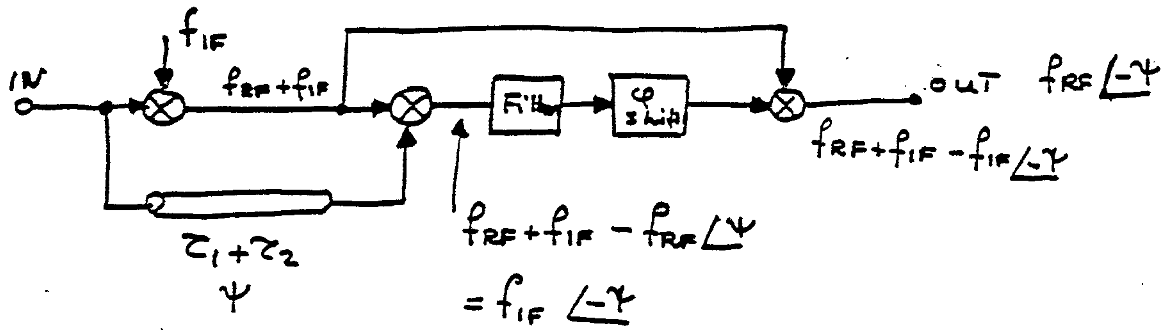
$$\varphi_{out} - \varphi_{in} = -\omega(\tau_2 + \tau_1) + \text{constant}$$

only for sinusoidal signal → ~~no~~ negative delay!

- one possible technique:



The AGS solution:

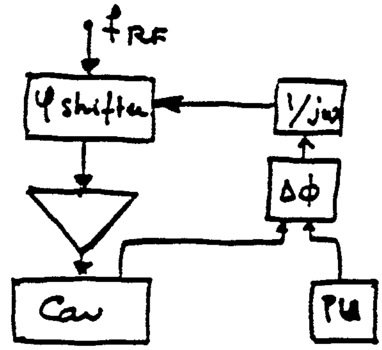


- Filters to select the proper side bands
- Phase shifter works at fixed frequency
- If $f_{IF} \gg f_{RF_{max}}$ no tunable filters are needed.

Like the Phase Locked Loop it is a non linear system.
Overall delay : $2(\tau_1+\tau_2)$

In the phase loop technique equal lengths of cable from PU and cavities (ϕ measured indep of f_{RF})

<u>Another damping technique</u>



integrator

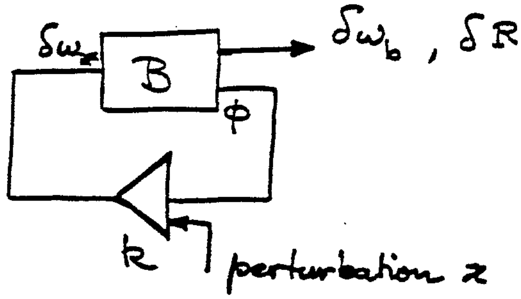
VCO is replaced by integrator + φ shifter

But limited to small k (limited range of φ shifter + integrator)

Useful to damp 1 bunch separately (multiplexer)

ISR, SPS.

Low frequency corrections

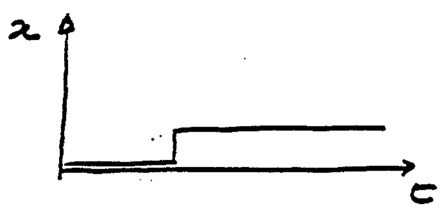


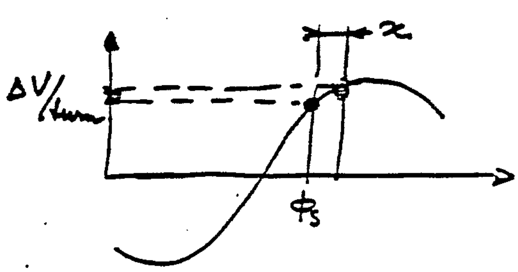
Transfer function $\delta\omega_b$ or δR $/ x$

$$\delta\omega_b = \frac{\omega_s^2}{\omega_s^2-\omega^2}\,\delta\omega_{RF} \qquad \delta\omega_{RF} = \frac{-kx}{1+k\frac{j\omega}{\omega_s^2-\omega^2}}$$

$$\delta\omega_b = \frac{-kx\,\omega_s^2}{\omega_s^2-\omega^2+jk\omega} \simeq \frac{1}{j\omega}\,\omega_s^2 x$$

↑ integrator

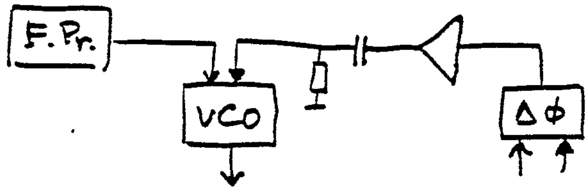




slope ~ independent of loop parameters

→ need corrections

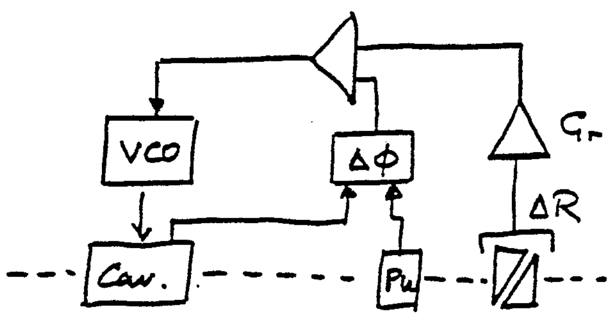
1st solution . AC coupling of phase loop



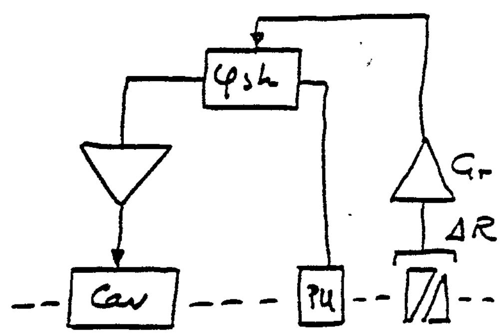
OK if F. Program precise enough
never true at transition

example. PS Booster

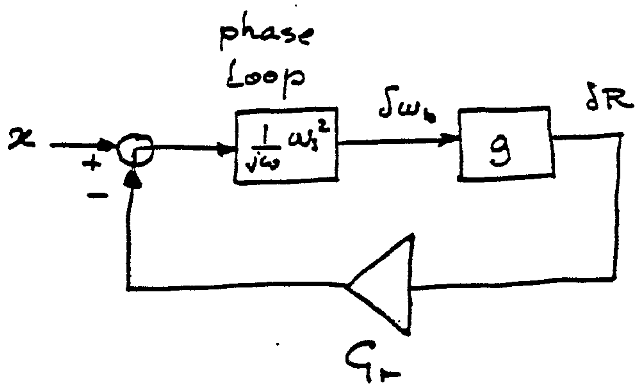
2nd Solution
Radial correction



Phase loop type



AGS type

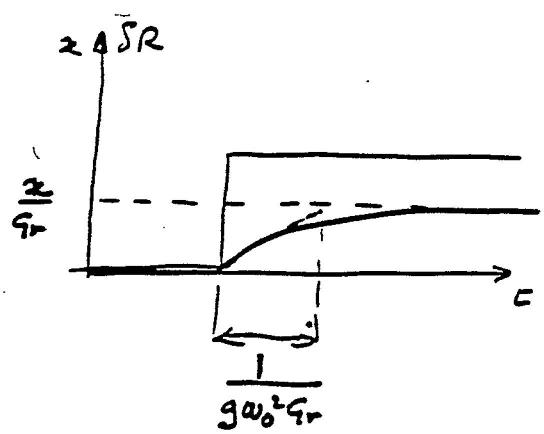


$$\frac{\delta R}{R} = \frac{\gamma^2}{\gamma_{tr}^2 - \gamma^2}\,\frac{\delta f}{f}$$

g

$$\delta R = g\,\frac{\omega_s^2}{j\omega}\,(x - G_r\,\delta R)$$

$$\boxed{\delta R = \frac{1}{G_r}\;\frac{x}{1+\dfrac{j\omega}{g\,\omega_s^2 G_r}}}$$



- δR limited
- single pole rolloff for G_r real

To minimise x = phase offsets → stable phase program

$$\phi_s = \text{Arcsin}\left(\frac{2\pi R e \dot{B}}{V_{RF}}\right)$$

changes sign at transition

→ frequency correction program (corrects differences in cable lengths)

Transition: g changes sign → G_r must change sign.

Frequency loop.

If a precise frequency program is available, measure

$$\delta f = f_{RF} - f_{prog} = f_b - f_{prog}$$

and use instead of δR

4. Instabilities

- Coasting beam, microwave, negative mass.
- Robinson instability, coupled bunch instabilities.
- Coupled loop instabilities. Landau damping.

δf measurement not affected by beam intensity like δR

→ low intensity beams (pilot $p\bar{p}$, heavy ions).

SPS Xtal f. discri, noise problems

→ at transition $\delta R \to \infty$ for $\delta f \neq 0$

need fast crossing (PS)

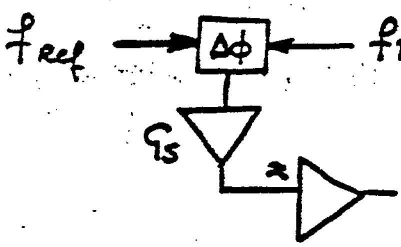
precise program

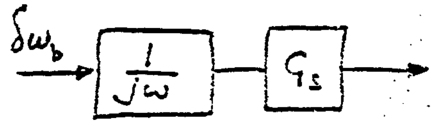
<u>No change of sign</u>

<u>Synchronization loop</u>

locks the beam (and RF) on an external frequency

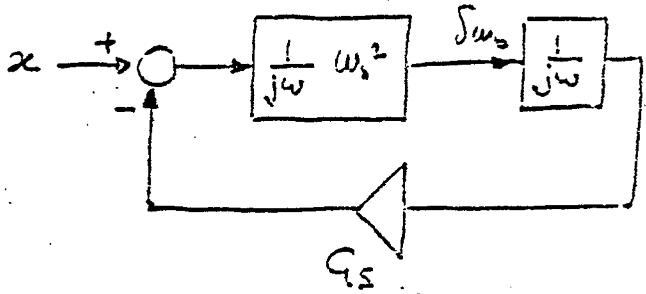
transfers from one machine to the next.



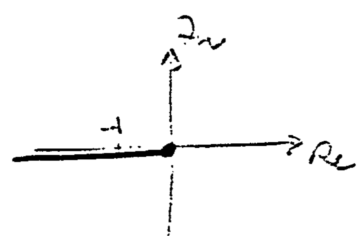


freq to phase conversion

phase loop   freq → phase

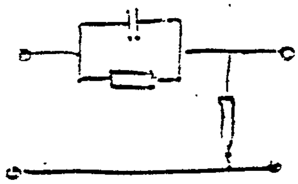


If G_S real : system unstable (2 integrators)

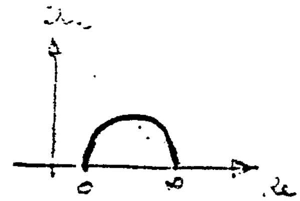


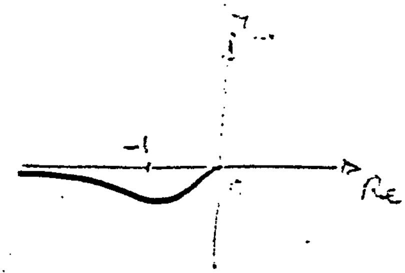
Need a phase advance network :

#2



Phase advance network





Corrected loop

QUADRUPOLE MODE DAMPING.

dipole mode → phase oscillation at f_{RF}

quadrupole mode → amplitude oscillation at $f \sim 1/\text{bunch length}$
→ oscillation of peak bunch current.

Similar analysis

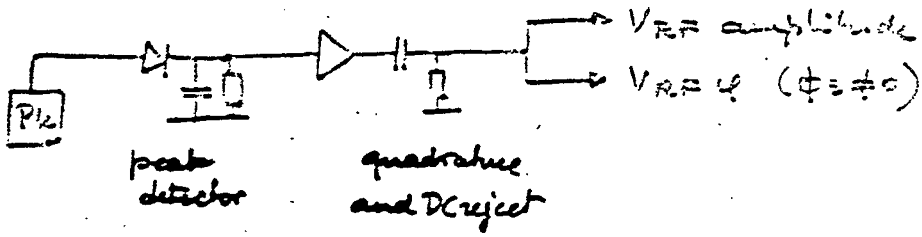
amplitude modulation of V_{RF} $\xrightarrow{\text{transfer function}}$ peak current oscillation

or, if $\phi_s \neq 0$

phase modulation of V_{RF}

$$\frac{\alpha}{(2\omega_s)^2 - \omega^2}$$

Damping of peak oscillation is effected in quadrature



The perturbation approach to beam damping.

Leave the RF waveform unchanged, but synthesize the required perturbation.

Damping of dipole mode is obtained if:

$$\delta\omega_{RF} = j\omega\,\phi_{RF} = k\,\phi = k\,(\phi_b - \phi_{RF})$$

$$\phi_{RF} = \frac{k}{k + j\omega}\,\phi_b \simeq \frac{k}{j\omega}\,\phi_b \qquad \text{(small damping)}$$

→ ϕ_{RF} must have a quadrature component / ϕ_b

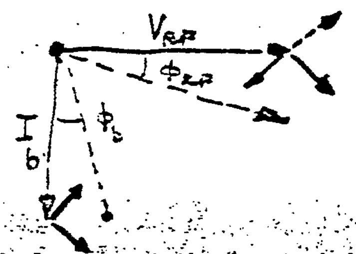
Represent ϕ_b and ϕ_{RF} (phase modulation at ω) with carrier + sidebands.

Beam side bands $+\omega, -\omega$, should be transformed by an equivalent impedance into RF side bands such that quadrature ϕ is obtained.

Carrier transmission unimportant

4

A possible solution: $Z_{equivalent}$ Real and changes sign at f_{RF}



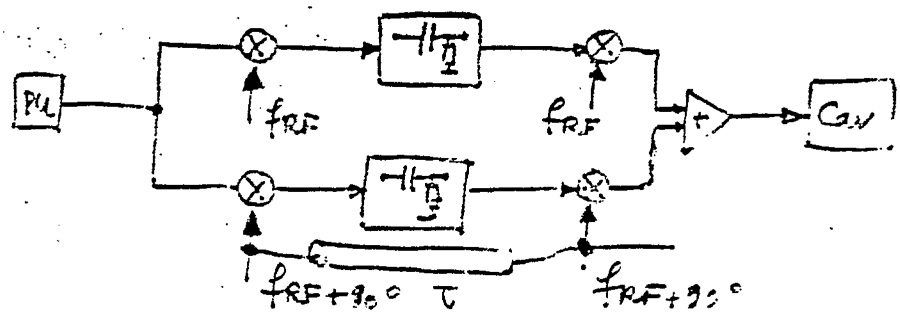
→ $+\omega$ side band

→ $-\omega$ side band (changes sign)

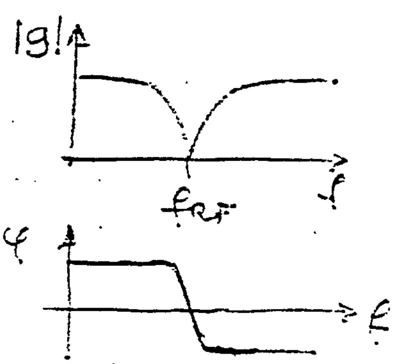
$\left.\begin{array}{l}\phi_b \text{ max} \rightarrow \phi_{RF} = 0 \\ \phi_b = 0 \rightarrow \phi_{RF} \text{ max}\end{array}\right\}$ quadr.

Circuit synthesis

$I_b \longrightarrow Z_{equiv} \longrightarrow V_{R.F}$



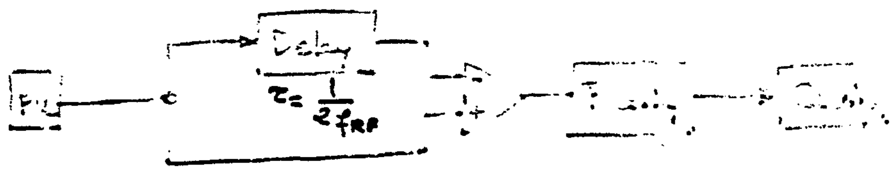
Transfer impedance:

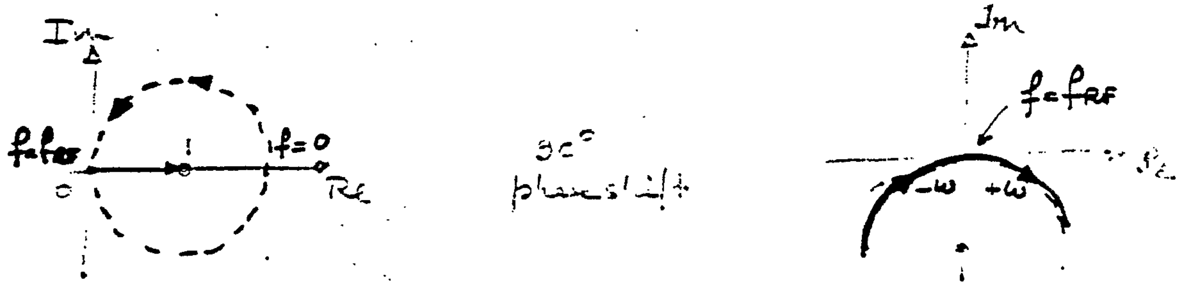


P45

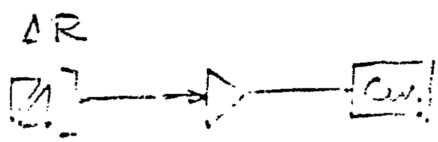
- [illegible] of the [illegible]
- technique similar to S.S.B. [illegible]
- [illegible]

<u>Another technical solution</u>





Application for stochastic cooling ([illegible])

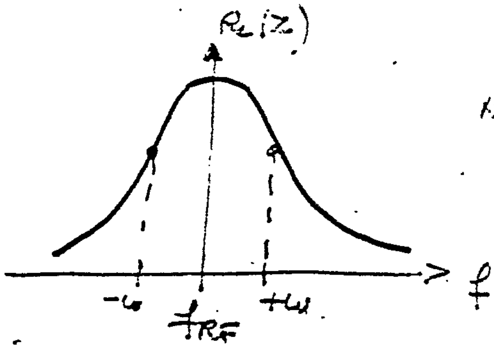


- ΔR [illegible]
- [illegible]

(Palmer cooling)

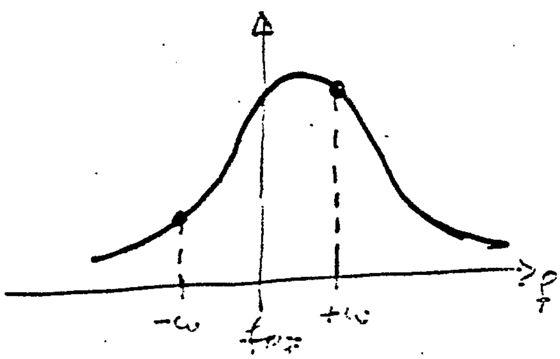
INSTABILITIES

Robinson instability, first example.

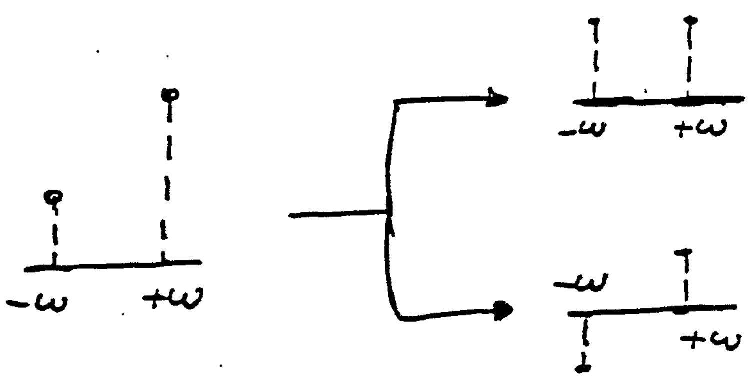


RF cavity impedance

Detune the cavity:



→ 2 unequal sidebands.



change of ω_s (real)

damping or antidamping

↓

INSTABILITY

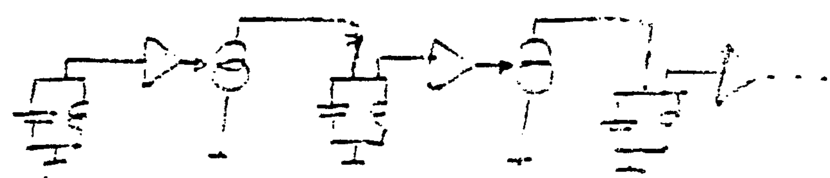
stability if $\omega_{RF} < \omega_c$

For proton machines phase loop takes care of it.

MULTIBUNCH INSTABILITIES.

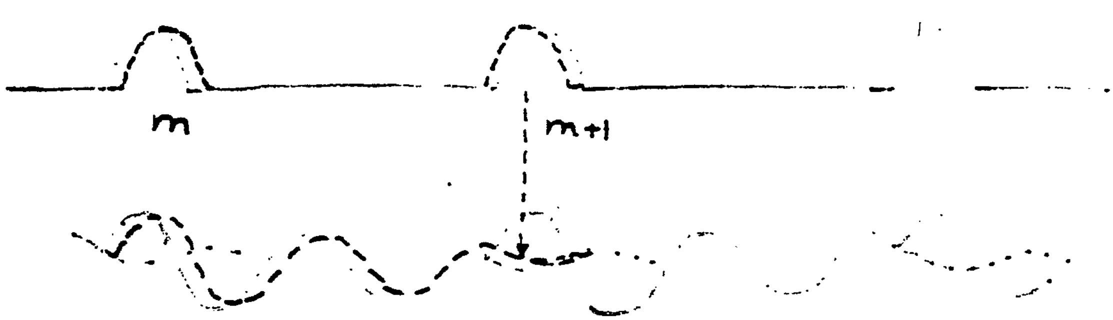
If M bunches in the machine, M RC equivalent circuits

Any coupling from one bunch to the next:



Looks like an oscillator circuit: phase shift/stage = $2\pi/M$ → INSTABILITY

Simplified analysis for dipole mode.



— steady state

— displaced bunch n° m

— displaced bunch n° m+1

[illegible]

[illegible]

$$\boxed{\ddot{\Phi}_m + \Omega^2 \Phi_m + \beta \Phi_{m+1} = 0}$$

looking for solutions

$\Phi_m = \Phi_m \cos \omega t$; [illegible]

For branch m: $\boxed{\Phi_m (\Omega^2 - \omega^2) + \beta \Phi_{m+1} = 0}$

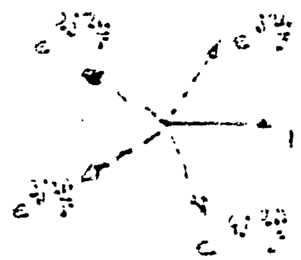
[illegible]

$$\begin{cases} (\Omega^2-\omega^2)\Phi_1 \quad [illegible] = 0 \\ \beta \Phi_1 + (\Omega^2 - \omega^2)\Phi_2 = 0 \\ \beta \Phi_2 + (\Omega^2-\omega^2)\Phi_3 = 0 \\ \beta \Phi_3 + (\Omega^2 - \omega^2)\Phi_4 = 0 \\ \cdots \quad (\Omega^2-\omega^2)\Phi_n = 0 \end{cases}$$

Set of [illegible] equations.

Non zero solutions if $\underline{\mathrm{Det} = 0}$

$$\frac{\Omega^2-\omega^2}{-\beta} = (1)^{\frac{1}{5}} = \exp j\,n\,\frac{2\pi}{5}$$



of either sign

$$\Delta\omega = \Omega-\omega \simeq \frac{\beta}{2\Omega}\left(\cos n\,\frac{2\pi}{5} + j\sin n\,\frac{2\pi}{5}\right)$$

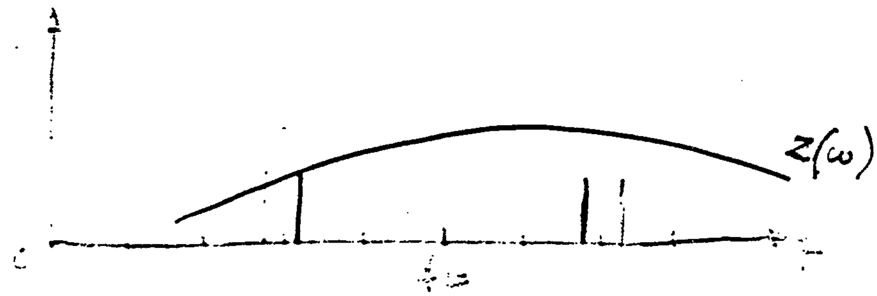
real shift in f_s

imaginary shift in f_s

damping or antidamping

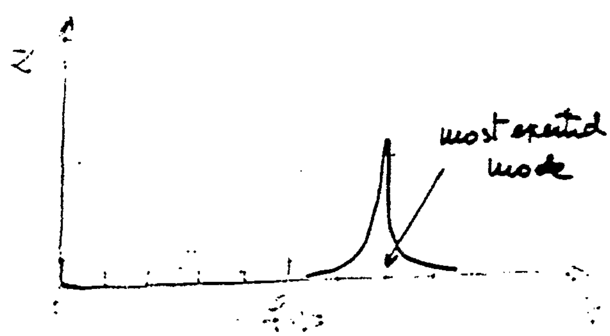
Growth time:

$$\tau = \frac{1}{\Delta\omega} = \frac{2\Omega}{\beta\sin\frac{2\pi}{5}} \qquad (n=1)$$



Side bands of f...

$$f_{RF} \pm (n f_0 - m f_s)$$ mode $k-n$ $(-n)$



II

Possible cures:

1) Fix the impedance: [illegible]

[illegible]

Usually: higher order modes of RF cavities [illegible] electron rings).

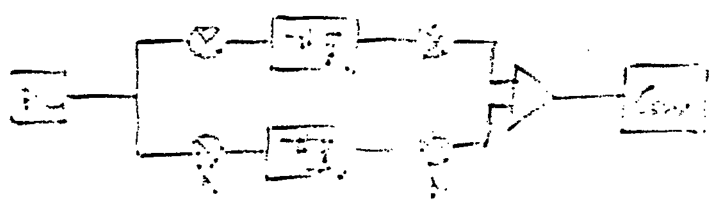
2) Detune the resonator: (coupling loops) [illegible] frequency between two f. lines (fixed RF frequency).

Feedback:

1) Bunch-by-bunch: damp each bunch individually [illegible]

2) [illegible]: damp each mode separately

By [illegible] the [illegible] [illegible]
the damped modes [illegible]



[illegible]

[illegible]

[illegible] :

No [illegible] needed to [illegible] [illegible]

[illegible]

Consequence: [illegible]

[illegible]

FREQUENCY SPLITTING

Different f_s to different bunches : [illegible]

Stability criterion (approximate):

$\boxed{[illegible]}$

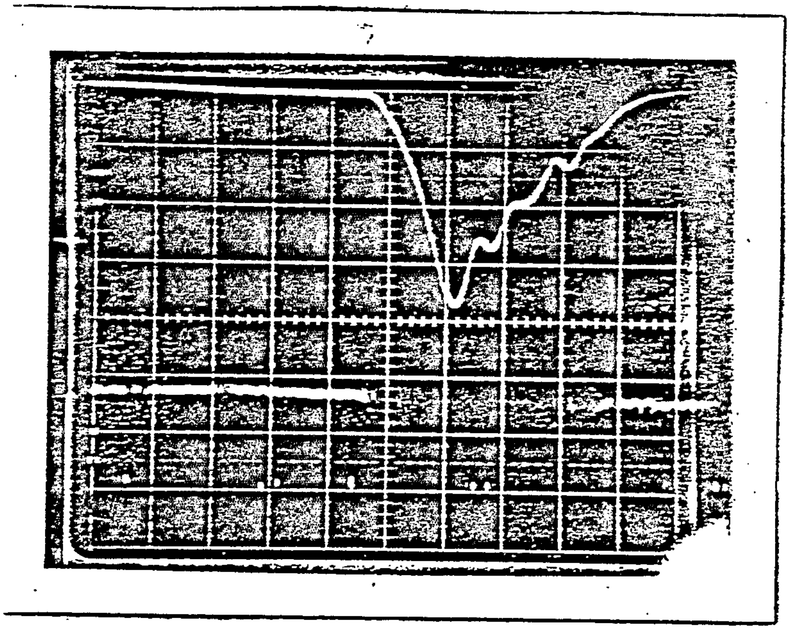
spread in [illegible] [illegible]

[illegible] [illegible]

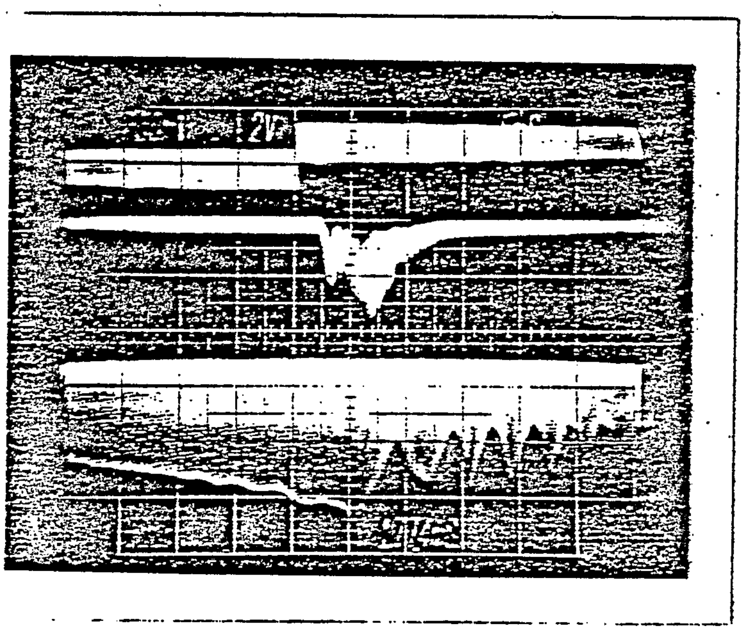
– [illegible] ω_s [illegible]

$$\frac{\Delta\omega_s}{\omega_s} = \frac{1+ [illegible]}{[illegible]}$$

Limited efficiency ([illegible]



Coasting beam instability in AGS. RF [illegible]



Microwave instability at transition in AGS

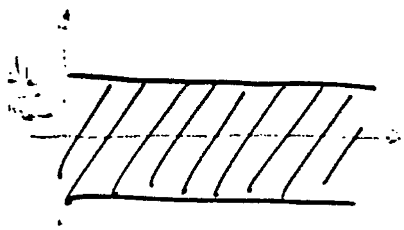
13

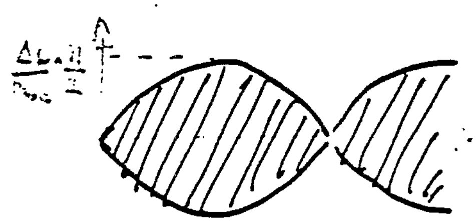
COASTING BEAM INSTABILITY

Density modulation [illegible] Ring impedance → [illegible] [illegible]
$\bar{I}_b$ [illegible] Z [illegible]

[illegible] [illegible] [illegible]
Self [illegible] : Z [illegible]) → [illegible]
proton ring : many harmonics

Threshold





Same beam, [illegible] [illegible]
[illegible]

Voltage produced by modulated beam $V = Z\,I_{b\omega}$ [illegible]

[illegible] [illegible] : $\sqrt{\ }\ \sqrt{\ }\ \frac{\ }{\ }\ \frac{1}{\sqrt{\ \eta E}}$

Self sustained situation (Keil Schnell) if:

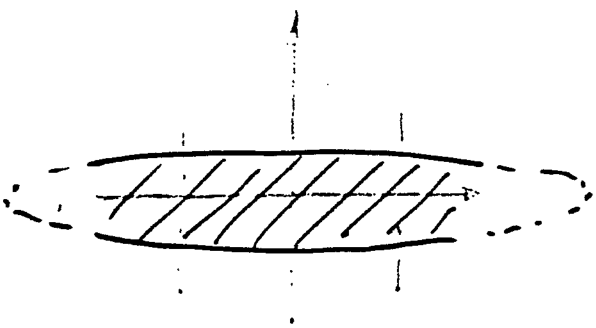
$$\sqrt{\ }\ \sqrt{\ }\ \frac{\ }{\ }\ \frac{1}{\ \eta E} > \frac{\Delta}{\ }\ \frac{\ }{\ }$$

[illegible]

Stability criterion, [illegible]

- Reduce Z of RF cavities [illegible]
[illegible]

Bunched beam case



looks like a coasting beam of [illegible]

- frequencies >> [illegible]
- fast growth

Apply coasting beam criterion to local values [illegible]

→ [illegible]

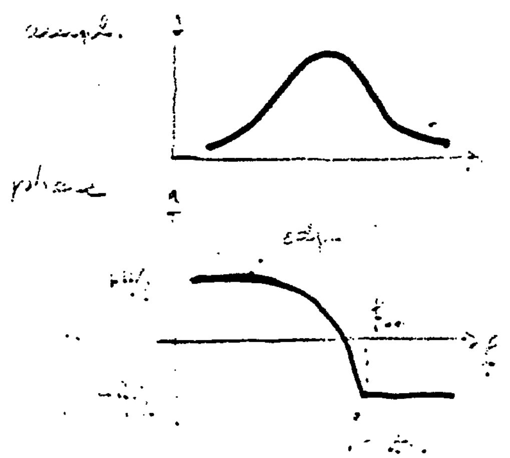
- example: Gaussian ($\eta \to 0$), [illegible]

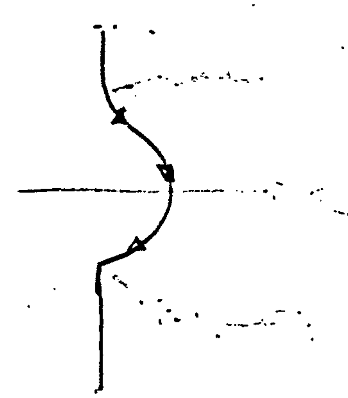
Importance of $\boxed{Z/n}$ at very high frequencies [illegible]

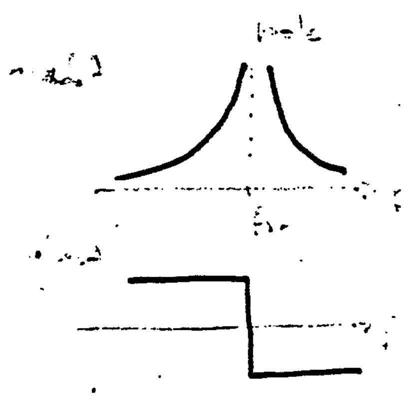
impedance / [illegible]
(bellows, [illegible])

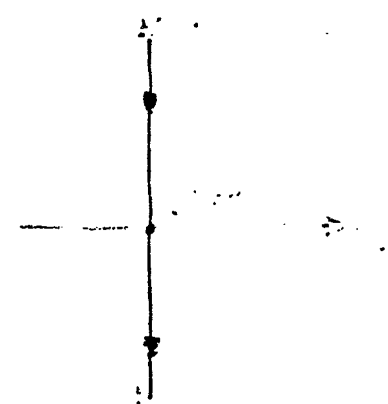
5. Two examples of beam loading compensation at CERN

- Narrow band (CPS) against coupled loop instabilities.
- Broad band (SPS) against coupled bunch instabilities.



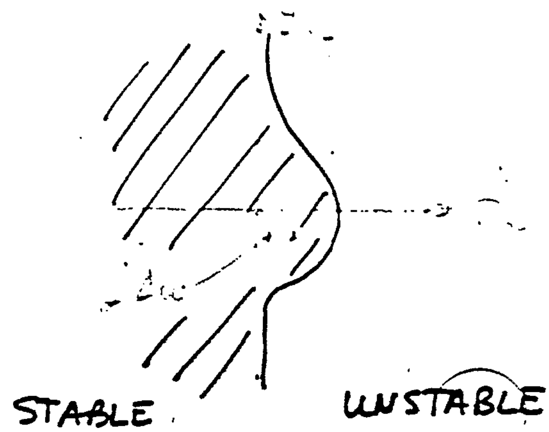






$$\frac{1}{B} = \frac{\omega_0^2-\omega^2}{j\omega}$$

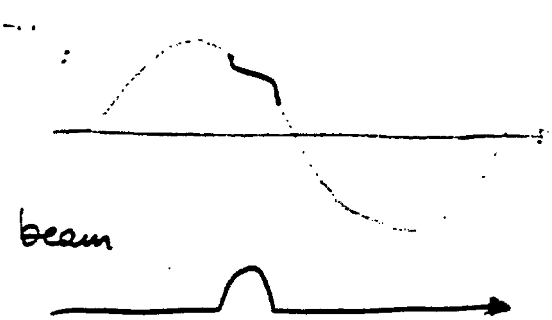
Beam is thus against instabilities if [illegible] lies [illegible] on the left of the stability diagram



<u>Approximate threshold</u> (Sacherer)

$$\boxed{\text{spread in } \omega_s > \frac{4}{[illegible]} \, |\Delta\omega_{[illegible]}|}$$

<u>The effect of the [illegible] field</u> [illegible]

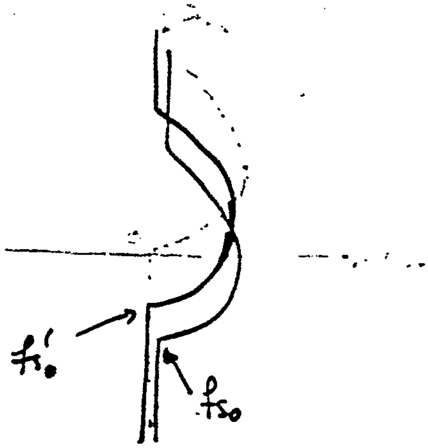


[illegible]

[illegible]

3

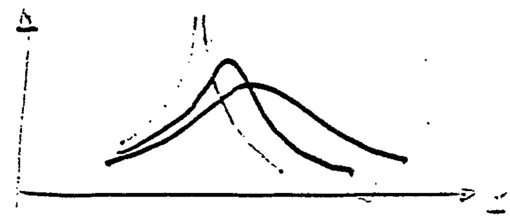
[illegible]

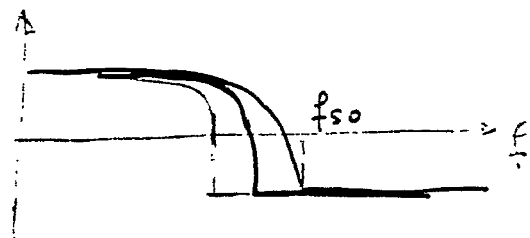


— zero beam current (f_{s0})
— non zero beam current (f_{s0}')

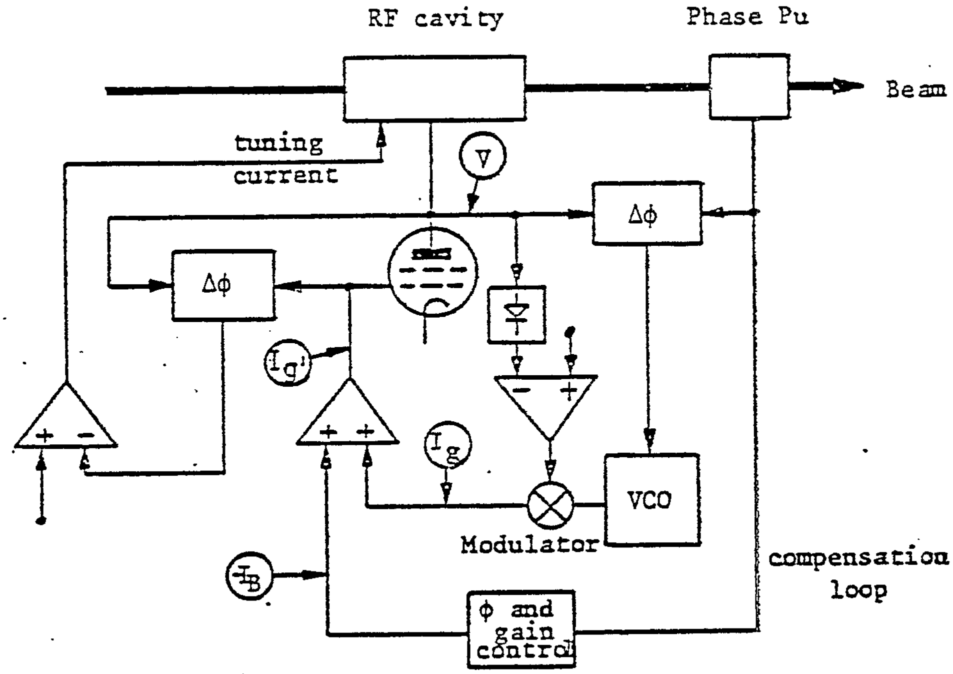
[illegible]

Importance of [illegible] for [illegible] instabilities [illegible]

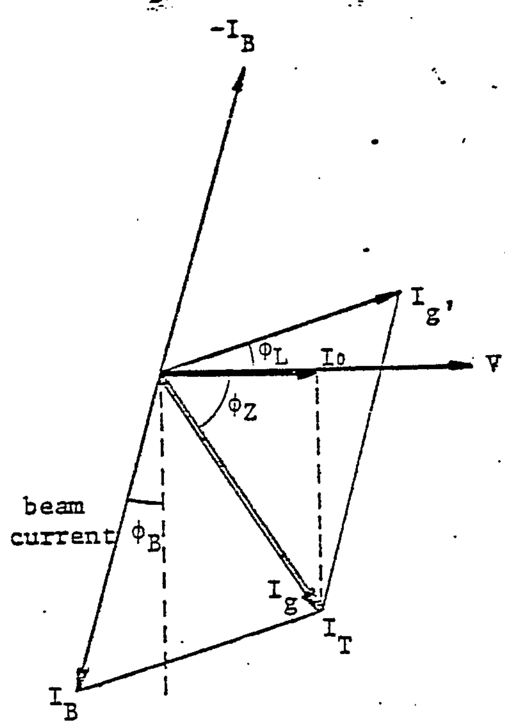




increasing [illegible]



<u>Fig. 1</u> The RF loops including compensation



$I_T = I_B + I_{g'}$

$I_{g'} = I_g - I_B$

$I_T = I_g$

<u>Fig. 2</u> Vector diagram of the compensated case

5.

[illegible] Z [illegible] ?

[illegible]

[illegible]

Transfer functions

Example : gap Volt φ = generator current phase × G [illegible]

generator current ampl = C_a × gap volt. [illegible]

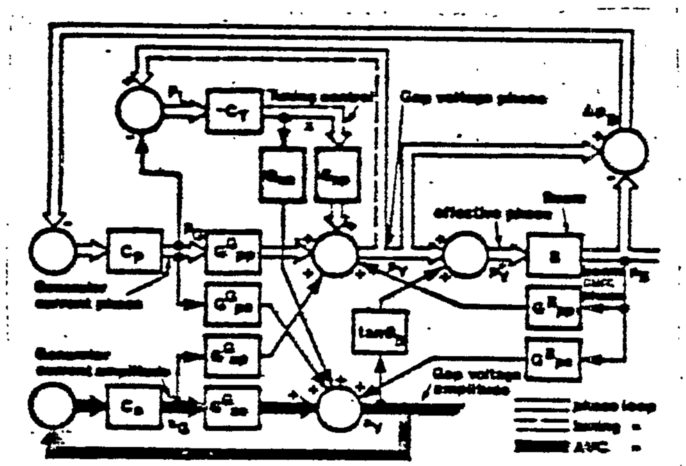


Fig. 3 : Small signal model showing transmissions between generator current, gap voltage, beam current and tuning control.

6

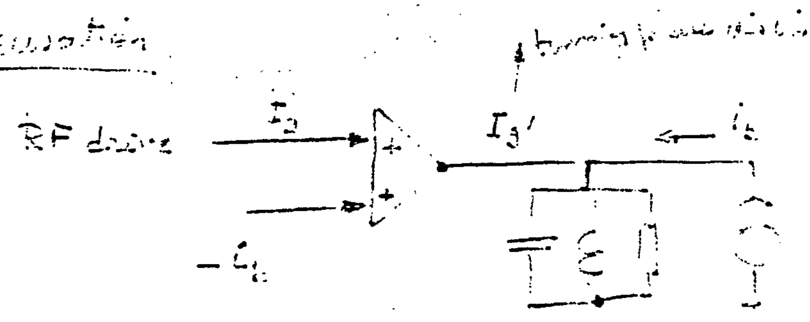
Transfer functions are determined by cavity impedance (bandwidth + detuning angle) [illegible]
($\vec{I}_g$ or $\vec{I}_b$) and $\vec{I}_T$

example: [illegible] [illegible]

Even if $B(j\omega)$ = beam transfer function is [illegible], the system may become <u>unstable</u> because of the [illegible] G_{pa}, G_{ap}

→ Observation in the [illegible]

<u>Compensation</u>



Consequences:

1) $I_g \approx I_T$ → new transfer function [illegible]

2) New transfer functions between $\vec{I}_g$ and tuning phase [illegible]

Transient response of the system (RF voltage at the gap)

Top trace	$I_p = 2 \times 10^{12}$
Bottom trace	$I_p = 1.1 \times 10^{13}$
Sweep	50 μs/div

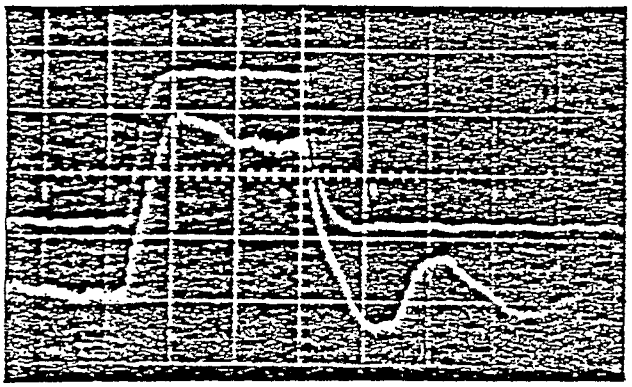


Photo 1

No compensation

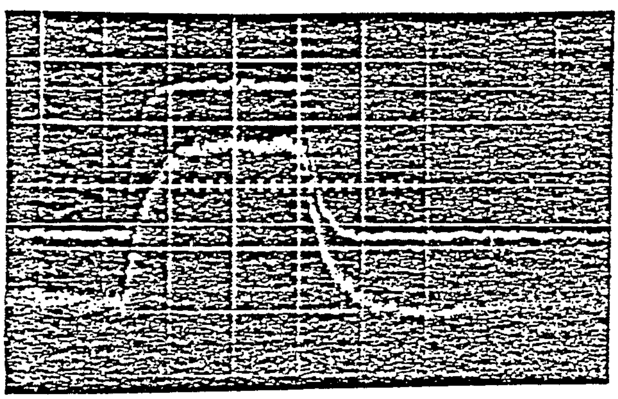


Photo 2

With compensation

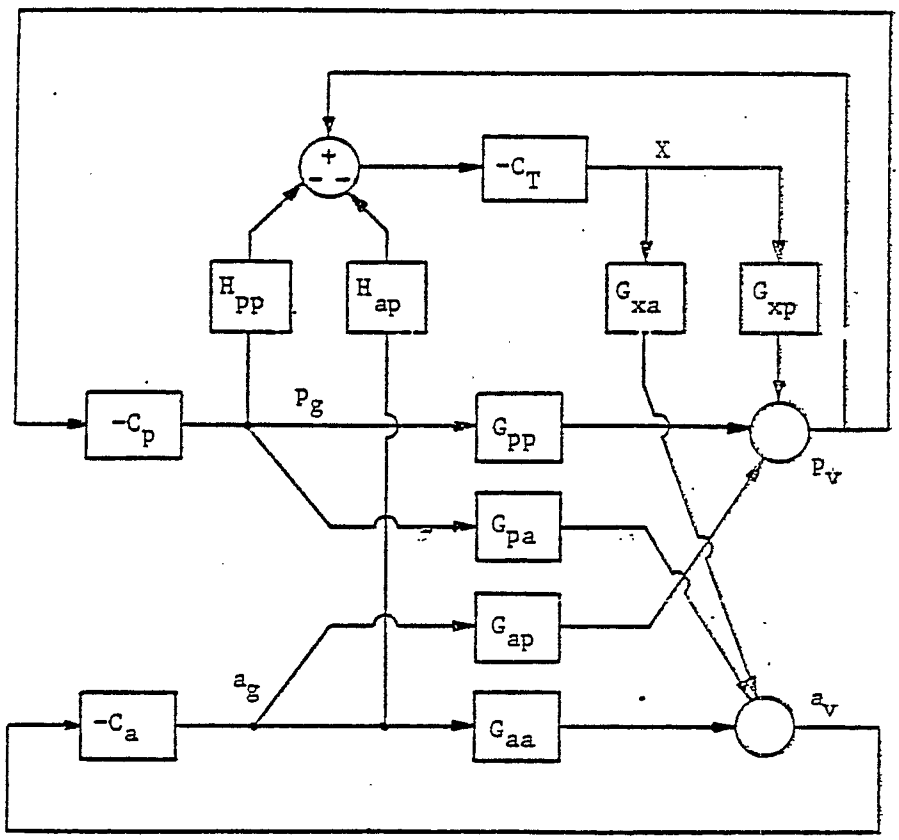


Fig. 3 Small signal model with compensation. Compared to the normal case, compensation modifies the G_{pp}, G_{pa}, G_{ap}, G_{aa} and H_{pp} transfer functions and introduces an extra connection (H_{ap}).

For frequencies < cavity bandwidth, the Gap, $g_{FR} = 0$

compensation requires exact coupling transfer functions

- Suppression of the instability.
- Tube works in the same condition (no extra power needed)
- Synthesizes a zero impedance cavity (at f_{RF})

Implementation in the PS

- exact compensation at all frequencies difficult (phase and amplitude control)

- Few fixed points: 800 MeV injection
  1 GeV int. flat top (blow-up)
  10 GeV flat top (debunching)

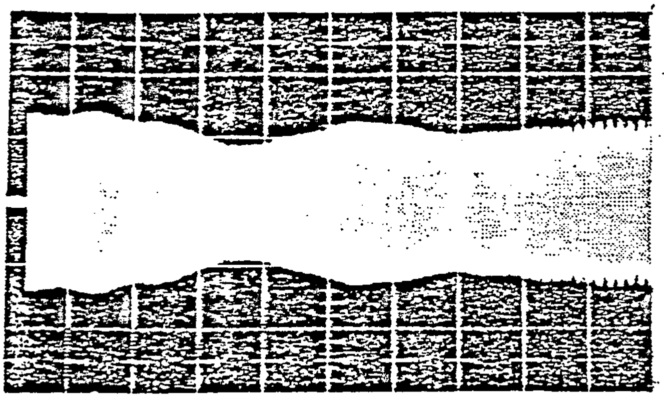
Outside: coarse correction.

- Resistive wall PU: better than electrostatic (I_b directly)

- Setting up: RF drive off → minimize beam loading voltage on cavity to be adjusted.
  Z decreased by > 20 dB

COMPENSATION PERMANENTE - RESULTATS DU 28.3.79
TENSION RELEVEE SUR LE "GAP" DE LA CAVITE 96

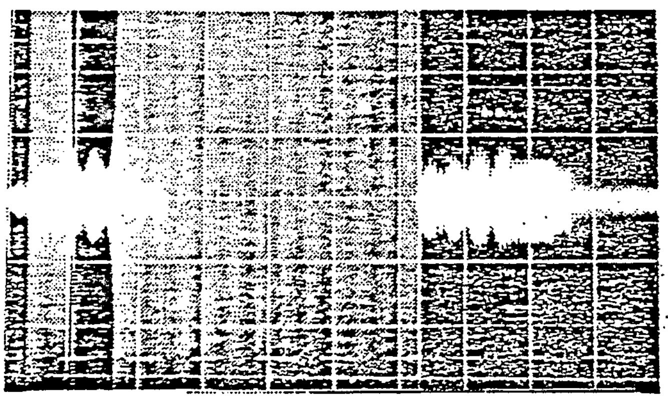
$I_p \approx 1.1 \cdot 10^{13}$ ppp



Sans compensation

X : Trigger C235 - 20 ms/div

Y : 50 mV/div

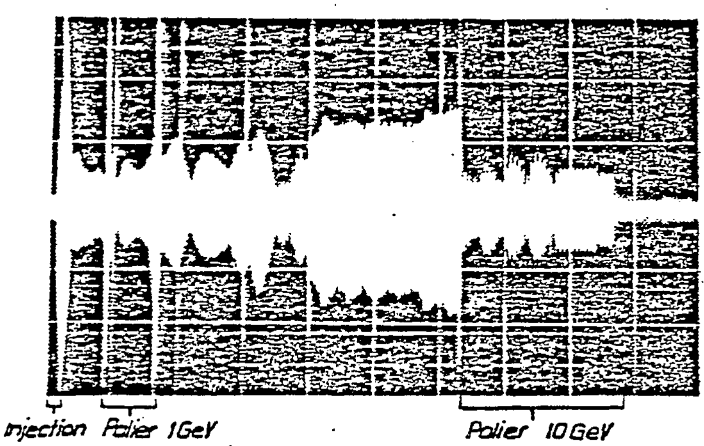


Avec compensations fixes

(Injection - 1 GeV - 10 GeV)

X : Trigger C235 - 50 ms/div

Y : 10 mV/div

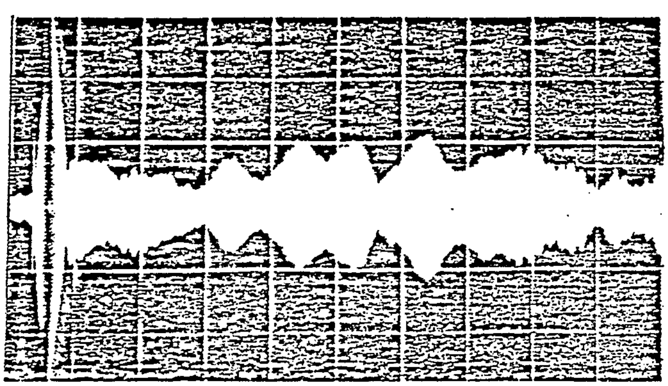


Avec compensations fixes

+ compensation permanente

X : Trigger C235 - 50 ms/div

Y : 10 mV/div



Avec compensation fixe à l'injection

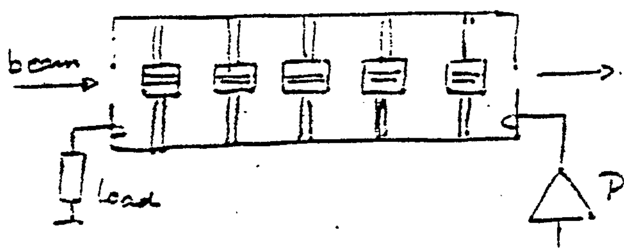
+ compensation permanente

X : Trigger C235 - 20 ms/div

Y : 10 mV/div

THE RF FEEDBACK SYSTEM IN THE SPS

Accelerating cavities = travelling wave structures 200 MHz.



- No tuning
- RF amplifier sees a matched load

  transfer function $I_g \to V_{RF}$ Z_1 real $\sim |\sin \tau/2| / (\tau/2)$

- Beam sees $\sim$ RLC circuit $I_b \to V_{RF}$ Z_2 complex

$$V = Z_1 I_g + Z_2 I_b$$

$Z_2(\omega)$ extends over many f_0 lines

$\longrightarrow$ transient beam loading (not corrected by AGC loops)

$\longrightarrow$ coupled bunch instabilities at injection

$(Re\ Z_2(\omega_{RF} + n\omega_0) \neq Re\ Z_2(\omega_{RF} - n\omega_0))$

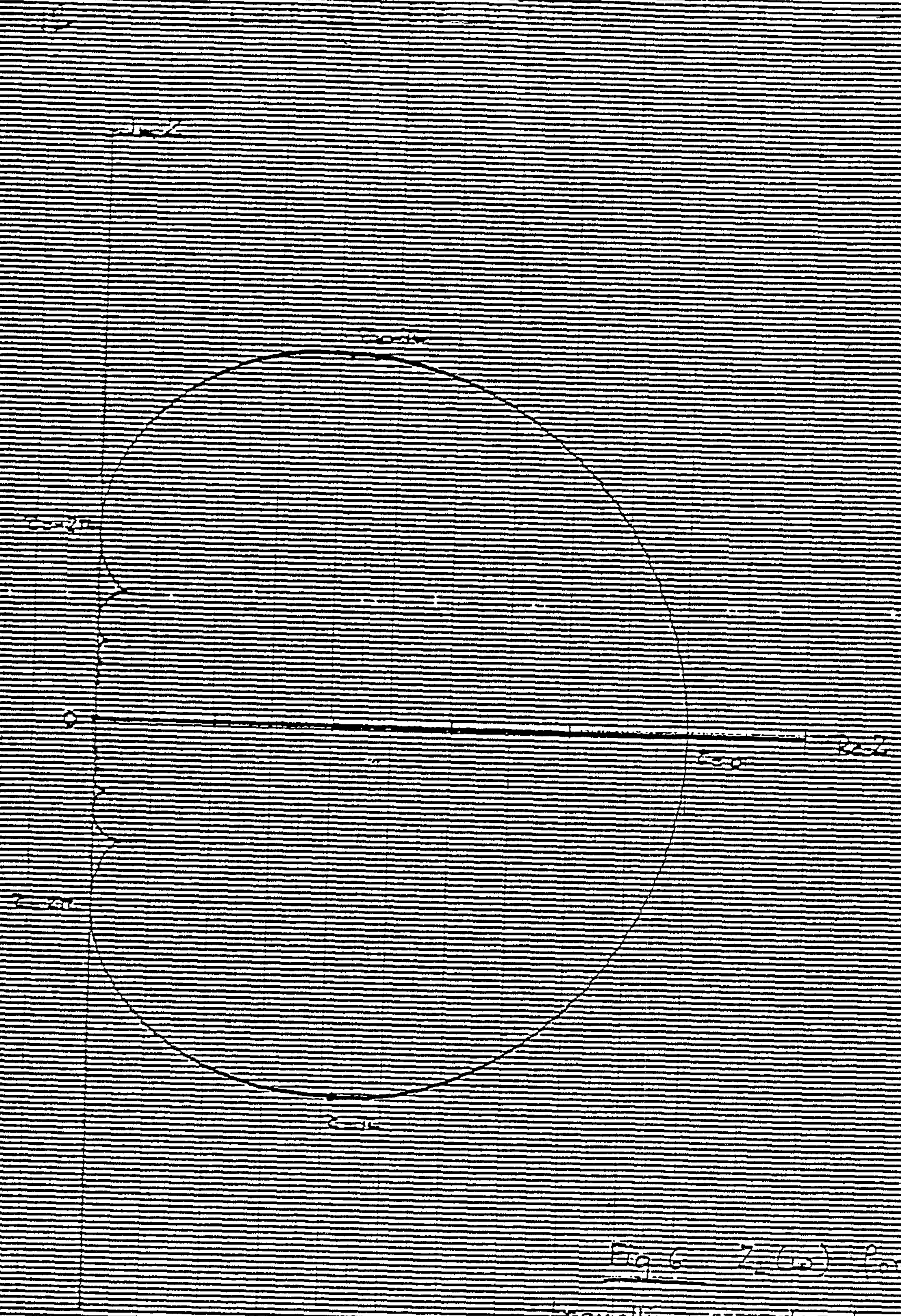
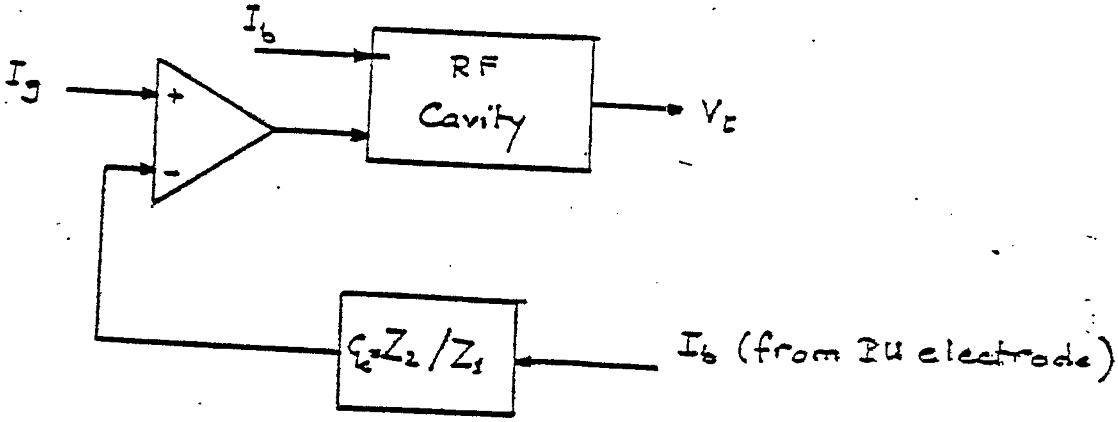
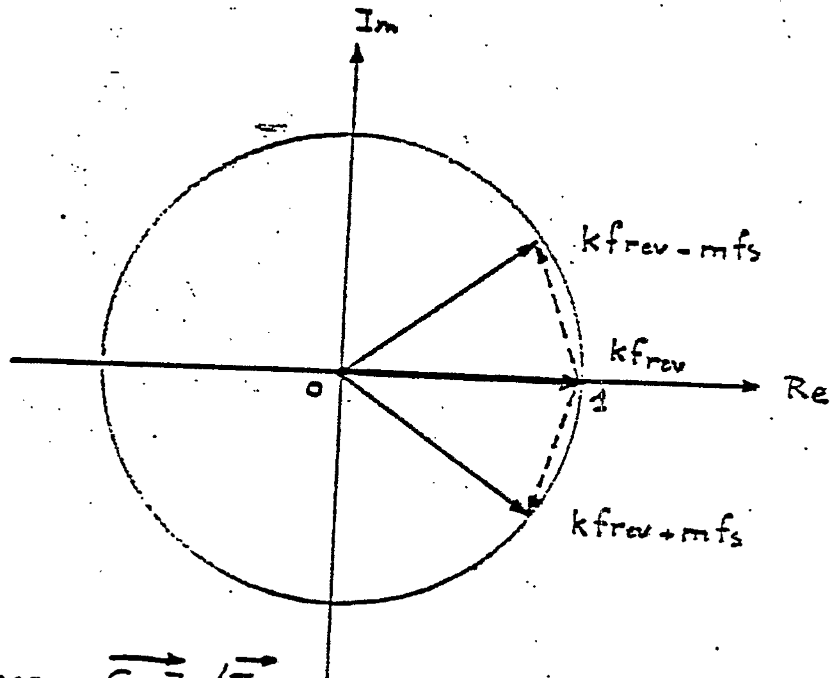


Fig 6 $Z(\omega)$ for a travelling wave structure



$$V_t = Z_1\left(I_g - \frac{Z_2}{Z_1} I_b\right) + Z_2 I_b = Z_1 I_g$$

Fig 7. Principle of cavity compensation.
need to synthesize a transfer function $(Z_2/Z_1) \times \frac{1}{\text{RF amplifier gain}}$



Solid lines: $\overrightarrow{G_c Z_1}/\overrightarrow{Z_2}$ at $k f_{rev} \pm m f_s$

Dotted lines: residual impedance at $k f_{rev} \pm m f_s$

Fig 8. The inherent delay of the system (1 turn) makes cancellation imperfect at the synchrotron sidebands.

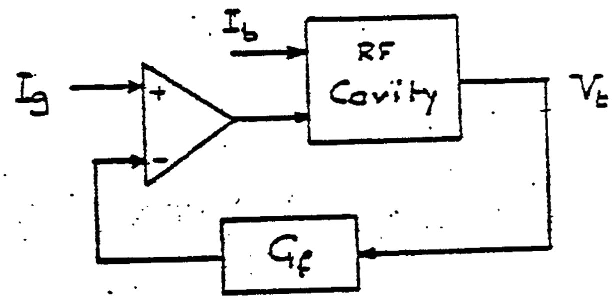


Fig 11 Principle of RF cavity feedback

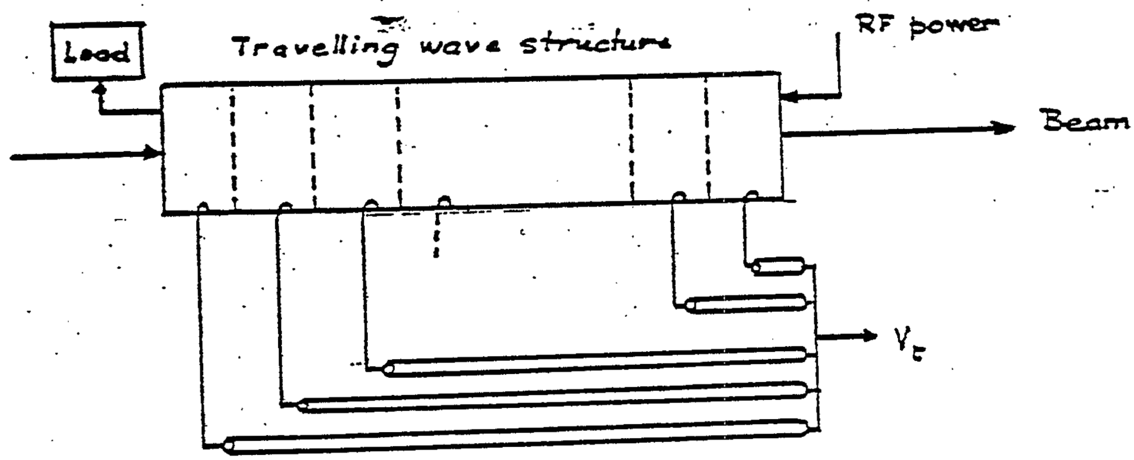
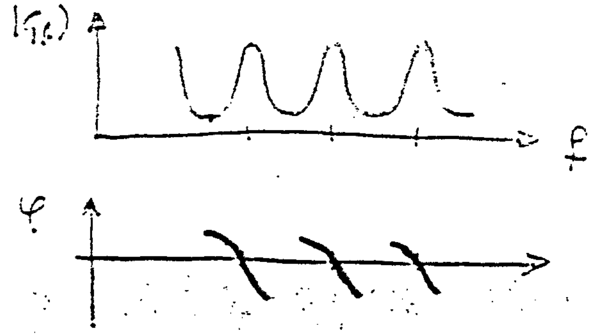


Fig 12. Synthesis of V_t for a travelling wave structure

Due to the delay between amplif. and cavity, the bandwidth of the system would be very small.

Shape the transfer function G_f



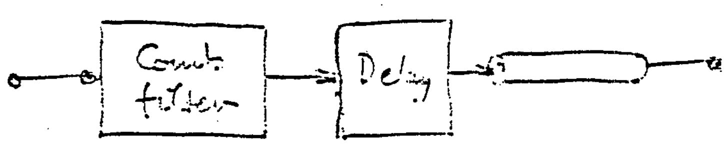
loop gain is high, phase is right near $n \times f_{rev}$
low not right " $(n+\frac{1}{2}) f_{rev}$

Stability can be achieved, but the feedback is efficient only near $n f_{rev}$, where the beam components are

at $n f_{rev}$ → transient beam loading

at $n f_{rev} \pm m f_s$ → instabilities.

<u>Synthesis of the filter G_f</u>



amplitude condition

Total delay = 1 turn

phase condition

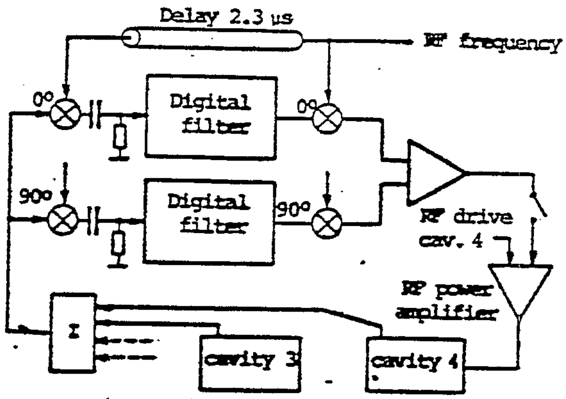


Fig. 3 Layout of the RF feedback system

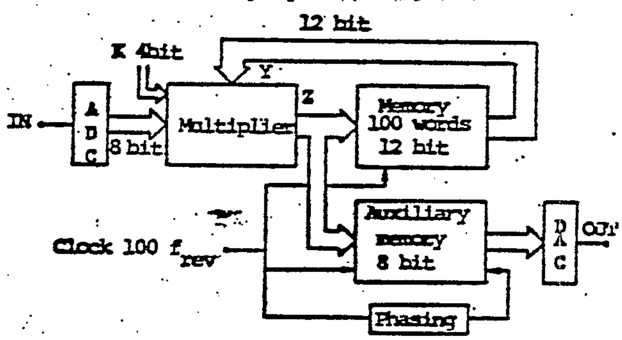


Fig. 4 The digital filter and delay

$G_{max} = \frac{1}{1-K}$

$G_{min} = \frac{1}{1+K}$ → stability limit

K defines the bandwidth

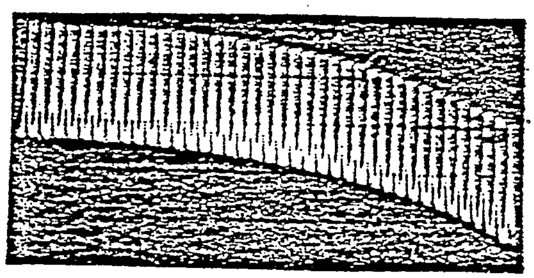


Fig. 5 Digital filter amplitude response 10 dB/div

Test of the RF feedback system without beam (cavity 3)

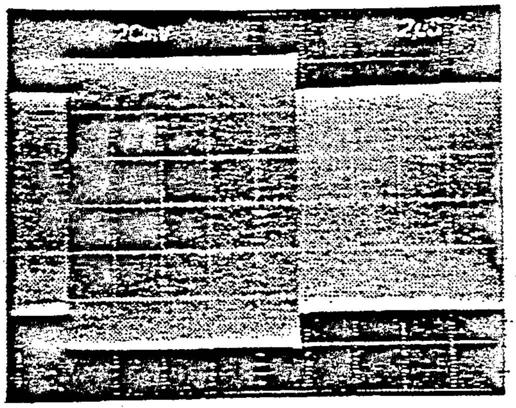


Fig. 3 - Perturbed RF wave at the revolution frequency (no feedback)

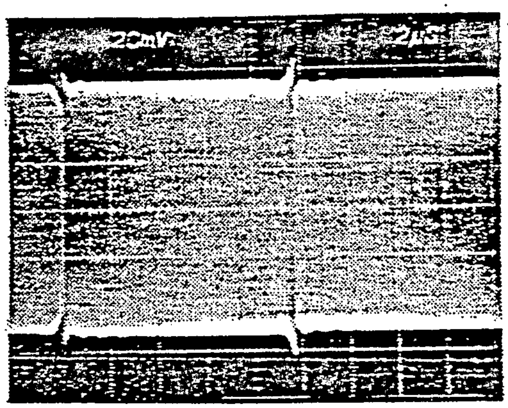


Fig. 4 - Corrected RF wave by the RF feedback system

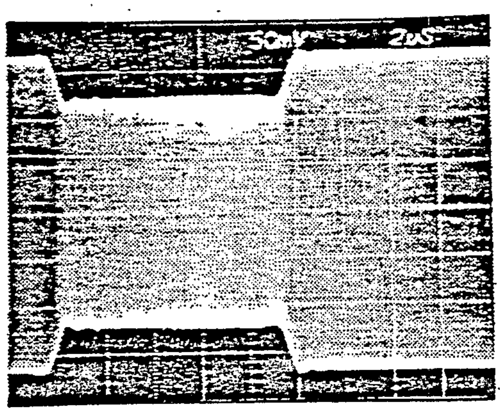


Fig. 5 - Correcting signal (input power to cavity 3)

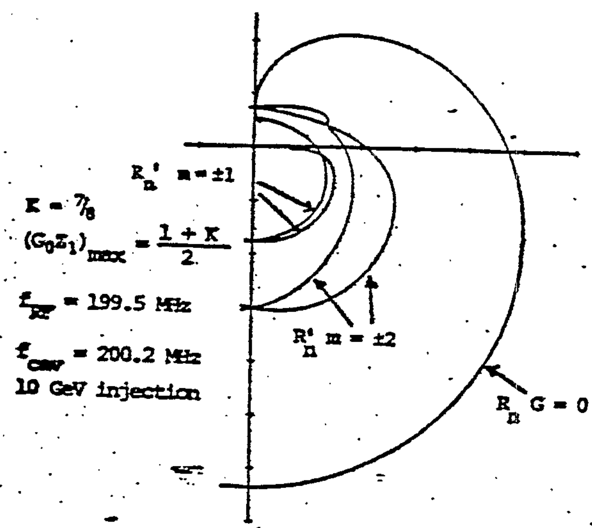


Fig. 2 Synchrotron frequency shift with and without RF feedback

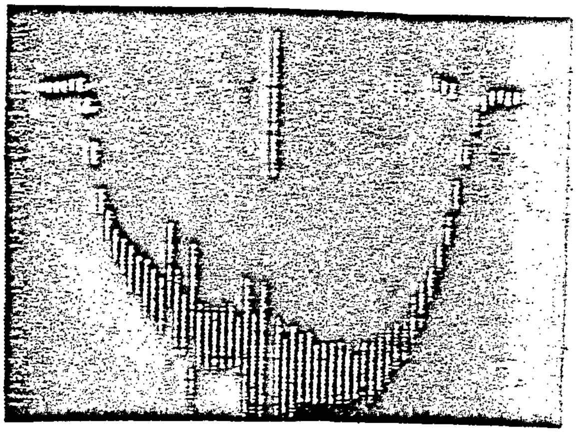


Fig. 6

f_{RF} = 200.0 MHz

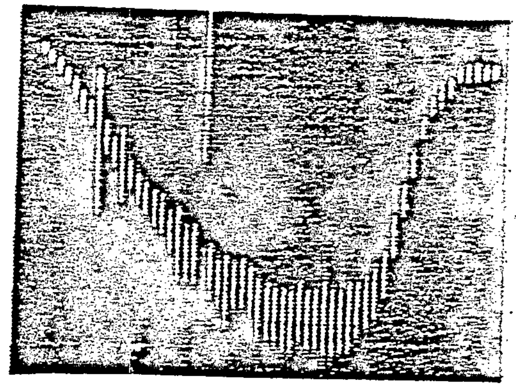


Fig. 7

f_{RF} = 199.525 MHz + n f_{rev}

Reduction of effective impedance at the n f_{rev} lines (Fig. 6 and 7) and at n f_{rev} + 2 kHz lines (Fig. 8)

Vertical : 2.5 dB/div

Horizontal: f_{rev} spacing between lines

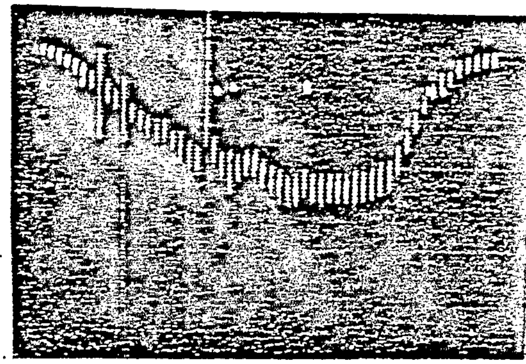


Fig. 8

f_{RF} = 199.525 MHz + n f_{rev} + 2 f_s (10 GeV.

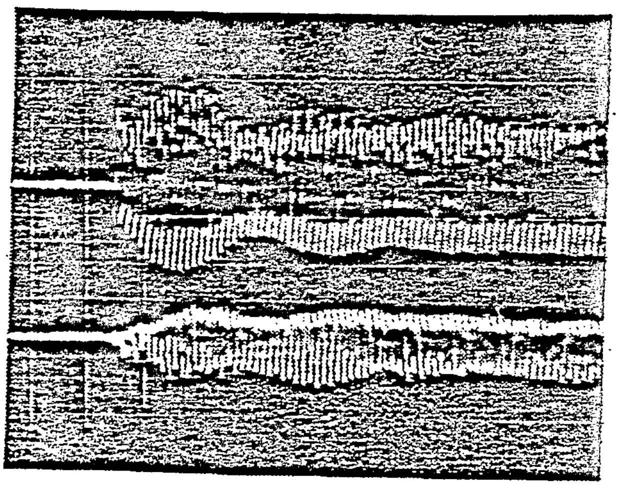


Fig. 9

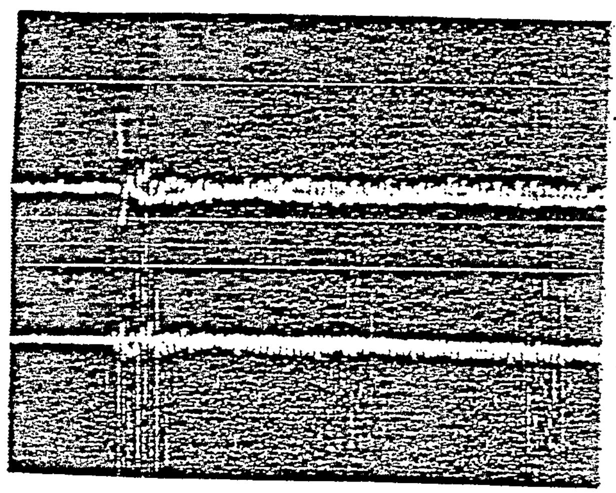


Fig. 10

Input of the two digital filters 200 µs/div. 1st injection

RF feedback off

RF feedback on

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