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AGS Studies Report

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Reported by L. Ahrens

Subject Polarized Proton Quad - E.O. Position Measurement

Observations and Conclusion

Objectives

To minimize transverse emittance growth when jumping depolarizing resonances using the fast quadrupoles it is necessary to have the quads centered on the equilibrium orbit (E.O.). Following the main magnet moves (10, 11 June 1987) a series of measurements of the quad - E.O. offsets were taken and are reported here. Prior to accelerating polarized protons, these offsets will be remeasured and the quad positions adjusted to eliminate the offsets.

Procedure:

The technique to measure offsets is as follows. A reference orbit is taken at the desired momentum and radius. Then a given quad is pulsed at this momentum and the charge in the due to the quad is measured. The quad pulse has a flat top of a few hundred μs duration, orbit acquisition takes ~100μs. The difference orbit (Fig. 1) has a cusp at the quad and an amplitude related to the (undistorted orbit - quad) offset. The connection is described in more detail in the appendix. The observed amplitude depends on the offset, but also on the machine tune, and of course on the quad current.

While accelerating 2×10^{12} protons, each of the ten quads was pulsed at 21300 GC counts (~10.7 GeV/c) and 1500 amps current at two radii. The radial shift gives a large charge in the observed result in the horizontal plane due both to large position and tune shifts, but affects the vertical plane only through the slight vertical tune variation with momentum.

The resulting difference orbits are fit to a sine wave with a cusp at the quad being pulsed, with the tune, amplitude, and an overall offset as variables to minimize chi squared. For fitting, the PUE points are assigned the standard deviations observed over five samples.

The observed variation in PUE measurement of orbits which determines the measurement signal to noise ratio was large probably due to the low intensity (2×10^{12}) at which the data was taken.

Results:

The reference orbits, one of vertical and one of horizontal at each of the two radii are given in figures 2 & 3. Figure 1 gives the difference orbits for the two radii for D15 horizontal. Table 1 gives the fit results for amplitude and tunes for the horizontal work. The expected tunes (from S.N. #182) for the normal machine at this momentum and radii are 8.78 and 8.60 at -0.7 and +0.91 cm respectively. These tunes (incremented by the 0.15 shift due to the quad itself) are consistent with the tunes returned by the fit and are used in the analysis. The deviations of the fit tunes from the expected value (Fig. 4 & Table 1) is an indication of the signal disappearing into the noise. This occurs for reported amplitudes less than about 0.05 cm for this data. No vertical data is reported because the amplitude found in the vertical fits were less then 0.03 cm and very sensitive to the starting values for the fit. For the vertical the true signal was lost in the PUE noise. In a sense this is good - the quads were nearly centered on the E.O. - but the sensitivity could have been improved.

The procedure outlined in the appendix can be applied to the amplitudes given in Table 1 to generate the horizontal quad - orbit offsets for the two radii. These results are given in Table 2. The quads, if shifted by these amounts would then be aligned horizontally to the E.O. at this momentum and radius. (These shifts would have to be increased by the calibration factor for the PUE's namely 1.2.)

Table 2 gives the position of the E.O. relative to the quad center lines for two radii. Now a check can be made of these results. The difference between these positions is just the movement of the undistorted E.O. at the two radial positions. This motion can be extracted independently using the PUE orbits given in Figure 2, in particular from the difference in orbit positions at the PUE's located in straight sections (S.S.) 14 and 18 (the quads are in straight section 15). To approximate the #15 S.S. we assume a linear interpolation between the #14 and #18 PUE's, and then reduce the difference by the square root of the ratio of betatron functions between PUE's (β_{ave}) and quad (β_{min}). Table 3 gives this data as well as the final comparison in the two right most columns. The errors assigned the fit result came from the fitting routine, the PUE error is that seen over five measurements. The two methods give results which are close but not always within the expected errors.

Conclusions:

A procedure has been worked out and tested to use the beam to measure the offset of the polarized proton quads from the equilibrium orbit. The results given are reasonable. To get the highest precision, a number of experimental techniques are relevant. The beam intensity should be well above that necessary to get good orbit ($>5 \times 10^{12}$). The distortion amplitude (A of the Appendix) for a given offset should be as large as possible. This implies that the quads should have as large a current as reasonable, and should be focussing in the plane being measured. Also the tune should be as close to 9 as possible. This last desire can not be achieved by doing anything that charges the E.O. of the machine however since the point is to measure the undistorted E.O. in the quads. Indeed the machine should be set up with all parameters that affect the E.O. as close to polarized proton operating condition as possible before making the final measurement. This set of parameters includes the radius at which the machine will be operated.

TABLE 1

Fit results - Amplitudes and Tunes

Quad	<PUE> = -0.69 cm		<PUE> = 0.91 cm	
	Amp.(cm)	Tune	Amp.(cm)	Tune
A15	0.090(0.005)	8.78	-0.111	8.57
B15	0.01 (0.02)	8.82	-0.152	8.53
C15	0.03	9.1	-0.187(0.02)	8.66(0.06)
D15	0.143	8.85	-0.114(0.02)	8.59
G15	0.169	8.81	-0.09	8.66
H15	0.94 (0.01)	8.82	-0.176	8.60
I15	0.150(0.005)	8.78	-0.116	8.57
J15	0.43	8.97	-0.124	8.59
K15	-0.032(0.005)	8.77	-0.198	8.62
L15	0.05	8.70	-0.155	8.67
Unperturbed Predicted tune horiz.: (S.R. #182)		8.78		8.60
$\Delta\nu_H$ due to pulse:		0.015		0.015
Tune for calculation:		8.795		8.615

TABLE 2

Calculated Quad - E.O. Offset

Quad	<PUE> = -0.69 cm cm		<PUE> = 0.91 cm cm		Δ cm	
A15	0.470	(0.03)	-0.805		1.275	(0.03)
B	0.052	(0.11)	-1.102		1.154	(0.11)
C	0.157		-1.356	(0.15)	1.513	(0.15)
D	0.746		-0.827	(0.15)	1.573	(0.15)
G	0.882		-0.653		1.535	
H	0.491	(0.06)	-1.276		1.767	(0.06)
I	0.783	(0.03)	-0.841		1.624	(0.03)
J	0.224		-0.899		1.123	
K	-0.167	(0.03)	-1.436		1.269	(0.03)
L	0.261		-1.124		1.385	

$x_0 = A\ (5.22)$
$(\nu = 8.795)$
$k = 0.226$
$\frac{1 - k \cos \pi \nu}{k} = 5.22$

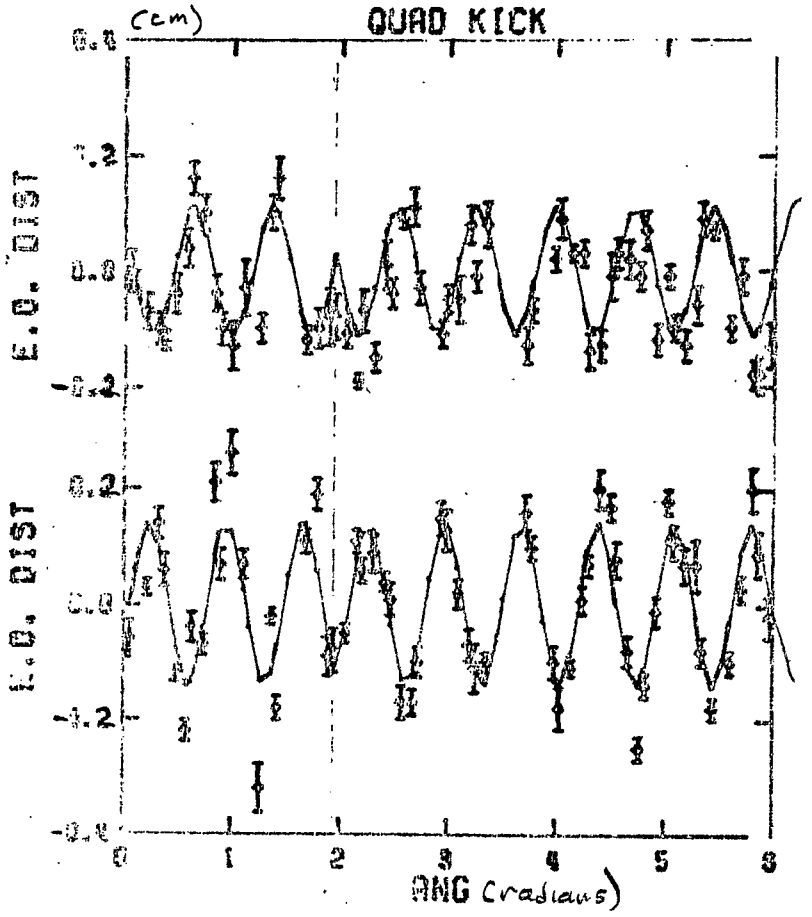
$x_0 = A\ (7.25)$
$(\nu = 8.615)$
$k = 0.145$
$\frac{1 - k \cos \pi \nu}{k} = 7.25$

TABLE 3

PUE Shift and Shift from Fits

Quad	(cm) inside <r> = -0.69 cm			(cm) outside <r> = 0.91 cm			(cm) Δ from PUE $(\beta_m/\beta_{ave})x^{1/2}$		(cm) Δ from fits
	14	18 →	15	14	18 →	15	15 out-	15 in	(Table 1)
A15	0.98	-1.20	-1.04	0.62	0.38	0.56	1.28	(0.04)	1.28(0.03)
B	0.58	-1.11	-0.71	0.84	0.58	0.78	1.19	"	1.15(0.11)
C	-0.09	?	?	1.26	?	?	—		1.51(0.15)
D	-1.08	-0.56	-0.95	0.65	1.11	0.77	1.38	"	1.57(0.15)
G		—			—		—		1.54
H	-1.0	-0.65	-0.91	0.84	0.87	0.85	1.41	"	1.77(0.06)
I	-1.34	-0.98	-1.25	0.39	0.79	0.49	1.39	"	1.62(0.03)
J	-0.64	-1.31	-0.81	0.65	0.18	0.53	1.07	"	1.12
K	0.35	-0.24	0.20	1.69	0.96	1.51	1.04	"	1.27(0.03)
L	-1.01	-0.47	-0.88	0.67	1.14	0.79	1.34	"	1.39

Difference Orbits (data & fit) D15 Quad On/Off
(error bars reduced by x3 for clarity)



$\langle r \rangle_{PUE}$ = +0.905 cm

Amp. = 0.11
ν_f = 8.59

$\langle r \rangle_{PUE}$ = −0.69 cm

Amp = 0.143
ν_f = 8.85

D15 Quad (position of kick)

Same data
(Norb output)

(Full error bars, and includes "L" s.p., line connects pts.)

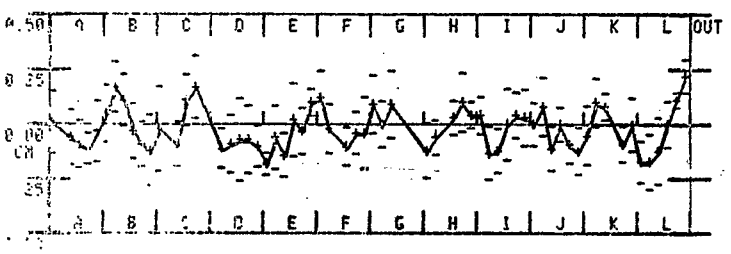
DATA FROM REFERENCE FILE DQ05
11-JUN-87 10:29 HOR. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 222
AVERAGE POS= -0.028

A	0.00		-0.07	-0.10	-0.12	-0.04
B	0.04	0.16	0.10	-0.04	-0.10	-0.13
C	-0.03		-0.10	0.10	0.16	
D		-0.12	-0.10	-0.08	-0.08	-0.11
E	-0.15	-0.07	-0.15	0.01	-0.04	0.09
F	0.11	-0.03		-0.11	-0.05	-0.05
G	0.00	-0.01	0.08			
H	-0.13	-0.07		0.02	0.09	0.03
I	0.03	-0.14	-0.13	-0.01	0.03	0.02
J	-0.01	0.07	-0.12	-0.01	-0.10	-0.13
K	-0.06	0.03	0.07		-0.10	-0.01
L	-0.16	-0.18	-0.13	-0.01	0.10	0.21

HARMONIC ANALYSIS

	COS	SIN	AMP
8	0.06	-0.08	0.059
9	0.04	-0.06	0.068
10	0.01	0.01	0.019
11	-0.00	0.00	0.006
12	0.01	-0.02	0.020

$\langle r \rangle_{pue}$ = +.905



DATA FROM REFERENCE FILE DQ01
11-JUN-87 10:22 HOR. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 222
AVERAGE POS= -0.003

A	-0.06		0.03	0.14	0.06	-0.12
B	-0.11	-0.04	-0.07	0.21	0.07	0.26
C	0.07		-0.32	-0.02	-0.18	
D		0.11	0.19	-0.09	-0.08	-0.05
E	0.11	0.05	0.09	0.04	0.01	-0.17
F	-0.17	-0.10		0.15	0.13	0.02
G	-0.07	-0.11	-0.10			
H	0.16	0.10		-0.10	-0.18	-0.11
I	0.01	0.07	0.20	0.17	0.08	-0.08
J	-0.25	-0.14	-0.01	0.18	0.11	0.07
K	0.07	-0.09	-0.18		-0.10	0.03
L	0.20	0.03	-0.02	-0.07	-0.04	-0.16

HARMONIC ANALYSIS

	COS	SIN	AMP
8	-0.04	-0.00	0.041
9	-0.06	0.12	0.135
10	-0.01	-0.01	0.011
11	0.01	-0.01	0.016
12	-0.02	-0.00	0.023

$\langle r \rangle_{pue}$ = −.69

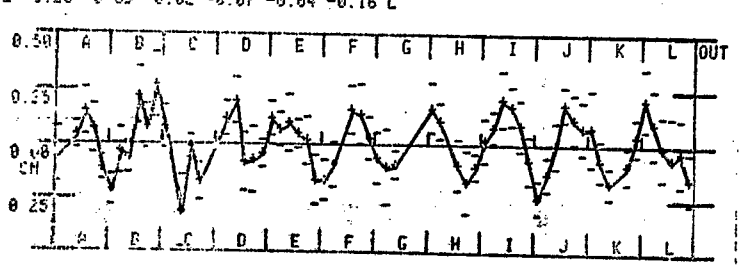


Figure 1.

Unperturbed Horizontal Orbits
(with the radius shifted in and out)

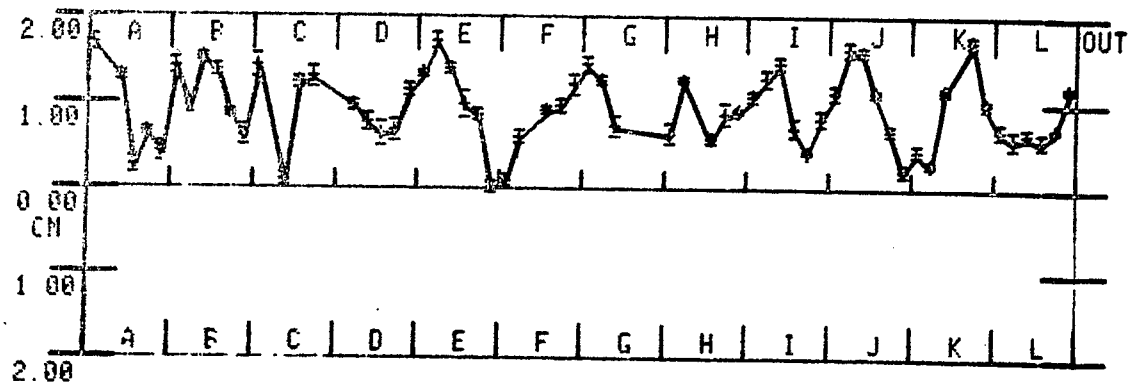
DATA FROM REFERENCE FILE JQ05
11-JUN-87 08:57 HOR. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 210
AVERAGE POS= 0.905 cm

A	1.64		1.26	0.22	0.62	0.38	A
B	1.38	0.92	1.49	1.33	0.84	0.58	B
C	1.39		0.06	1.19	1.26		C
D		0.94	0.74	0.61	0.65	1.11	D
E	1.30	1.68	1.36	0.95	0.83	-0.02	E
F	0.05	0.57		0.88	0.93	1.18	F
G	1.41	1.23	0.70				G
H	0.61	1.23		0.55	0.84	0.87	H
I	1.04	1.25	1.43	0.68	0.39	0.79	I
J	1.09	1.60	1.56	1.08	0.65	0.18	J
K	0.41	0.25	1.12		1.69	0.96	K
L	0.66	0.56	0.62	0.55	0.67	1.14	L

HARMONIC ANALYSIS

	COS	SIN	AMP
8	0.31	0.03	0.311
9	0.20	0.18	0.270
10	0.12	-0.06	0.137
11	-0.03	-0.12	0.127
12	-0.04	0.18	0.182

$\langle pue \rangle = .905$ cm
$\langle int \rangle = 2.1 \times 10^{12}$



DATA FROM REFERENCE FILE JQ01
11-JUN-87 08:39 HOR. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 234
AVERAGE POS= -0.690

A	0.22		-0.17	-1.35	-0.98	-1.20	A
B	-0.18	-0.45	-0.04	-0.11	-0.58	-1.11	B
C	-0.22		-1.46	-0.38	-0.09		C
D		-0.68	-1.09	-0.95	-1.08	-0.56	D
E	-0.25	0.27	-0.26	-0.78	-1.00	-1.78	E
F	-1.57	-0.91		-0.64	-0.55	-0.48	F
G	-0.43	-0.71	-1.04				G
H	-0.94	-0.53		-1.43	-1.00	-0.65	H
I	-0.12	-0.07	-0.15	-0.99	-1.34	-0.98	I
J	-0.57	-0.05	0.17	-0.18	-0.64	-1.31	J
K	-1.48	-1.52	-0.85		0.35	-0.24	K
L	-0.83	-1.02	-1.13	-1.21	-1.01	-0.47	L

HARMONIC ANALYSIS

	COS	SIN	AMP
8	0.24	0.05	0.243
9	0.34	0.34	0.477
10	0.14	-0.08	0.161
11	-0.01	-0.11	0.111
12	-0.01	0.16	0.155

$\langle pue \rangle = -.690$ cm
$\langle int \rangle = 2.3 \times 10^{12}$

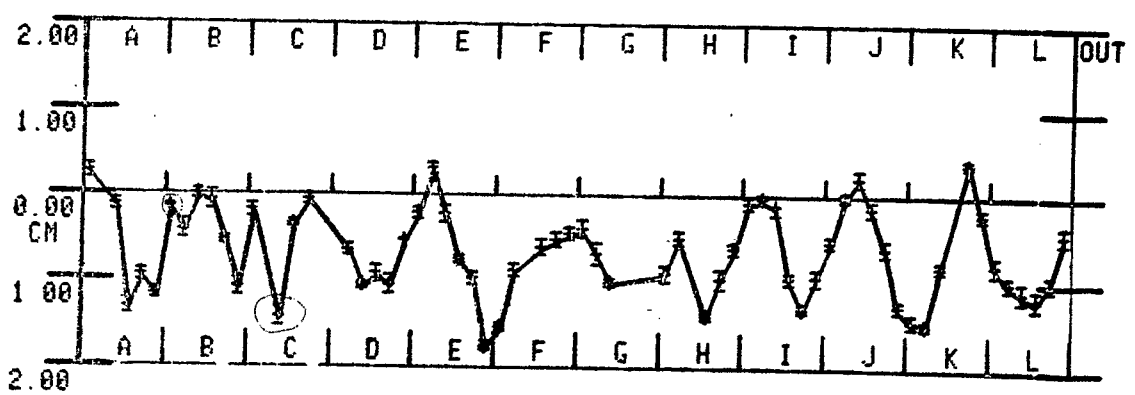


Figure 2.

Unperturbed Vertical Orbits
(with the radius shifted in and out)

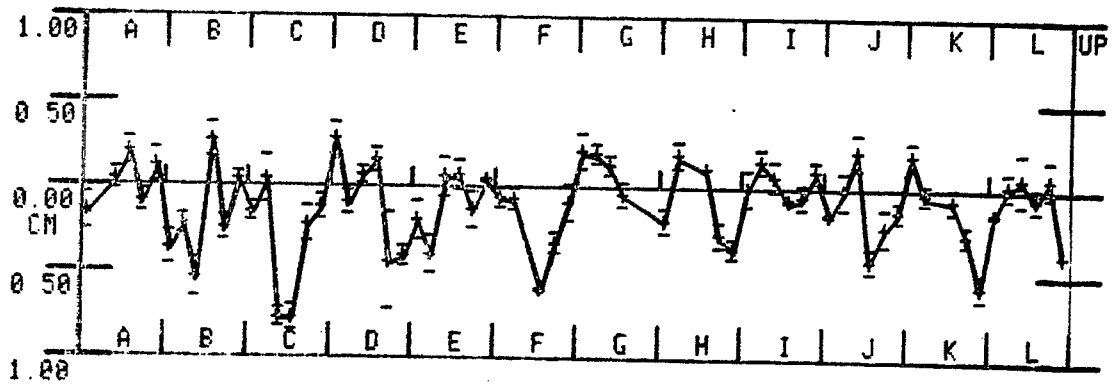
DATA FROM REFERENCE FILE JQ05
11-JUN-87 08:57 VER. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 210
AVERAGE POS= -0.120

A	-0.17		0.02	0.18	-0.11	0.10	A
B	-0.38	-0.24	-0.55	0.25	-0.26	0.02	B
C	-0.17	0.02	-0.78	-0.78	-0.24	-0.14	C
D	0.26	-0.11	0.05	0.14	-0.45	-0.42	D
E	-0.21	-0.41	0.04	0.05	-0.15	0.02	E
F	-0.08	-0.09		-0.61	-0.34	-0.11	F
G	0.19	0.18	0.11	-0.06			G
H	-0.21	0.17		0.09	-0.29	-0.37	H
I	-0.02	0.14	0.05	-0.09	-0.07	0.08	I
J	-0.17	-0.03	0.20	-0.43	-0.24	-0.15	J
K	0.18	-0.05		-0.08	-0.28	-0.58	K
L	-0.16	0.01	0.05	-0.07	0.05	-0.39	L

HARMONIC ANALYSIS

	COS	SIN	AMP
8	0.07	0.07	0.101
9	-0.18	-0.02	0.178
10	-0.04	0.00	0.043
11	-0.06	-0.09	0.108
12	-0.02	0.09	0.089

$\langle pu_e \rangle_r = .905$ cm



DATA FROM REFERENCE FILE JQ01
11-JUN-87 08:39 VER. ORBIT @ 2 CMPTR NAVEN 5
NORMALIZED BY L20CT = 234
AVERAGE POS= -0.096

A	-0.10		0.00	0.20	-0.04	0.08	A
B	-0.32	-0.21	-0.51	0.10	-0.19	0.01	B
C	-0.09	0.25	-0.67	-0.77	-0.29	-0.22	C
D	0.19	-0.01	0.06	0.12	-0.04	-0.41	D
E	-0.25	-0.44	-0.04	0.11	-0.08	0.09	E
F	-0.14	-0.10		-0.67	-0.36	-0.04	F
G	0.40	0.33	0.18	0.06			G
H	-0.22	0.25		0.06	-0.24	-0.48	H
I	-0.13	0.06	-0.09	0.00	0.01	0.14	I
J	-0.21	-0.07	0.10	-0.39	-0.27	-0.16	J
K	0.28	0.01		-0.19	-0.33	-0.36	K
L	-0.20	0.06	0.10	0.05	0.06	-0.39	L

HARMONIC ANALYSIS

	COS	SIN	AMP
8	0.05	0.08	0.094
9	-0.21	-0.06	0.221
10	-0.02	0.03	0.029
11	-0.05	-0.07	0.088
12	-0.01	0.08	0.078

$\langle pu_e \rangle_r = -.690$ cm

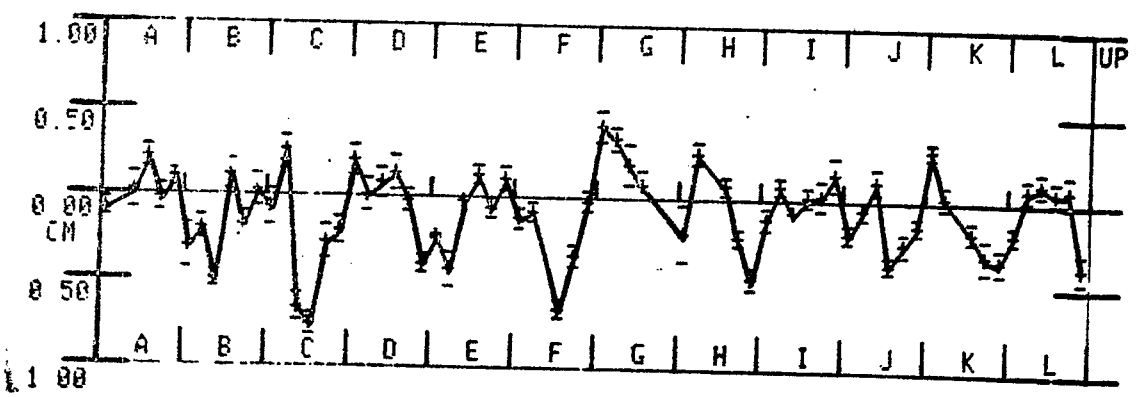


Figure 3.

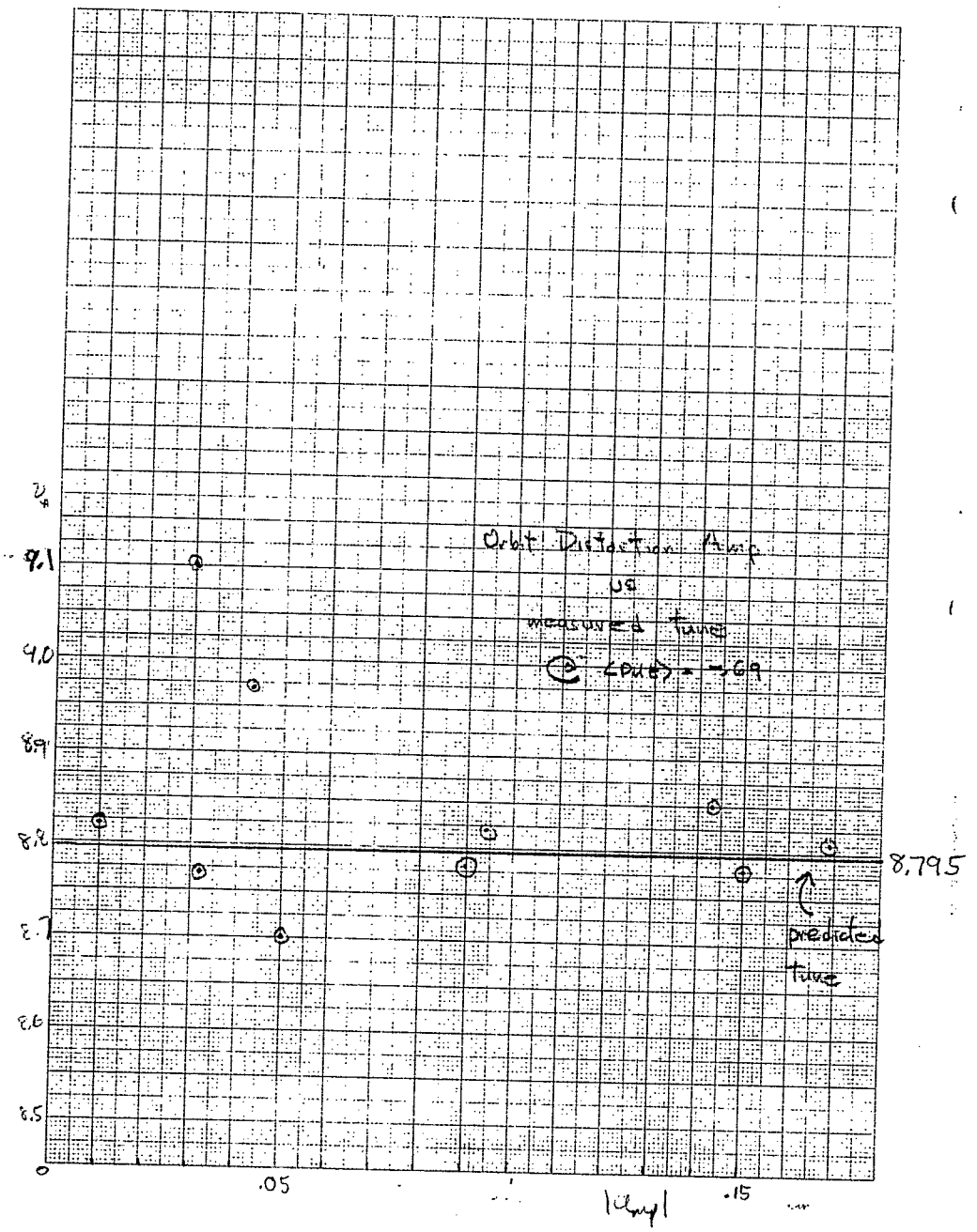


Figure 4. Horizontal Tune (from fit)
vs.
Fit Orbit Distortion

Appendix

The exercise is to extract the Quadrupole - Equilibrium Orbit (E.O.) distance (offset) from the amplitude of the distortion to the E.O. observed when the quad is powered.

Given an E.O., adding a point dipole ($\Delta B\ell$) causes a sinusoidal E.O. distortion with a cusp at the kick described by

$$u(s) = 1/2\ \Phi\ [\beta(s)\ \beta_k]^{1/2}\ \ [\sin \pi\nu]^{-1}\ \ \cos[\pi\nu - \mu(s)] \qquad (1)$$

or $u(s) = A \cos(\pi\nu - \mu)$ which defines A

where
- s is positioned around the ring
- k refers to the kick
- u is the transverse motion in real space
- β is the betatron function

- ν is the tune of the machine ($8.5 < \nu < 9$)
- μ is the betatron phase, $\mu=0$ at the kick
- Φ is the kick strength = $\Delta B\ell/(B\rho)$

in particular the amplitude at the kick ($\mu=0$) is

$$A \cos(\pi\nu)$$

and half way around the ring ($\mu=\pi\nu$) the amplitude is A (including the sign).

Knowing $\beta(s)$ and ν, a measure of the amplitude A gives the dipole strength at the kick or the orbit offset if this kick is due to a quadrupole. The offset includes the distortion however and what we need is the offset without the distortion since the goal is to move the quads on to the undistorted orbit.

From eq. 1 and Figure A1

$$A = kx, \quad k = 1/2\ \Phi\ [\beta(s)\beta_k]^{1/2}\ \ (\sin \pi\nu)^{-1} \qquad (2)$$

where Φ is the kick and is proportional to the quad current and inversely proportional to momentum. $\beta(s) = \beta_{PUE} = \beta_{ave}$ since all measurements are made at PUE's located at average betas.

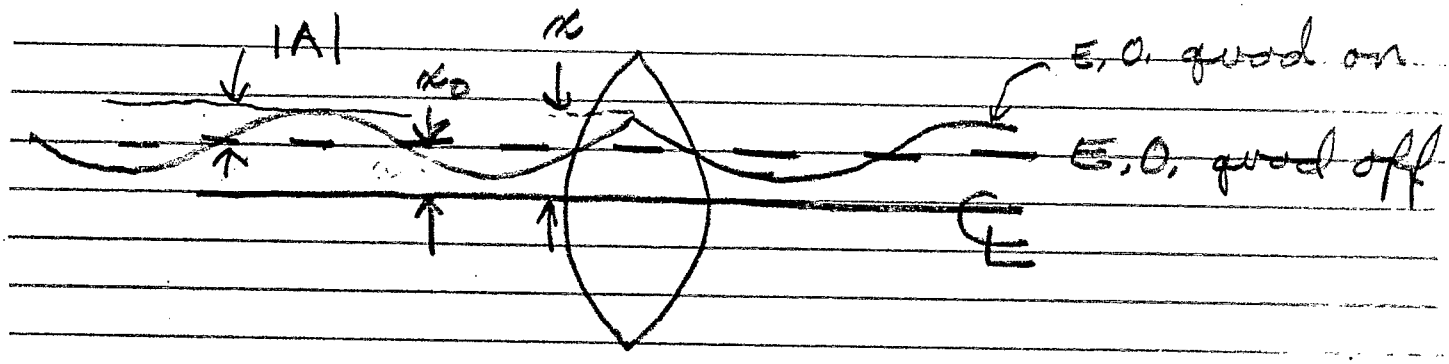


Figure A1.

Since the amplitude at the kick is A cos $\pi\nu$, $x=x_o + A \cos \pi \nu$ (3)

therefore

$$A = k\,(x_o + A \cos \pi \nu) \qquad (2 \,\&\, 3)$$

or

$$A = (k)/(1 - k \cos \pi \nu)\, x_0$$

or

$$x_o = A\left(\frac{1 - k \cos \pi \nu}{k}\right)$$

for the AGS ($8.5 \leqslant \nu \leqslant 9$) $\cos \pi \nu < 0$; $k < 0$ for focussing quad (> 0 for defoc) therefore for a given offset x_0, the amplitude measured is increased for focussing and decreased for defocussing.

Now all that is left is to evaluate k for the experimental situation and x_o can be extracted from A & ν.

$$k = \frac{1/2\ \Phi}{x}\ [\beta_{PUE}\beta_k]^{1/2}\ (\sin \pi\ \nu)^{-1} \qquad \text{(from 2)}$$

$$\Phi = \frac{\Delta(B\ell)}{B\rho} = (qIx)/B\rho$$

where q has units (Gauss/Meter) $\frac{\text{meter}}{\text{amp}}$ and is a property of the quadrupole gradient and length.

$$k = \frac{1/2\ (qI)}{B\rho}\quad (\beta_{PUE}\beta_k)^{1/2}\quad (\sin \pi\ \nu)^{-1}$$

this proportionality was extracted experimentally in an earlier study giving

$$k = 0.179 \quad @\ \nu_0 = 8.625$$
$$I_0 = 1980\ \text{A}$$
$$G_0 = 20000\ \text{Gauss counts}$$

hence

$$k_H = k_0\ \left(\frac{\sin \pi\ \nu_0}{\sin \pi\ \nu_H}\right)\ (I/I_0)\quad (G_0/G)$$

and

$$k_V = k_o\ \sqrt{2}\ \frac{\sin \pi\ \nu_0}{\sin \pi\ \nu_V}\quad \left(\frac{I}{I_o}\right)\ \left(\frac{G_0}{G}\right)$$

the $\sqrt{2}$ in the vertical formula reflects the fact that $\beta_k = \beta_{max} \simeq 2 \times \beta_{min}$ vertically and $\beta_k = \beta_{min}$ horizontally.

This value for k_o results in the offsets given in Table 2 of the text. k_o can also be extracted from the tune shift produced by the quads namely $\Delta\nu_V = 0.025$ @4.5 GeV/c and 542 A

$$\Delta\nu = \frac{\beta_k}{4\pi}\ \Delta k\ell = \frac{\beta_k}{4\pi}\ \frac{qI}{B\rho} \quad \rightarrow \quad q = 3.95\ \frac{G}{A}$$

this gives $k_o = .203$ (instead of the 0.179 used).

Some further work is needed to resolve the 10% discrepancy, the check described in the text favors the larger k_o.